Neural Networks 54 (2014) 85–94

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

# Impulsive synchronization schemes of stochastic complex networks with switching topology: Average time approach

Chaojie Li [a], Wenwu Yu [b], Tingwen Huang [c,*]

[a] Platform Technologies Research Institute, RMIT University, Melbourne, VIC 3001, Australia
[b] Department of Mathematics, Southeast University, Nanjing 210096, PR China
[c] Texas A&M University at Qatar, Doha, P.O. Box 23874, Qatar

ARTICLE INFO

*Article history:*
Received 1 November 2013
Received in revised form 29 January 2014
Accepted 27 February 2014
Available online 12 March 2014

*Keywords:*
Impulsive synchronization
Stochastic complex networks
Switching topology
Average dwell time
Average impulsive interval

ABSTRACT

In this paper, a novel impulsive control law is proposed for synchronization of stochastic discrete complex networks with time delays and switching topologies, where average dwell time and average impulsive interval are taken into account. The side effect of time delays is estimated by Lyapunov–Razumikhin technique, which quantitatively gives the upper bound to increase the rate of Lyapunov function. By considering the compensation of decreasing interval, a better impulsive control law is recast in terms of average dwell time and average impulsive interval. Detailed results from a numerical illustrative example are presented and discussed. Finally, some relevant conclusions are drawn.



## 1. Introduction

Consensus has received increasing attention since the collective dynamical behaviors of complex networks have been a subject of intensive research with potential applications in physical, social, biological, and technological fields. In general, complex networks are modeled by a graph with non-trivial topological features where every node is an individual element of the whole system with certain pattern of connections, which are neither entirely regular nor entirely random (Barabási & Albert, 1999; Strogatz, 2001; Watts & Strogatz, 1998). These features do not occur in the mathematical models of the networks that have been studied in the past, such as lattices or random graphs, but they do truly exist in nature. However, the phenomenon of synchronization of large populations is a challenging problem and requires different hypothesis to be solved. Synchronization processes in populations of locally interacting elements are in the focus of intensive research. The analysis of synchronizability has benefited not only from the advance in the understanding of complex networks, but also has it contributed to the understanding of general emergent properties of networked systems, such as the Internet, Human and Robot interaction networks, human collaboration networks, etc. As a consequence, many results for synchronization of different complex networks have been extensively studied in the physics and mathematics literature; e.g. Wang and Chen (2002a), Wu (2003), Olfati-Saber, Fax, and Murray (2007), Yu et al. (2009), Yu, Chen, and Lü (2009), Yu, Chen, Lü, and Kurths (2013), Wang, Wang, and Liu (2010), Wang, Ho, Dong, and Gao (2010), Liang, Wang, Liu, and Liu (2008), Liang, Wang, and Liu (2009), Zhang, Tang, Fang, and Wu (2012), Lu and Ho (2010), Lu, Ho, and Cao (2010), Mahdavi, Menhaj, Kurths, and Lu (2013), Shen, Wang, and Liu (2011), Shen, Wang, and Liu (2012), Gan (2012), Lü and Chen (2005), Wen, Bao, Zeng, and Huang (2013), Huang, Li, Yu, and Chen (2009), Wang and Chen (2002b), Chen, Liu, and Lu (2007), Liu, Lu, and Chen (2011), Guan, Wu, and Feng (2012), Yang, Cao, and Lu (2012), Liu, Lu, and Chen (2013), Hu, Yu, Jiang, and Teng (2012), Zhao, Hill, and Liu (2011) and references therein.

