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Event-triggered synchronization strategy for complex dynamical networks with the Markovian switching topologies

Aijuan Wang, Tao Dong*, Xiaofeng Liao

College of Electronics and Information Engineering, Southwest University, Chongqing, 400715, PR China

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ABSTRACT

This paper concerns the synchronization problem of complex networks with the random switching topologies. By modeling the switching of network topologies as a Markov process, a novel event-triggered synchronization strategy is proposed. Unlike the existing strategies, the event detection of this strategy only works at the network topology switching time instant, which can significantly decrease the communication frequency between nodes and save the network resources. Under this strategy, the synchronization problem of complex network is equivalently converted to the stability of a class of Markovian jump systems with a time-varying delay. By using the Lyapunov–Krasovskii functional method and the weak infinitesimal operation, a sufficient condition for the mean square synchronization of the complex networks subject to Markovian switching topologies is established. Finally, a numerical simulation example is provided to demonstrate the theoretical results.

1. Introduction

In recent years, complex networks have attracted much attention in science and engineering (Chen, Lu, & Lin, 2013; Chen, Lu, Yu, & Lin, 2013; Dong, Liao, & Wang, 2015; En Liu, Jian Ji, Ming Xie, & Wang, 2015; Jian Ji, Lin, & Sheng Yu, 2015; Strogatz, 2001). The synchronization of all dynamical nodes is an important and interesting phenomena mostly because the synchronization phenomena has been found both in natural and man-made systems, such as internet, social systems, electrical power grids, distributed computing systems, chaos-based communication network, and so on. Consequently, the synchronization of complex networks has been studied widely (Beard, McLain, Goodrich, & Anderson, 2002; Chen, Liu, & Lu, 2007; Fax & Murray, 2004; Li, Liao, & Wong, 2004; Li, Yu, & Huang, 2014; Lu & Chen, 2005; Lu, Li, & Rong, 2010; Millerioux & Daafouz, 2001, 2003; Olfati-Saber, 2006; Rakkiyappan, Sakthivel, & Cao, 2015; Reynolds, 1987; Vicsek, Czirok, Ben-Jacob, Cohen, & Shochet, 1995; Xie, Chen, & Bollt, 2002).

In the practical networks, the connectivity of network topology might be unfixed, even randomly changing. For example, in the marine oil pervasion monitoring systems deployed in an oceanic area (Guo, 2010), the topologies of the mobile sensor network may vary (switch) with the coverage area and the spatial distribution of the pervading oil, which may be affected by some random factors such as wind, sea wave and temperature. In this case, it is suitable to model the randomly switching network topologies as a Markov process. Thus, how to solve the synchronization problem of complex networks with the Markovian switching topologies becomes an important, interesting and challenging issue. Recently, there are many studies in this field (Ding & Guo, 2015; Guo, 2010; Guo & Zhong, 2015; Hao, Park, Guang, & Qiang, 2015; Hua, Huo, & Sheng, 2015; Huang, Dey, Nair, & Manton, 2010; Jin, Liang, Mei, Qing, & Sheng, 2012; Jun, De, & Qiang, 2015; Matei, Martins, & Baras, 2009; Zhao, Ren, Yuan, & Chen, 2012). Yet, all existing synchronization strategies of complex networks with stochastic switching topologies are continuous. Continuous synchronization strategies can result in frequent communication between nodes, which cause the network congestion and waste the network resources. In order to overcome the conservativeness of continuous synchronization strategies, the event-triggered strategy is proposed, where the update of controller are only determined by certain events that are triggered depending on the agents dynamic behavior. Up to now, great efforts have been made on event-based synchronization strategy in complex networks (Hua, De, Feng, & Xiao, 2015; Lian Lu, Juan Han, & Ping, 2015; Qing et al., 2015; Zhou, feng Liao, wen Huang, & Chen, 2015). In Zhou et al. (2015), pinning exponential synchronization of complex networks via event-triggered communication with combinational measurements was studied. In Qing et al. (2015), the authors developed an event-triggered asynchronous intermittent communication strategy, under which synchronization of complex dynamical networks was obtained. In Hua,

* Corresponding author. Tel.: +86 23 68250048.
E-mail address: david_312@126.com (T. Dong).

