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Non-fragile H_∞ synchronization of memristor-based neural networks using passivity theory

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ABSTRACT

In this paper, we formulate and investigate the mixed H_∞ and passivity based synchronization criteria for memristor-based recurrent neural networks with time-varying delays. Some sufficient conditions are obtained to guarantee the synchronization of the considered neural network based on the master–slave concept, differential inclusions theory and Lyapunov–Krasovskii stability theory. Also, the memristive neural network is considered with two different types of memductance functions and two types of gain variations. The results for non-fragile observer-based synchronization are derived in terms of linear matrix inequalities (LMIs). Finally, the effectiveness of the proposed criterion is demonstrated through numerical examples.

1. Introduction

Memristor is a two-terminal electrical circuit element that has attracted immense interests from both industry and academia ever since an operational device was postulated by Chua in 1971 (Chua, 1971). He predicted that besides the resistor, capacitor and inductor, there should be a new fourth fundamental two-terminal circuit element called a memristor (a contraction of memory and resistor). This device acts as a variable resistance that "remembers" how much current has flowed through it by changing the voltage across its terminals. In 2008, the Hewlett–Packard (HP) Lab team announced their realization of a prototype of the memristor, see Strukov, Snider, Stewart, and Williams (2008) and Tour and He (2008). Using memristors one can achieve circuit functionalities that are not possible to establish with resistors, capacitors and inductors. In addition, it has been shown that memristors are proposed to work as synaptic weights in artificial neural networks Hu and Wang (2010), Li, Rakkiyappan, and Velmurugan (2015) and Pershin and Ventra (2012). Due to this feature, the model of memristor-based neural networks (MNNs) can be built to emulate the human brain where synapses are implemented with memristors. Memristor-based recurrent neural networks (MRNNs) is a ground-breaking concept, which are used to understand the behavior of many physical, biological, social and technical systems. Based on physical symmetry arguments, MNN is a memristor bridge weighting circuit that can perform analog multiplication, i.e., the MNN is a neuromorphic computing system consisting of some identical memristors. Therefore, we can use memristors instead of resistors to build a new model of neural networks to emulate the human brain. In recent years, there has been number of results proposed in literature, for example, in Ito and Chua (2009), memristive neural networks have been designed by replacing with memristors instead of the resistors in the primitive neural networks. The problems of robust stability analysis and robust stabilization of uncertain memristive neural networks have been addressed in Wang, Li, and Huang (2014). Exponential stability for a class of memristive neural networks with time-varying delays have been investigated in Wang, Li, Huang, and Duan (2014). The problem of finite-time stability of fractional-order complex-valued MNNs with time delays have been dealt in Rakkiyappan, Velmurugan, and Cao (2014). The study of time-delay systems plays an important role, because the existence of delays is a main source of systems instability or divergence or oscillation and poor performance. Thus, in the past years substantial efforts have been made to the memristive system with time-delays, see Cao and Wan (2014), Hu and Wang (2010), Wu, Li, Wei, Zhang, and Yao (2015) and references therein.

Moreover, under different pinched hysteresis loops, the evolutionary tendency or process of memristive systems evolves into

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different forms. It is generally known that the pinched hysteresis loop is due to the nonlinearity of memductance functions. So, it is useful to deal the memristive systems with the different types of memductance functions. The dynamic flows for memristor-based wavelet neural networks with continuous feedback functions and discontinuous feedback functions in the presence of different memductance functions has been investigated in Wu, Zeng, and Xiao (2013). The problem of the existence, uniqueness and uniform stability of memristor-based fractional-order neural networks with two different types of memductance functions has been investigated in Rakkiyappan, Velmurugan, and Cao (2015). On the other hand, the dynamical behaviors of coupled systems achieve the same time spatial state known as synchronization. Because of its potential features in many different areas including secure communication, biological system, optics, and information processing, synchronization has been received greater importance among the researchers Gan (2013), Gao, Zhang, and Lu (2013), Wu, Wen, and Zeng (2012) and Wu, Zeng, and Huang (2012). In Sun, Shen, Yin, and Xu (2013), the authors discussed the compound synchronization of four memristor chaotic oscillator systems, which shows memristor-based nonlinear hybrid system plays an important role in the secure communication. Exponential synchronization of delayed memristor-based chaotic neural networks based on the periodically intermittent control has been studied in Zhang and Shen (2014). The concept of exponential synchronization for memristive Cohen–Grossberg neural networks with mixed delays has been derived in Yang, Cao, and Yu (2014). The synchronization issues for chaotic memristive neural networks with time delays using sampled-data feedback controller is studied in Wu et al. (2015). Recently, synchronization of memristor-based recurrent neural networks with two delay components based on second-order reciprocally convex approach has been discussed in Chandrasekar, Rakkiyappan, Cao, and Lakshmanan (2014). The periodicity and synchronization of the coupled memristive neural networks with supremums and time-varying delays was investigated in Wan and Cao (2015). The design of controller on synchronization of memristor-based neural networks with time-varying delays has been proved in Wang and Shen (2015).