Theoretically, synchronization of a network is mainly contributed by the nodes' dynamical behaviors connections among the nodes. Many results have been devoted to the structural characterization and evolution of complex networks. In Wang and Chen (2002a), the authors studied robustness and fragility of synchronization of scale-free networks through the spectral properties of the underlying structure. In terms of graph based theoretical bounds to synchronizability, Wu (2003) mainly focused on the bounds of its extreme eigenvalues with graph. Global and local synchronization of coupled networks were discussed in Wen et al. (2013) and Yu et al. (2009) where different techniques were employed to address time delays in the system level. The authors

* Corresponding author. Tel.: +974 44230174.
*E-mail address:* tingwen.huang@qatar.tamu.edu (T. Huang).

http://dx.doi.org/10.1016/j.neunet.2014.02.013
0893-6080/© 2014 Elsevier Ltd. All rights reserved.

of Wang, Ho et al. (2010) and Wang, Wang et al. (2010) studied synchronization of discrete complex networks when the case involves randomly occurred nonlinearities and mixed time-delays. The bounded $H_\infty$ state estimation problem was studied in Shen et al. (2011) via RLMI method. In addition, synchronizability for general dynamical networks and fuzzy complex dynamical networks were discussed in Lu and Ho (2010) and Mahdavi et al. (2013).

Meanwhile, to design a control law for synchronization of complex network, many efforts have been made for this purpose. In the sense of continuous control law, Lü and Chen (2005) described a remarkable idea of a time-varying complex dynamical network model and designed its controlled synchronization criteria. Stability schemes of delayed neural networks were studied by Zhang et al. (2012) in which time-varying impulses have been modeled. A set of sampled-data synchronization controllers is designed by utilizing both the Gronwall's inequality and the Jenson integral inequality in Shen et al. (2012). Taking account into the consideration of discontinuous control approach, periodic intermittent controller (Gan, 2012) and intermittent controller (Hu et al., 2012) for synchronization of networks with time delays were investigated through different methods. Lu et al. (2010) first established a novel concept, namely average impulsive interval, for the synchronizability analysis of complex networks. By introducing intermittent linear state feedback, Huang et al. (2009) studied problems of cluster synchronization of linearly coupled complex networks and synchronization of delayed chaotic systems with parameter mismatches, respectively. Hybrid adaptive and impulsive control (Yang et al., 2012) was exploited for stochastic synchronization of complex networks. In addition, as known, certain complex networks display a scale-free feature, of which the connectivity distributions have the power-law form. The pinning scheme of the most highly connected nodes induced that a significant reduction in required controllers as compared to the traditional control scheme, see Wang and Chen (2002b). More general cases of pining control can be found in Yu et al. (2009), Yu et al. (2013). A single controller of pinning synchronization was designed, for example, without assuming symmetry, irreducibility, or linearity of the couplings, the authors (Chen et al., 2007) proved that a single controller can pin a coupled complex network to a homogeneous solution. By considering the average impulsive interval, some generic mean square stability criteria of synchronization control were derived in Lu et al. (2010). Obviously, these methods are effective for the synchronization control of different complex networks.

In the practical network environment, information exchange of each node through communication channels often experiences link failures and transmission delay, which can be modeled by a sequence of switching interconnections with time delays as shown in Liu et al. (2011) and Olfati-Saber et al. (2007). Synchronization problems with respect to switched systems were studied by typical techniques, for instance, the multiple $\upsilon$-Lyapunov function method (Zhao et al., 2011). As aforementioned in Lu et al. (2010) and Yang et al. (2012), by average impulsive interval, it can be derived that a unified synchronization criterion of complex networks is less conservative no matter desynchronizing or synchronizing impulses. Likewise, by average dwell time (Vu & Morgansen, 2010; Zhang & Shi, 2009), the stability criterion of switched system is less conservative. If we can obtain the result by virtue of the relationship between average impulsive interval and average dwell time, it would be much less conservative than the previous results. The research undertaken so far in order to understand how synchronization phenomena are affected by impulsive control law and the topological substrate of interactions, especially, when complex networks are involving noisy disturbances. The main goal of this paper is to examine the effect of time delays in switching topologies exerting on synchronization through Lyapunov–Razumikhin technique. Correspondingly, the major contributions in this paper are twofold. First, the relationship between average impulsive interval(AII) of control law and average dwell time(ADT) of switched topologies is established. Second, the average time approach presented in this paper offers attractive features that are potentially useful for controlling different categories of complex networks.

