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Projective synchronization of nonidentical fractional-order neural networks based on sliding mode controller

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ABSTRACT

This paper investigates global projective synchronization of nonidentical fractional-order neural networks (FNNs) based on sliding mode control technique. We firstly construct a fractional-order integral sliding surface. Then, according to the sliding mode control theory, we design a sliding mode controller to guarantee the occurrence of the sliding motion. Based on fractional Lyapunov direct methods, system trajectories are driven to the proposed sliding surface and remain on it evermore, and some novel criteria are obtained to realize global projective synchronization of nonidentical FNNs. As the special cases, some sufficient conditions are given to ensure projective synchronization of identical FNNs, complete synchronization of nonidentical FNNs and anti-synchronization of nonidentical FNNs. Finally, one numerical example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

Fractional calculus, which is a generalization of integer-order integration and differentiation to its non-integer counter part, has been intensively investigated in many fields, covering dynamics of complex materials or porous media (Carpinteri, Cornetti, & Kolwankar, 2004), fluid mechanics (Tripathi, Pandey, & Das, 2010), bioengineering (Magin, 2004, 2010; Magin & Ovadia, 2008), viscoelasticity (Soczkiewicz, 2002), etc. As fractional calculus has infinite memory, and has proven to be an excellent tool for the description of memory and hereditary properties of various materials and processes (Chen, Ye, & Sun, 2010; Isfer, Lenzi, Teixeira, & Lenzi, 2010; Kilbas & Marzan, 2005; Szabo & Wu, 2000), it is easy to see that the incorporation of a memory term into a neural network model is an extremely important improvement. In recent years, fractional-order neural networks (FNNs) have attracted attentions of many researchers, and various dynamical behaviors of FNNs have been widely investigated, such as Chen, Chai, Wu, Ma, and Zhai (2013), Kaslik and Sivasundaram (2012), Wang, Yu, Wen, and Zhang (2013), Yu, Hu, and Jiang (2012), Zhou, Li, and Zhu (2008), Zou, Qu, Chen, Chai, and Yang (2014).

Since Pecora and Carroll (Pecora & Carroll, 1990) firstly put forward chaos synchronization in 1990, more and more researchers pay enough attentions to studying synchronization. The increasing interest in researching synchronization stems from its potential applications in cryptography, secure communication, optimization of nonlinear systems performance, modeling brain activity, and chemical reaction (Boccaletti, Kurths, Osipov, Valladares, & Zhou, 2002; Chen & Dong, 1998; Ojalvo & Roy, 2001; Ott, Grebogi, & Yorke, 1990; Sprott, 2003; Yang & Chua, 1997). So far, various types of fractional-order synchronization results emerge in large numbers, such as complete synchronization (Ding, Shen, & Wang, 2016; Yan & Li, 2007), generalized synchronization (Wu, Lai, & Lu, 2012), phase synchronization (Erjaee & Momani, 2008), lag synchronization (Zhu, He, & Zhou, 2011), etc. As pointed out in Wang and He (2008), projective synchronization, amongst all kinds of synchronization, can obtain faster communication for its proportional feature in application to secure communications. Subsequently, a series of results about projective synchronization appear, such as Bao and Cao (2015), Hu, Xu, and Yang (2008), Si, Sun, Zhang, and Chen (2012), Xin, Chen, and Liu (2011), Yu, Hu, Jiang, and Fan (2014).

However, to the best of our knowledge, most reports are concerned with the projective synchronization problem for identical fractional-order systems. In practice, due to the mismatched parameters and functions which are unavoidable in real implementation, the drive system and response system are not identical. From the point of view of engineering, it is very difficult to keep the two systems to be identical all the time (Huang & Feng, 2009). Therefore, it is significant to study projective synchronization problem of nonidentical FNNs.