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De et al. (2015), cluster synchronization of complex networks via event-triggered strategy under stochastic sampling was investigated. In Lian Lu et al. (2015), the authors employed the event-triggered strategy in both coupling configuration and pinning control terms to realize stability in coupled dynamical systems with Markovian switching couplings and pinned node set. However, most of the studies have not considered the event-triggered synchronization strategy of complex networks with Markovian switching topologies. So the observation provides us the motivation of this paper to design a event-triggered synchronization strategy of complex networks with the Markovian switching topologies.

In this paper, we investigate the event-triggered synchronization strategy problem for complex networks with the Markovian switching topologies. By considering the network topology switching frequently is the character of networks with the Markovian switching topologies, a novel event-triggered synchronization strategy is proposed. Unlike the existing event trigger strategy, the event detection of our strategy only works at the network topology switching time instant, which significantly decreases which can significantly decrease the communication frequency between nodes and save the network resources. Moreover, a sufficient condition is presented by employing Lyapunov–Krasovskii functional method and the weak infinitesimal operation, which can ensure the synchronization of complex network can be globally exponentially achieved in mean-square sense.

The rest of this paper is organized as follows. Some preliminary results of graph theory and a new event-triggered synchronization strategy are briefly reviewed in Section 2. The main results are established in Section 3. In Section 4, a numerical example with stimulation is presented to illustrate the effectiveness of theoretical results. Conclusions are finally drawn in Section 5.

Notations: $R^{m\times n}$ denotes the family of $m \times n$ dimensional real matrices. I_m denotes the $m \times m$ dimensional identity matrix. $\otimes$ denotes the Kronecker product. For a given vector or matrix X, X^T denotes its transpose. Denote $\|\cdot\|$ the Euclidean norm for vectors in R^N or the induced 2-norm for matrices in $R^{m\times n}$. For a square non-singular matrix X, X^{-1} denotes its inverse matrix. **E** denotes the mathematical expectation.

2. Problem statement

2.1. Graph theory

In this subsection, some basic notions of graph theory are briefly introduced. We introduce the following terms. Let $\gamma(t)$ be a continuous-time Markov process with values in a finite set $S = \{1, 2, \dots, q\}$. Let $G = (\nu, \varepsilon, \Delta)$ be a weighted digraph with the set of vertices $\nu = \{1, 2, \dots, n\}$ and the set of edges $\varepsilon \subseteq \nu \times \nu$. In G, the ith vertex represents the ith node, and a directed edge from i to j is denoted as an ordered pair $(i, j) \in \varepsilon$ which means that node j can directly receive information from node i. In this case, the vertex i is called the parent vertex and the vertex j is called the child vertex. The set of neighbors of the ith node is denoted by $N_i = \{j \in \nu \mid (j, i) \in \varepsilon\}$. $\Delta = \left(w_{ij}(\gamma(t))\right) \in R^{n\times n}$ is called the weighted adjacency matrix of G with nonnegative elements, and $w_{ij}(\gamma(t)) = 0$, $w_{ij}(\gamma(t)) > 0 \Leftrightarrow j \in N_i$. The degree matrix of subgraph G is denoted by $\Xi = diag\{\bar{w}_1, \bar{w}_2, \dots, \bar{w}_n\}$, where the diagonal element is represented as $\bar{w}_i = \sum_{j=1}^{n} w_{ij}$ (which is also called as the in-degree of node). Correspondingly, the Laplacian matrix of the directed graph G is defined as $L = \Xi - \Delta$. If there is a sequence of edges of the form $(i, j_1), (j_1, j_2), \dots, (j_m, j) \in \varepsilon$ composing a directed path beginning with i and ending with j in the directed graph g with distinct nodes j_k, $k = 1, 2, \dots, m$, then the node j is said to be reachable from node i. A directed graph is strongly connected if for any distinct nodes i and j, there exists a directed path from node i to node j. The union graph of a collection of directed graph G_r $(r = 1, 2, \dots, s)$, which is denoted by $\bigcup_{r=1}^{s} G_r$, is a directed graph with node set ν and the edge set equal to the union of the edge sets of all of the graphs G_r in the collection.