The paper is organized as follows. We first introduce the basic mathematical descriptions of discrete complex networks that will be used henceforth. Next, we focus on the synchronization analysis of discrete complex networks through impulsive control in the sense of average dwell time. The effect of time delays is tackled by Lyapunov–Razumikhin technique. Section 4 is devoted to the analysis of the conditions for the synchronization of complex networks in the sense of average interval time. The relationship is established by means of the results in Section 3. In Section 5, we present a numerical example and the discussion on the tradeoff between AII and ADT is given. Finally, the last section rounds off the paper by giving our conclusions.

## 2. Preliminary

Let $\mathbb{R}^d$ denote the $d$-dimensional Euclidean space and $\| \bullet \|$ be the Euclidean norm in $\mathbb{R}^d$. Denote $\mathbb{Z}_+ = \{1, 2, \ldots\}$, $\mathbb{Z}_\tau = \{-\tau, -\tau+1, -\tau+2, \ldots, -1, 0\}$, the finite set $\mathscr{H} = \{1, 2, \ldots, H\}$ with finite positive integer $H$, $\mathscr{N} = \{0, 1, 2, \ldots\}$, $I$ be the identity matrix, and matrix $X > 0$ $(<, \geq, \leq)$ means that $X$ is a symmetric positive definite matrix (negative definite, positive semi-definite, negative semi-definite, respectively). Denote the maximum eigenvalue and minimum of the matrix by $\lambda_{\max}(\bullet)$ and $\lambda_{\min}(\bullet)$. The superscript $T$ stands for matrix transposition. The Kronecker product of matrices $Q \in \mathbb{R}^{m\times n}$and $P \in \mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ denoted as $Q \otimes P$. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The notation $\lceil x \rceil$ stands for the minimal integer not less than $x$. $A \doteq B$ means $A$ is denoted by $B$. As the definition in Liang et al. (2009, 2008), let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all -null sets and is right continuous) and by Brownian motion$\{\omega(s) : 0 \leq s \leq t\}$, where we associate $\Omega$ with the canonical space generated by $\omega(t)$, and denote $\mathcal{F}$ the associated $\sigma$-algebra generated by $\omega(t)$ with all the probability measure $\mathcal{P}$. $L^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^d)$ is the family of all $\mathcal{F}_0$-measurable $C([-\tau, 0], \mathbb{R}^d)$-valued random variable. $E\{\bullet\}$ stands for the mathematical mean expectation operator with respect to the given probability measure $\mathcal{P}$.

Consider the following stochastic discrete time-varying complex networks with $N$ identical nodes involving time delays and switching topology:

$$\mathbf{x}_i(k+1) = f(k, \mathbf{x}_i(k)) + \sum_{j=1}^{N} l_{ij.\sigma(k)} \Gamma \mathbf{x}_j(k - \tau(k)) + g_i(k, \mathbf{x}_i(k))\omega(k), \quad (1)$$

where $\mathbf{x}_i(k) = (x_{i,1}(k), x_{i,2}(k), \ldots, x_{i,d}(k))^T \in \mathbb{R}^{Nd}$ represents the state vector of the $i$th node at each instant of time $k$ and $d$ denotes the number of nodes affiliated to each sub-networks. $f : \mathbb{Z}_+ \times \mathbb{R}^d \to \mathbb{R}^d$ is a nonlinear vector function which stands for different dynamical behavior of each node. $\Gamma = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_d\} > 0$ is the inner coupling matrix between two connected nodes. $\tau(k)$ is a nonnegative constant which represents the time-delay of the signal transmitted from the network to the $i$th node, where the coupling time-delay $\tau(k)$ satisfies $0 \leq \tau(k) \leq \tau$ for a constant $\tau > 0$. $g_i : \mathbb{Z}\times\mathbb{R}^d \to \mathbb{R}^d$ is the noise intensity function vector. $\omega(k)$ is a scale Wiener process (Brownian motion) defined on $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\mathbb{E}\{\omega(k)\} = 0, \qquad \mathbb{E}\{\omega(k)^2\} = 1,$$
$$\mathbb{E}\{\omega(i)\omega(j)\} = 0, \quad (i \neq j). \quad (2)$$