However, in practice, the controllers should be robust with respect to system uncertainties (Shi, Yin, Liu, & Zhang, 2014), because in some situations they may be very sensitive to their own uncertainties (implementation errors) and this is called fragility problem of controllers. In many practical systems, the measurements of the states of are not feasible in some situations, so the immeasurable states are generally estimated using the available measurements and knowledge of the physical systems. Hence, in such situation for improving the unstable systems performance or in other words to stabilize it, the observer-based control schemes are probably well suited. Further the non-fragile control schemes are concerned to design a feedback control that will be insensitive to some error or gains variation in feedback loop (Dorato, 1998). In recent years, the design of observer-based state estimation or synchronization for neural networks with time-varying delays has received considerable research attention (Fang & Park, 2013b; Lien, 2007; Phat, Fernando, & Trinh, 2014). However, it is well known that, the H_∞ approach was proposed to restrict the effect of the disturbance input on the desired output of the system to a prescribed level (Wu, Karimi, & Shi, 2013). On the other hand, the passivity analysis is one of major tools for analyzing stability of nonlinear dynamical system. The passive properties of a system can keep the system internally stable, by using input–output characteristics. The robust non-fragile H_∞ control problem has been investigated for uncertain stochastic time-delay systems containing sector nonlinearities in Tian, Xie, and Chen (2007). In recent years, the new type of objective function based on mixed H_∞ and passivity concept got considerable attention among the researchers. For example, the mixed H_∞ and passivity based state estimation problem has been discussed in Zhang, Cai, and Wang (2014), where the sufficient condition has been derived such that the estimation error system is exponentially stable in the mean square sense and achieves a prescribed mixed H_∞ and passivity performance level. The problem of mixed H_∞ and passive filter design for Markovian jump impulsive networked control systems with norm bounded uncertainties and random packet dropouts has been discussed in Mathiyalagan, Park, Sakthivel, and Marshal Anthoni (2014). The problem of mixed H_∞ and passive filtering for singular systems with time delays is studied in Wu, Park, Su, Song, and Chu (2013). However, the study of synchronization criteria for MRNNs with non-fragile controller and mixed H_∞ and passivity approach has not been investigated and remains challenging.

Based on the above discussion and facts, in this paper, an LMI based approach is developed to study the mixed H_∞ and passivity synchronization for MRNNs by with uncertain delay. The MRNNs is considered with continuous feedback functions and discontinuous feedback functions in the presence of different memductance functions. The main contribution and novelty of this paper are as follows:

(1) The mixed H_∞ and passivity synchronization is addressed first time for memristive neural networks.
(2) The memristive neural network is considered with two different types of memductance functions and two types of gain variations.
(3) The randomness of controller gain fluctuation has been taken into account when studying the non-fragile synchronization.

The delay-dependent sufficient conditions are derived by constructing an appropriate Lyapunov–Krasovskii functional (LKF) and using LMI approach to guarantee the master system synchronizes with the slave system with a guaranteed mixed H_∞ and passivity performance level. Finally, two examples are given to show the effectiveness and conservativeness of our method.

2. Problem formulation and preliminaries

2.1. Model description

First we describe the circuit of a general class of recurrent neural networks (RNNs). Taking the ith subsystem as the unit of analysis, in order to simplify illustration we describe its dynamics by using the Kirchhoff current law as follows:

$$\begin{aligned}
\dot{x}_i(t) &= -\frac{1}{\mathcal{C}_i}\Big[\Big(\sum_{j=1}^{n}\left(\frac{1}{\mathcal{R}_{ij}}+\frac{1}{\mathcal{F}_{ij}}\right)\times \mathrm{sign}_{ij}+\frac{1}{\hat{\mathcal{R}}_i}\Big)x_i(t) \\
&\quad+\sum_{j=1}^{n}\frac{f_j(x_j(t))}{\mathcal{R}_{ij}}\times \mathrm{sign}_{ij}+\sum_{j=1}^{n}\frac{f_j(x_j(t-\tau(t)))}{\mathcal{F}_{ij}} \\
&\quad\times \mathrm{sign}_{ij}+\mathcal{I}_i(t)\Big],
\end{aligned} \tag{1}$$

$$p_i(t)=\grave{h}_i x_i(t),$$

where $x_i(t)$ is the voltage of the capacitor $\mathcal{C}_i$; $\mathcal{R}_{ij}$ represents the resistor between the feedback function $f_i(x_i(t))$ and $x_i(t)$; $\mathcal{F}_{ij}$ represents the memristor between the feedback function $f_i(x_i(t-\tau_i(t)))$ and $x_i(t)$ satisfying $f_i(0) = 0$, and $\hat{\mathcal{R}}_i$ represents the parallel-resistor corresponding to the capacitor $\mathcal{C}_i$. $I_i(t)$ denotes the external input or bias. Also $\tau(t)$ represents the time-varying delay satisfying $0 \le \tau(t) \le \tau + \Delta_\tau(t)$ with $\dot{\tau}(t) \le \mu$, where $\Delta_\tau(t)$ is the time-varying uncertain element of the transmission delay satisfying $|\Delta_\tau(t)| \le \delta_\tau$ and τ, μ are known positive constants. The feedback function $f(\cdot)$ is assumed to satisfy the following assumption: