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Delay-distribution-dependent state estimation for discrete-time stochastic neural networks with random delay

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ABSTRACT

This paper is concerned with the state estimation problem for a class of discrete-time stochastic neural networks (DSNNs) with random delays. The effect of both variation range and distribution probability of the time delay are taken into account in the proposed approach. The stochastic disturbances are described in terms of a Brownian motion and the time-varying delay is characterized by introducing a Bernoulli stochastic variable. By employing a Lyapunov–Krasovskii functional, sufficient delay-distribution-dependent conditions are established in terms of linear matrix inequalities (LMIs) that guarantee the existence of the state estimator which can be checked readily by the Matlab toolbox. The main feature of the results obtained in this paper is that they are dependent on not only the bound but also the distribution probability of the time delay, and we obtain a larger allowance variation range of the delay, hence our results are less conservative than the traditional delay-independent ones. One example is given to illustrate the effectiveness of the proposed result.

1. Introduction

Various classes of neural networks have been increasingly studied in the past few decades, due to their practical importance and successful applications in many areas such as combinatorial optimization, signal processing and communication (Elanayar & Shin, 1994; Fantacci, Forti, Marini, & Pancani, 1999; Haykin, 1998; Joya, Atencia, & Sandoval, 2002). These applications greatly depend on the dynamic behaviors of the underlying neural networks. In reality, time-delay systems are frequently encountered in various areas, and a time delay is often a source of instability and oscillators in a system. So, dynamics in a neural network often have time delays due to the finite switching speed of amplifiers in electronic neural networks or to the finite signal propagation time in biological networks. As a result, delay-independent and delay-dependent sufficient conditions have been proposed to verify the asymptotical or exponential stability of delayed neural networks.

In recent years, the dynamic analysis of discrete-time neural networks (DNNs) have received increasing research interest than their continuous-time counterpart when implementing the control laws in a digital way (Gao, Lam, Wang, & Wang, 2004; Tang, Fang, Xia, & Yu, 2009; Wang, Wei, & Feng, 2009; Wang, Liu, Wei, & Liu, 2010; Wang, Wang, & Liu, 2010). In reality, the synaptic transmission is a noisy process brought on by random fluctuation from the release of neuron transmitters and other probabilistic causes in real nervous systems (Gao et al., 2004; Sun, Cao, & Wang, 2007). So stochastic perturbation should be considered when investigating DNNs.

It is worth pointing out that when investigating discrete-time stochastic neural networks (DSNNs) only the deterministic time-delay case was concerned, and the stability criteria were derived based only on the information of variation range of the time delay. Actually, the time delay in some neural networks is often existent in a stochastic fashion (Blythe, Mao, & Liao, 2001; Hirasawa, Mabu, & Hu, 2006; Hopfield, 1982; Ray, 1994; Tang et al., 2009; Wang, Yang, Ho, & Liu, 2006; Wang, Wei et al., 2009; Wang, Liu et al., 2010; Wang, Wang et al., 2010; Yue, Zhang, Tian, & Peng, 2008; Zhang, Yue, & Tian, 2009), and its probabilistic characteristic, such as Poisson distribution or normal distribution, can often be obtained by statistical methods. It often occurs in real systems that some values of the delay are very large but the probabilities of the delay taking such large values are very small. In this case, if only the variation range of time delay is employed to derive the criteria, the results may be somewhat more conservative. Hence, DSNNs with random delays should be considered.

On the other hand, as pointed out in Wang, Ho, and Liu (2005), the neuron states are not often completely available in the network output in many applications. Therefore, the state estimation problem of neural networks becomes significant for many applications (such as Habtom and Litz (1997), He, Wang, Wu, and Lin (2006), Liang, Wang, and Liu (2009), Liu, Wang, and Liu (2007), Liu, Wang, Liang, and Liu (2008a), Liu, Wang, and Liu (2008b), Mou, Gao, Qiang, and Fei (2008), Salam and Zhang