2.2. A new event-triggered synchronization strategy

Considering a group of directed graph $G(\gamma(t)) \in (G_1, G_2, \dots, G_q)$, we defined the transition probabilities as follows:

$$prob\{\gamma(t+\Delta t) = s \mid \gamma(t) = r\} = \begin{cases} \pi_{rs}\Delta t + o(\Delta t), & r \neq s \\ 1 + \pi_{rr}\Delta t + o(\Delta t), & r = s \end{cases} \tag{1}$$

where $\Delta t > 0$, $o(\Delta t) \to 0$ as $\Delta t \to 0$ and π_{rs} is the transition rate from mode r to mode s, which satisfies $\pi_{rs} > 0$ for $r \neq s$ and $\pi_{rr} = 1 - \sum_{s=1, s\neq r}^{q} \pi_{rs}$ for $r \in S$. Then, it is easily known that $L(\gamma(t)) \in \{L_1, L_2, \dots, L_q\}$.

Due to the topologies of complex network frequently switch, we design a novel event-triggered synchronization strategy based on this network features, which is as follows:

$$\dot{x}_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} w_{ij}(\gamma(t_k))\left(x_i\left(t_k^i\right) - x_j\left(t_k^j\right)\right), \tag{2}$$

where $t \in \left[t_k^i, t_{k+1}^i\right)$, $x_i(t) \in R^N$ is the state, $f(x_i(t)) \in R^N$ is a continuously differentiable nonlinear function, c is internal coupling gain, $x_i\left(t_k^i\right)$ is the state of node i at event-triggered time instant, t_k is the Markovian switching time instant, t_k^i is the event-triggered time instant of node i. The event detection of this strategy only works at the switching topologies time instant. The event-triggering threshold error is defined as $e_i(t) = x_i\left(t_k^i\right) - x_i(t_k)$, $t \in \left[t_k^i, t_{k+1}^i\right)$. And, the distributed triggering event is defined as

$$\begin{aligned} E_i(t) &= \|e_i(t)\| - \beta_i\sum_{j=1}^{N}\left\|\left(x_i\left(t_k^i\right) - x_j\left(t_k^j\right)\right)\right\|, \\ \beta_i &> 0, \qquad t_{k+1}^i = \inf\left\{t : t > t_k^i, E_i(t) > 0\right\} \end{aligned} \tag{3}$$

where β_i is the control gain for each node, the event can analytically determine the node's individual time instant. Therefore, the communication between neighbors' nodes is asynchronous.

Remark 1. At the switching topologies time instant t_k, if $\|e_i(t)\| > \beta_i\sum_{j=1}^{N}\left\|\left(x_i\left(t_k^i\right) - x_j\left(t_k^j\right)\right)\right\|$, the state of event-triggered time instant is updated at the switching topologies time instant t_k, namely, $t_k^i = t_k$, otherwise, the event-triggered time instant is the same as the switching topologies time instant, namely, $t_k \neq t_k^i$.

Remark 2. Since the inter-event time is no less than bounded by the topology switching interval $h_k > 0$, which implies that Zeno behavior is absolutely excluded.

Remark 3. Compared with the previous synchronization strategy of complex networks with the switching topologies (Guo & Zhong, 2015; Hao et al., 2015; Hua, Huo et al., 2015; Jin et al., 2012; Jun et al., 2015), our synchronization strategy may be more feasible, the control will be updated only when the time is at switching instants and the event-triggered condition is satisfied, which significantly decreases the number of communication among nodes.

Remark 4. Compared with the previous event-triggered strategy (Hua, De et al., 2015; Lian Lu et al., 2015; Qing et al., 2015; Zhou et al., 2015), the event-triggered strategy proposed in this paper makes full use of the character of networks with the Markovian switching topologies, the event detection of this strategy only works at the network topology switching time instant.