Neural Networks 54 (2014) 1–10

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

An improved robust stability result for uncertain neural networks with multiple time delays

Sabri Arik

Isik University, Department of Electrical and Electronics Engineering, 34980 Sile, Istanbul, Turkey

ARTICLE INFO

Article history:
Received 14 November 2013
Received in revised form 6 February 2014
Accepted 16 February 2014
Available online 28 February 2014

Keywords:
Neural networks
Delayed systems
Lyapunov functionals
Stability analysis

ABSTRACT

This paper proposes a new alternative sufficient condition for the existence, uniqueness and global asymptotic stability of the equilibrium point for the class of delayed neural networks under the parameter uncertainties of the neural system. The existence and uniqueness of the equilibrium point is proved by using the Homomorphic mapping theorem. The asymptotic stability of the equilibrium point is established by employing the Lyapunov stability theorems. The obtained robust stability condition establishes a new relationship between the network parameters of the system. We compare our stability result with the previous corresponding robust stability results derived in the past literature. Some comparative numerical examples together with some simulation results are also given to show the applicability and advantages of our result.

1. Introduction

In recent years, dynamical neural networks have received a great deal of attention due to their potential applications in solving various classes of engineering problems such as image and signal processing, associative memory, pattern recognition, parallel computation, control and optimization. When neural networks are employed to solve practical engineering problems, their dynamics must exhibit some certain behaviors depending on the intended applications. Therefore, equilibrium and stability properties of neural networks are of great importance in the design of dynamical neural networks. For instance, if a neural network is designed to solve the problems in areas of control, optimization and signal processing, then, this neural network must have a unique and globally asymptotically stable equilibrium point in the presence of some certain inputs in order to avoid the risk of suboptimal responses. Therefore, it is important to design neural networks with desired stability properties. On the other hand, when a neural network is applied to solve a real time engineering problem on an electronically implemented neural network model-based chips, establishing the desired equilibrium and stability properties of neural networks becomes more important. It is known that in the VLSI implementation of neural networks, time delays are unavoidably encountered during the processing and transmission of signals, which can significantly affect the dynamical behaviors of neural networks. On the other hand, some deviations in the neural network parameters may happen due to the existence of modeling errors, external disturbance, and parameter fluctuations, which would cause the parameter uncertainties. Therefore, we must take into account the time delays and parameter uncertainties when studying stability of neural networks, in which case, one must deal with the robust stability of delayed neural networks. Recently, global robust stability of various classes of delayed neural networks have been extensively investigated. In particular, Chen, Cao, and Huang (2005); Deng, Hua, Liu, Peng, and Fei (2011); Ensari and Arik (2010); Guo and Huang (2009); Han, Kao, and Wang (2011); Huang, Li, Mohamad, and Lu (2009); Kwon and Park (2008); Li, Chen, and Huang (2007); Luo, Zhong, Wang, and Kang (2009); Mahmoud and Ismail (2010); Pan and Cao (2012); Raja and Samidurai (2012); Shao, Huang, and Wang (2011); Shen and Zhang (2007) Shengyuan et al. (2005), Singh (2007); Wang, Liu, Liu, and Shi (2010); Wu, Su, Chu, and Zhou (2009); Yang, Gao, and Shi (2009); Zhang, Liu, and Huang (2010); Zheng, Fei, and Li (2012); Zhou and Wan (2010); Zhu and Shen (2013) have studied the robust stability of neural networks with discrete and distributed with constant delays and time varying delays and obtained the stability conditions in terms linear matrix inequalities (LMIs). Robust stability of Cohen–Grossberg neural network models with discrete and distributed time delays has also been studied in Balasubramaniam and Ali (2010); Bao, Wen, and Zeng (2012); Huang (2011); Kao, Guo, Wang, and Sun (2012); Li (2009); Su and Chen (2009); Zhang, Wang, and Liu (2008); Zhang and Zhou (2009). In some recent papers (Faydasicok & Arik, 2012, 2013a, 2013b; Liao

E-mail addresses: ariks@istanbul.edu.tr, ariks@isikun.edu.tr.

http://dx.doi.org/10.1016/j.neunet.2014.02.008
0893-6080/© 2014 Elsevier Ltd. All rights reserved.