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To deal with synchronization of FNNs, there exist different control approaches, such as linear feedback control (Chen, Zeng, & Jiang, 2014; Zhang, Yu, & Wang, 2015), adaptive control (Bao & Cao, 2015; Yu et al., 2014), and so on. Sliding mode control, as a robust variable structure control method, is an important control technique. Its advantages include fast response, low sensitivity to external noises, robustness to the system uncertainties and easy realization. The main idea of sliding mode control is forcing the system state trajectories to some predefined sliding surfaces by using a discontinuous controller, and the system on sliding surfaces has desired properties such as stability. In addition, sliding mode controller includes an equivalent control part that describes the behaviors of the system when the trajectories stay over the sliding surface, and a variable structure control part that enforces the trajectories to reach the sliding surface and remain on it evermore.

Motivated by the above discussions, in this paper, we focus our attentions on the global projective synchronization of nonidentical FNNs in the sense of Caputo fractional derivation. Firstly, by considering the measured output of system, a fractional-order integral sliding surface is properly constructed. To guarantee the existence of the sliding motion, we design a sliding mode controller by using sliding mode control theory. Then, based on fractional Lyapunov direct methods and the properties of Caputo fractional-order derivative, reachability of the specified sliding surface is analyzed, and some sufficient criteria for the global projective synchronization of nonidentical FNNs are presented. Besides, the global projective synchronization of identical FNNs are proved. Finally, by selecting different projection coefficient, the obtained results can be used to achieve globally asymptotically complete synchronization and globally asymptotically anti-synchronization of nonidentical FNNs.

The organization of this paper is as follows. The system and some preliminaries are introduced in Section 2. In Section 3, sufficient criteria are established based on a fractional-order integral sliding mode controller, sliding mode control theory and fractional Lyapunov direct methods. Then, numerical simulations are given to demonstrate the effectiveness of the obtained results in Section 4. Finally, conclusions are drawn in Section 5.

2. Preliminaries

Notations. Through this paper, $\mathbb{R}$ is the space of real number, $\mathbb{N}_+$ is the set of positive integers, $\mathbb{C}$ is the space of complex number, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m \times n$ real matrices. $\operatorname{sgn}(\cdot)$ is symbolic function. $[\cdot,\cdot]$ represents the interval and $diag\{\cdots\}$ denotes a block-diagonal matrix. If not stated explicitly, matrices are assumed to have compatible dimensions for algebraic operations. In addition, $C^r\big([t_0,+\infty),\mathbb{R}\big)$ denotes the space consisting of r-order continuous differentiable functions from $[t_0,+\infty)$ into $\mathbb{R}$.

In order to investigate FNNs, we firstly recall some definitions about fractional calculation and introduce some useful lemmas in this section.

2.1. Caputo fractional-order derivative

Definition 1 (*Kilbas, Srivastava, & Trujillo, 2006; Podlubny, 1999*). The fractional-order integral of order α for an integrable function $f(t):[t_0,+\infty)\to\mathbb{R}$ is defined as

$$ {}_{t_0}I_t^{\alpha} f(t) = \frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}f(\tau)d\tau, $$

where $\alpha > 0$, and $\Gamma(\cdot)$ is the Gamma function which is defined by

$$ \Gamma(z) = \int_0^{\infty} e^{-t}t^{z-1}dt, \quad \big(Re(z)>0\big), $$

where $Re(z)$ is the real part of z.

Definition 2 (*Kilbas et al., 2006; Podlubny, 1999*). The Caputo fractional-order derivative of order α for a function $f(t) \in C^{n+1}\big([t_0,+\infty),\mathbb{R}\big)$ is defined as

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)}\int_{t_0}^{t}\frac{f^{(n)}(\tau)}{(t-\tau)^{\alpha-n+1}}d\tau, $$

where $t \ge t_0$ and n is a positive integer such that $n-1<\alpha<n$. Particularly, when $0<\alpha<1$,

$$ {}_{t_0}D_t^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)}\int_{t_0}^{t}\frac{f'(\tau)}{(t-\tau)^{\alpha}}d\tau. $$

Several important properties about Caputo fractional-order derivative are listed below (Aghababa, 2013a; Li & Deng, 2007; Podlubny, 1999).

