Neural Networks 76 (2016) 46–54

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Finite-time robust stabilization of uncertain delayed neural networks with discontinuous activations via delayed feedback control

Leimin Wang, Yi Shen *, Yin Sheng

School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China
Key Laboratory of Image Processing and Intelligent Control of Education Ministry of China, Wuhan 430074, China

ARTICLE INFO

Article history:
Received 9 September 2015
Received in revised form 17 December 2015
Accepted 13 January 2016
Available online 21 January 2016

Keywords:
Delayed neural networks (DNNs)
Finite-time robust stabilization
Discontinuous activations
Parameter uncertainties
Delayed controller
Settling time

ABSTRACT

This paper is concerned with the finite-time robust stabilization of delayed neural networks (DNNs) in the presence of discontinuous activations and parameter uncertainties. By using the nonsmooth analysis and control theory, a delayed controller is designed to realize the finite-time robust stabilization of DNNs with discontinuous activations and parameter uncertainties, and the upper bound of the settling time functional for stabilization is estimated. Finally, two examples are provided to demonstrate the effectiveness of the theoretical results.

© 2016 Elsevier Ltd. All rights reserved.

1. Introduction

In recent years, stability of neural networks (NNs) has received great attention for its potential applications in pattern recognition, parallel computation, associative memory and control optimization (Cao & Wang, 2005; Cohen & Grossberg, 1983; Forti & Tesi, 1995; Shen & Wang, 2009; Zeng & Zheng, 2012). As is well known, during the hardware implementation of NNs, time delays are unavoidable due to the finite switching speed of the neuron amplifiers and they often cause undesirable dynamical behaviors such as instability or oscillation. On the other hand, the parameters of NNs may exhibit some deviations because of the existence of modeling errors, external disturbance, and parameter fluctuations, which would cause the parameter uncertainties. Therefore, it is of practical interest to take into account the time delays and parameter uncertainties when studying stability of NNs. In other words, the robust stability problem of delayed neural networks (DNNs) should be considered.

Recently, robust stability of DNNs has been widely investigated and various robust stability criteria have been proposed (Arik, 2014; Faydasicok & Arik, 2012; Feng, Yang, & Wu, 2015; Guo & Huang, 2009; Guo, Wang, & Yan, 2014; Huang, Li, Duan, & Starzyk, 2012; Shen & Wang, 2012; Wang, Li, & Huang, 2014; Wu, Wang, Huang, & Zuo, 2010; Xiao & Zeng, 2014; Yang, Gao, & Shi, 2009; Zhang, Wang, & Liu, 2008; Zuo et al., 2009). However, the neuron activations considered in Arik (2014), Faydasicok and Arik (2012), Feng et al. (2015), Guo et al. (2014), Huang et al. (2012), Shen and Wang (2012), Wang et al. (2014), Yang et al. (2009) and Zhang et al. (2008) are continuous, or even Lipschitzian. The authors in Forti and Nistri (2003) and Forti, Nistri, and Papini (2005) have demonstrated the interest in studying the stability problem of DNNs with discontinuous activation functions. The discontinuous activations are in the case that the gain of the neuron amplifiers is very high and the analysis of the ideal discontinuous case can better reflect the crucial features of the dynamics. It is shown that DNNs with discontinuous activations are frequently encountered in practice, and the analysis of the discontinuous case deserves the research interest in the potential applications of dry friction, switching in electronic circuits, and optimization problems (Forti, Grazzini, Nistri, & Pancioni, 2006; Forti & Nistri, 2003; Forti et al., 2005). Thus, it is necessary to consider the discontinuous activations for more general applications of DNNs.

Different from the asymptotical stabilization with infinite settling time, finite-time stabilization gives the convergence with finite settling time. It requires essentially that a control system is Lyapunov stable and its trajectories tend to zero in finite time under the designed controller. Previously, Bhat and Bernstein (2000) proved that there is a necessary and sufficient condition for finite-time stability of multi-dimensional continuous autonomous system. Since then, the problems of finite-time stability and

* Corresponding author at: School of Automation, Huazhong University of Science and Technology, Wuhan 430074, China. Tel.: +86 27 87543630; fax: +86 27 87543130.
E-mail address: yishen64@163.com (Y. Shen).

http://dx.doi.org/10.1016/j.neunet.2016.01.005
0893-6080/© 2016 Elsevier Ltd. All rights reserved.

stabilization have been widely studied (Efimov, Polyakov, Fridman, Perruquetti, & Richard, 2014; Forti et al., 2005; Hong & Jiang, 2006; Hu, Yu, & Jiang, 2014; Huang, Li, Huang, & He, 2014; Karafyllis, 2006; Liu, Ho, Yu, & Cao, 2014; Liu, Park, Jiang, & Cao, 2014; Moulay, Dambrine, Yeganefar, & Perruquetti, 2008; Sun, Feng, & Wang, 2014; Wang & Shen, 2015; Wang, Shen, & Ding, 2015; Wang & Xiao, 2010; Yang, 2014). However, so far few studies have been published concerning finite-time stability of time-delay systems (Moulay et al., 2008; Yang & Wang, 2012). This is because time-delay systems exhibit more complicated dynamic behaviors and they are more difficult to deal with than system without delays. As stated in Moulay et al. (2008), it is difficult to find a Lyapunov functional that satisfies the derivative condition for the finite-time stability of time-delay systems. Thus, the finite-time stability of time-delay systems, especially for the DNNs, is still an open problem that requires further investigation.

Motivated by the above discussions, we study the finite-time stability problem of NNs by taking into consideration the time delays, parameter uncertainties and discontinuous activations in our paper. To the best of authors' knowledge, so far there is few results for the finite-time stabilization of DNNs with discontinuous activations and parameter uncertainties. By using a designed delayed feedback controller, the finite-time robust stabilization for this system is realized. The contributions of this paper are as follows.

(1) Time delays, discontinuous activations and parameter uncertainties are considered in studying the finite-time stability problem.

(2) A delayed feedback controller is designed to realize the finite-time robust stabilization for a class of DNNs with discontinuous activations and parameter uncertainties.

(3) The NNs model in our paper is in the presence of parameter uncertainties and discontinuous activations, which makes our robust results more general than (robust) stability of neural networks with continuous activations in Arik (2014), Cao and Wang (2005), Faydasicok and Arik (2012), Feng et al. (2015), Guo et al. (2014), Liu, Wang, and Liu (2006), Shen and Wang (2012), Wang and Shen (2014) and Wang et al. (2014).

(4) The finite-time stabilization with finite settling time improve and extend the stabilization with infinite settling time in the previous works (Huang, Huang, Chen, & Qian, 2013; Phat & Trinh, 2010; Wang et al., 2014; Wen, Huang, Zeng, Chen, & Li, 2015; Wu & Zeng, 2012; Zhang & Shen, 2015).

(5) In Liu and Park et al. (2014) and Liu and Ho et al. (2014), finite-time stabilization was studied while the systems are without delays and parameter uncertainties. So our methods are general and can be used to study finite-time stability of other delayed systems with or without parameter uncertainties.

The remainder of this paper is organized as follows. Some preliminaries are introduced in Section 2. In Section 3, we design a delayed controller with which the finite-time robust stabilization of DNNs with discontinuous activations and parameter uncertainties is achieved. Besides, we estimate the upper bound of the settling time functional and provide the finite-time stabilization algebraic criteria for DNNs without parameter uncertainties. Then, two examples are provided to demonstrate the effectiveness and superiority of the obtained results in Section 4. Finally, conclusions are drawn in Section 5.

2. System description and preliminaries

The following notations will be used throughout this paper. $\mathcal{R}_+$, $\mathcal{R}^n$ and $\mathcal{R}^{n\times n}$ denote the set of all nonnegative real numbers, the n-dimensional Euclidean space and the set of all $n \times n$ real matrices, respectively. For all $x = (x_1, \dots, x_n)^T \in \mathcal{R}^n$, $\|x\| = \sqrt{x^T x}$ is the Euclidean norm and $\mathrm{sgn}(x) = (\mathrm{sgn}(x_1), \dots, \mathrm{sgn}(x_n))^T$ is the sign function. For a given square matrix $A = (a_{ij})_{n\times n} \in \mathcal{R}^{n\times n}$, $|A| = (|a_{ij}|)_{n\times n}$. $\mathcal{C}([a,b],\mathcal{R}^n)$ denotes the space of all continuous functions $\Psi : [a,b] \to \mathcal{R}^n$ with uniform norm $\|\Psi\| = \sup_{a\le s\le b} \|\phi(s)\|$. A continuous function $\nu : \mathcal{R} \to \mathcal{R}$ belongs to the class $\mathcal{K}$ if it is strictly increasing and $\nu(0) = 0$.

In this paper, we consider a class of DNNs as follows:

$$\dot{x}(t) = -Dx(t) + Af(x(t)) + Bg(x(t-\tau(t))), \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathcal{R}^n$ is the state vector. $D = \mathrm{diag}(d_1, d_2, \dots, d_n)$ is a diagonal matrix with $d_i > 0 (i = 1, 2, \dots, n)$. $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n} \in \mathcal{R}^{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively. $f(x(t)) = (f_1(x_1(t)), \dots, f_n(x_n(t)))^T \in \mathcal{R}^n$ and $g(x(t-\tau(t))) = (g_1(x_1(t-\tau_1(t))), \dots, g_n(x_n(t-\tau_n(t))))^T \in \mathcal{R}^n$ are the neuron activation functions. $\tau(t) = (\tau_1(t), \tau_2(t), \dots, \tau_n(t))^T$ is the time-varying delay, which satisfies $0 \le \tau_j(t) \le \tau$, $j = 1, 2, \dots, n$.

An important factor that affects the stability is the uncertainties of network parameters. In this paper, the parameter matrices $D = \mathrm{diag}(d_1, d_2, \dots, d_n)$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ of system (1) are assumed to be norm-bounded within the following ranges:

$$\begin{aligned} D_I &= [\underline{D}, \overline{D}] = \{D = \mathrm{diag}(d_i) : 0 < \underline{d}_i \le d_i \le \overline{d}_i, i = 1, 2, \dots, n\}, \\ A_I &= [\underline{A}, \overline{A}] = \{A = (a_{ij})_{n\times n} : \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}, i, j = 1, 2, \dots, n\}, \\ B_I &= [\underline{B}, \overline{B}] = \{B = (b_{ij})_{n\times n} : \underline{b}_{ij} \le b_{ij} \le \overline{b}_{ij}, i, j = 1, 2, \dots, n\}. \end{aligned} \tag{2}$$

The following assumptions are given for system (1).

(A1) For every $j = 1, 2, \dots, n$, $f_j, g_j : \mathcal{R} \to \mathcal{R}$ are continuous except on a countable set of isolate points $\{\rho_k^j\}$, where the finite right and left limits $f_j^+(\rho_k^j)$, $g_j^+(\rho_k^j)$ and $f_j^-(\rho_k^j)$, $g_j^-(\rho_k^j)$ exist, respectively.

(A2) For each $j = 1, 2, \dots, n$, suppose $0 \in K[f_j(0)]$, $0 \in K[g_j(0)]$ and there exist constants $h_j > 0$, $l_j > 0$, $r_j > 0$, $s_j > 0$, such that

$$\sup |\xi_j| \le h_j|u| + r_j, \qquad \sup |\zeta_j| \le l_j|v| + s_j, \tag{3}$$

for all $u, v \in R$, where $\xi_j \in K[f_j(u)] = [\min\{f_j^-(u), f_j^+(u)\}, \max\{f_j^-(u), f_j^+(u)\}]$, $\zeta_j \in K[g_j(v)] = [\min\{g_j^-(v), g_j^+(v)\}, \max\{g_j^-(v), g_j^+(v)\}]$.

Because of the presence of discontinuous activations, system (1) is discontinuous and its classical solution does not exist. Now we introduce the concept of Filippov solution (Filippov, 1988).

Definition 1 (*Filippov, 1988*)**.** For a system with discontinuous right-hand side:

$$\dot{x}(t) = F(t, x_t), \quad t \ge 0, \tag{4}$$

where $x(t) \in \mathcal{R}^n$, $x_t \in \mathcal{C}([-\tau, 0], \mathcal{R}^n)$ and $x_t(s) = x(t+s)$, $-\tau \le s \le 0$. $F : [0, +\infty) \times \mathcal{R}^n \to \mathcal{R}^n$ is Lebesgue measurable and locally essentially bounded. An absolutely continuous function $x(t)$, $t \in [0, T]$, $T > 0$ is said to be a Filippov solution of system (4) with initial condition $x(s)$, $-\tau \le s \le 0$, if it satisfies the differential inclusion:

$$\dot{x}(t) \in \Phi(t, x_t), \quad \text{for } a.a.\ t \in [0, T], \tag{5}$$

where the Filippov set-valued map $\Phi(t, x_t) : [0, +\infty) \times \mathcal{R}^n \to 2^{\mathcal{R}^n}$ is defined by

$$\Phi(t, x_t) \triangleq \bigcap_{\delta > 0} \bigcap_{\mu(N)=0} K[F(t, B(x_t, \delta) \setminus N)],$$

$K[E]$ is the closure of the convex hull of set E, $E \subset \mathcal{R}^n$, $B(x_t, \delta) = \{y_t : \|y_t - x_t\| < \delta, x_t, y_t \in \mathcal{R}^n, \delta \in \mathcal{R}_+\}$, and $N \subset \mathcal{R}^n$, $\mu(N)$ is the Lebesgue measure of set N.

Download English Version:

https://daneshyari.com/en/article/405426

Download Persian Version:

https://daneshyari.com/article/405426

Daneshyari.com