& Wong, 2004; Liao, Wong, Wu, & Chen, 2001; Li & Cao, 2004; Ozcan, 2011; Sun & Feng, 2003; Wang, Zhang, & Yu, 2007), robust stability of the class of neural networks with multiple time delays has been investigated and various sufficient conditions have been established for this type of stability. In case of multiple time delays, the stability results basically impose the M-matrix condition on the network parameters, or put some restraints on the norms of system matrices. In this paper, we present a new sufficient condition for the global robust asymptotic stability of neural networks with multiple time delays. We also make a detailed comparison of our new result with the previously reported corresponding results by giving some constructive numerical examples.

The class of neural networks with multiple time delays is described by the following set of ordinary differential equations

$$\frac{dx_i(t)}{dt} = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij})) + u_i, \quad i = 1, 2, \ldots, n \tag{1}$$

where n is the number of the neurons, $x_i(t)$ denotes the state of the neuron i at time t, $f_i(\cdot)$ denote activation functions, a_{ij} and b_{ij} denote the strengths of connectivity between neurons j and i at time t and $t - \tau_{ij}$, respectively; τ_{ij} represents the time delay required in transmitting a signal from the neuron j to the neuron i, u_i is the constant input to the neuron i, c_i is the charging rate for the neuron i.

The parameters $A = a_{ij}$ and $B = b_{ij}$ and $C = diag(c_i)$ of neural system (1) are assumed to be norm-bounded and satisfy the following conditions:

$$C_I = [\underline{C}, \overline{C}] = \{C = diag(c_i) : 0 < \underline{c}_i \le c_i \le \overline{c}_i, i = 1, 2, \ldots, n\}$$
$$A_I = [\underline{A}, \overline{A}] = \{A = (a_{ij})_{n\times n} : \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}, i, j = 1, 2, \ldots, n\} \tag{2}$$
$$B_I = [\underline{B}, \overline{B}] = \{B = (b_{ij})_{n\times n} : \underline{b}_{ij} \le b_{ij} \le \overline{b}_{ij}, i, j = 1, 2, \ldots, n\}.$$

The nonlinear activation functions f_i are assumed to be Lipschitz continuous and satisfy the following condition:

$$|f_i(x) - f_i(y)| \le \ell_i |x - y|, \quad i = 1, 2, \ldots, n, \ \forall x, y \in R, \ x \ne y$$

where $\ell_i > 0$ denotes a Lipschitz constant. This class of functions will be denoted by $f \in \mathcal{L}$.

The rest of this paper is organized as follows: In Section 2, some preliminaries are given. Section 3 presents the condition for the existence, uniqueness, and global robust stability of the equilibrium point for system (1). In Section 4, comparative numerical examples are given to illustrate the effectiveness of the proposed result and a comparison is made between our result and the previous literature results. The concluding remarks are given in Section 5.

2. Preliminaries

Throughout this paper, we will use the following notations: Let $v = (v_1, v_2, \ldots, v_n)^T$ be a real vector and $Q = (q_{ij})_{n\times n}$ be a real matrix. Then, $|v|$ will denote $|v| = (|v_1|, |v_2|, \ldots, |v_n|)^T$ and $|Q|$ will denote $|Q| = (|q_{ij}|)_{n\times n}$. The following norms will also be used:

$$\|v\|_1 = \sum_{i=1}^{n} |v_i|, \qquad \|v\|_2 = \left\{\sum_{i=1}^{n} |v_i|^2\right\}^{1/2},$$
$$\|v\|_\infty = \max_{1\le i\le n} |v_i|$$
$$\|Q\|_1 = \max_{1\le i\le n} \sum_{j=1}^{n} |q_{ji}|$$
$$\|Q\|_2 = [\lambda_{\max}(Q^T Q)]^{1/2}$$
$$\|Q\|_\infty = \max_{1\le i\le n} \sum_{j=1}^{n} |q_{ij}|.$$