Property 1. *${}_{t_0}D_t^{\alpha} c = 0$ holds, where c is any constant.*

Property 2. *For any constants ν_1 and ν_2, the linearity of Caputo fractional-order derivative is described by*

$$ {}_{t_0}D_t^{\alpha}\Big(\nu_1 f(t)+\nu_2 g(t)\Big) = \nu_1\, {}_{t_0}D_t^{\alpha} f(t) + \nu_2\, {}_{t_0}D_t^{\alpha} g(t). $$

Property 3. *${}_{t_0}D_t^{\alpha}\,{}_{t_0}I_t^{\beta} f(t) = {}_{t_0}D_t^{\alpha-\beta} f(t)$ where $\alpha \ge \beta \ge 0$. Especially, when $\alpha=\beta$, ${}_{t_0}D_t^{\alpha}\,{}_{t_0}I_t^{\alpha} f(t) = f(t)$.*

Property 4. *The Leibniz' s rule for fractional differentiation is given as:*

$$ {}_{t_0}D_t^{\alpha}(\phi(t)f(t)) = \sum_{k=0}^{\infty}\frac{\Gamma(1+\alpha)}{\Gamma(1+k)\Gamma(\alpha-k+1)}\phi^{(k)}(t)\, {}_{t_0}D_t^{\alpha-k} f(t), $$

if $\phi(t)$ and $f(t)$ and all their derivatives are continuous in the interval $[t_0,t]$, where $\phi^{(k)}(t)$ is the integer-order derivative of order k for function $\phi(t)$ and ${}_{t_0}D_t^{\alpha-k} f(t)$ is Caputo fractional-order derivative of order $\alpha-k$ for function $f(t)$.

Property 5. *If $x(t)\in C^1[0,T]$ for some $T>0$, then*

$$ {}_0D_t^{\alpha_1}\,{}_0D_t^{\alpha_2}x(t) = {}_0D_t^{\alpha_2}\,{}_0D_t^{\alpha_1}x(t) = {}_0D_t^{\alpha_1+\alpha_2}x(t), \quad t\in[0,T], $$

where $\alpha_1>0$, $\alpha_2>0$ and $\alpha_1+\alpha_2\le 1$.

In addition, some necessary lemmas about Caputo fractional-order derivative are given.

Lemma 1 (*Kilbas et al., 2006*). *Let $\Omega=[a,b]$ be an interval on the real axis $\mathbb{R}$, let $n=[\alpha]+1$ for $\alpha\notin\mathbb{N}_+$ or $n=\alpha$ for $\alpha\in\mathbb{N}_+$. If $x(t)\in C^n[a,b]$, then*

$$ {}_aI_t^{\alpha}\,{}_aD_t^{\alpha}x(t) = x(t) - \sum_{k=0}^{n-1}\frac{x^{(k)}(a)}{k!}(t-a)^k, \quad n-1<\alpha\le n, $$

where ${}_aI_t^{\alpha}$ is fractional-order integral of order α and ${}_aD_t^{\alpha}$ is Caputo fractional-order derivative of order α. In particular, if $0<\alpha\le 1$ and $x(t)\in C^1[a,b]$, then

$$ {}_aI_t^{\alpha}\,{}_aD_t^{\alpha}x(t) = x(t)-x(a). $$

Lemma 2 (*Zhang et al., 2015*). *If $h(t)\in C^1([0,+\infty],\mathbb{R})$ denotes a continuously differentiable function, the following inequality holds almost everywhere.*

$$ {}_0D_t^{\alpha}|h(t)| \le \operatorname{sgn}(h(t))\,{}_0D_t^{\alpha}h(t), \quad 0<\alpha<1. $$