The following lemma will play an important role in the proof of the main result of this paper:

Lemma 1. *Let A be any real matrix defined by*

$$A \in A_I = [\underline{A}, \overline{A}] = \{A = (a_{ij})_{n\times n} : \underline{a}_{ij} \le a_{ij} \le \overline{a}_{ij}, i, j = 1, 2, \ldots, n\}.$$

Then, for any two real vectors $x = (x_1, x_2, \ldots, x_n)^T$ and $y = (y_1, y_2, \ldots, y_n)^T$, the following inequality holds:

$$2x^T A y \le \beta \sum_{i=1}^{n} x_i^2 + \frac{1}{\beta} \sum_{i=1}^{n} p_i y_i^2$$

where β is any positive constant, and

$$p_i = \sum_{k=1}^{n} \left(\hat{a}_{ki} \sum_{j=1}^{n} \hat{a}_{kj}\right) \quad i = 1, 2, \ldots, n$$

with $\hat{a}_{ij} = max\{|\underline{a}_{ij}|, |\overline{a}_{ij}|\}, i, j = 1, 2, \ldots, n.$

Proof of Lemma 1. Let $A \in A_I$. Then, for any positive constant β and for any two real vectors $x = (x_1, x_2, \ldots, x_n)^T$ and $y = (y_1, y_2, \ldots, y_n)^T$, we can write

$$2x^T A y \le \beta x^T x + \frac{1}{\beta} y^T A^T A y$$

from which it can be derived that

$$\begin{aligned}
y^T A^T A y &= \sum_{i=1}^{n}\sum_{k=1}^{n} a_{ki}a_{ki} y_i^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\sum_{k=1}^{n} 2a_{ki}a_{kj} y_i y_j\right) \\
&\le \sum_{i=1}^{n}\sum_{k=1}^{n} |a_{ki}||a_{ki}| y_i^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\sum_{k=1}^{n} 2|a_{ki}||a_{kj}||y_i||y_j|\right) \\
&\le \sum_{i=1}^{n}\sum_{k=1}^{n} |a_{ki}||a_{ki}| y_i^2 + \sum_{i=1}^{n}\sum_{j=i+1}^{n}\left(\sum_{k=1}^{n} |a_{ki}||a_{kj}|(y_i^2 + y_j^2)\right) \\
&= \sum_{i=1}^{n}\sum_{k=1}^{n} |a_{ki}||a_{ki}| y_i^2 + \sum_{1=1}^{n}\left(\sum_{k=1}^{n} |a_{ki}| \sum_{\substack{j=1 \\ j\ne i}}^{n} |a_{kj}|\right) y_i^2 \\
&= \sum_{1=1}^{n}\left(\sum_{k=1}^{n} |a_{ki}| \sum_{j=1}^{n} |a_{kj}|\right) y_i^2
\end{aligned}$$

$A \in A_I$ implies that $|a_{ij}| \le \hat{a}_{ij}, i, j = 1, 2, \ldots, n$. Hence, we obtain

$$y^T A^T A y \le \sum_{1=1}^{n}\left(\sum_{k=1}^{n} |\hat{a}_{ki}| \sum_{j=1}^{n} |\hat{a}_{kj}|\right) y_i^2 = \sum_{i=1}^{n} p_i y_i^2$$

where

$$p_i = \sum_{k=1}^{n} \left(\hat{a}_{ki} \sum_{j=1}^{n} \hat{a}_{kj}\right), \quad i = 1, 2, \ldots, n.$$

Hence, it follows that

$$2x^T A y \le \beta \sum_{i=1}^{n} x_i^2 + \frac{1}{\beta} \sum_{i=1}^{n} p_i y_i^2.$$