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(2001), Wang et al. (2005) and Wang, Liu, and Liu (2009)). The main objective of the problem is to estimate the neuron states through available output measurements such that the dynamics of the error-state system is globally stable. Hence, it is desirable to study the state estimation problem of DSNNs with random delays. Recently, some papers have been engaged in the issue of state estimation of networks (Boyd, Ghaoui, Feron, & Balakrishnan, 1994; Elanayar & Shin, 1994; Habtom & Litz, 1997). Wang et al., laid fundamental works for state estimation problem of neural networks (Liang et al., 2009; Liu et al., 2007, 2008a, 2008b; Wang et al., 2005; Wang, Liu et al., 2009). Wang et al. firstly investigated the state estimation problem for neural networks with time-varying delays in Wang et al. (2005). In He et al. (2006), the problem of state estimation was addressed for delayed neural networks under a weak assumption that the time-varying delay was required to be differentiable. However, the proposed condition was expressed in terms of a matrix inequality, not an LMI, which corresponds to a nonlinear programming problem. The authors in Liu et al. (2007) and Wang, Liu et al. (2009), dealt with the state estimation problem for a class of neural networks with discrete and distributed delays. In Liang et al. (2009) and Liu et al. (2008a, 2008b), the sufficient conditions are formulated in terms of LMIs. However, the stochastic disturbance and random delay are not taken into account when dealing with the state estimation problem.

Motivated by the above discussion, the state estimation problem for a class of discrete-time stochastic neural networks (DSNNs) with random delays will be considered in this paper. By referring to the model of DSNNs in Yue et al. (2008) and the idea in Liang et al. (2009) and Wang, Liu, and Liu (2008), an estimator is designed to approximate the neuron states through available output measurement. The effect of both variation range and distribution probability of the time delay are taken into account in the proposed approach which is mainly different from the traditional methods and will lead to less conservative results and our results take some well studied models as special cases. We translate the distribution probability of the time delay into parameter matrices of the transferred systems. In the established model, the stochastic disturbances are described in terms of a Brownian motion and the time-varying delay is characterized by introducing a Bernoulli stochastic variable. By employing a Lyapunov–Krasovskii functional mild assumption, and a stochastic analysis technique, sufficient delay-distribution-dependent conditions are established in terms of linear matrix inequalities (LMIs) that guarantee the existence of the state estimator which can be checked readily by the Matlab toolbox.

The remainder of this paper is organized as follows. In Section 2 the model formulation and some preliminaries are given. The main results are stated in Section 3. One illustrate example is given to demonstrate the effectiveness of the proposed results in Section 4. Finally, the conclusions are presented in Section 5.

2. Notations and preliminaries

In this section, some elementary notations and lemmas are introduced which play an important role in the proof of the main result in Section 3.

Notation: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$, respectively, denote the n-dimensional Euclidean and the set of all $n \times m$ matrices. I is the identity matrix of appropriate dimensions. The superscript "T" denotes matrix transposition. The notation $X > 0$ (respectively, $X \geq 0$), where X is a real symmetric matrix, means X is positive definite (respectively, positive semi-definite). $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, $\|A\|$ denotes its operator norm. i.e. $\|A\| = \sup\{|Ax| : |x| = 1\} = \sqrt{\lambda_{\max}(A^TA)}$ where $\lambda_{\max}(A)$ (respectively $\lambda_{\min}(A)$) means the largest (respectively, smallest) eigenvalue of A. $\mathbb{Z}_{\geq 0}$ denotes the set including zero and positive integers. The asterisk $*$ in a matrix is used to denote term that is induced by symmetry. $\mathbb{E}\{\cdot\}$ denotes the expectation. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions. Denote by $\mathcal{L}^2_{\mathcal{F}_0}([-\tau, 0], \mathbb{R}^n)$ the family of all $\mathcal{F}_0$-measurable $C([-\tau, 0] : \mathbb{R}^n)$-valued random variables $\phi = \{\phi(s), -\tau \leq s \leq 0\}$ with the norm $\|\phi\| = \sup_{-\tau\leq s\leq 0} |\phi(s)|^2 < \infty$.

Consider the following n-neuron DSNNs with time delay:

$$x(k+1) = Ax(k) + B\tilde{F}(x(k)) + D\tilde{G}(x(k-\tau(k))) + \tilde{\sigma}(k, x(k))\omega(k) + J, \qquad (1)$$

where $x(k) = (x_1(k), x_2(k), \ldots, x_n(k))^T \in \mathbb{R}^n$ is the state vector associated with the n neurons, $A = \operatorname{diag}(a_1, a_2, \ldots, a_n)$ with $|a_i| < 1$; $B = (b_{ij})_{n\times n}$, $D = (d_{ij})_{n\times n}$ denote the connection weights matrix and the delayed connection weights matrix, respectively; $\tilde{F}(x(k)) = [\tilde{f}_1(x_1(k)), \tilde{f}_2(x_2(k)), \ldots, \tilde{f}_n(x_n(k))]^T$ and $\tilde{G}(x(k)) = [\tilde{g}_1(x_1(k)), \tilde{g}_2(x_2(k)), \ldots, \tilde{g}_n(x_n(k))]^T$ denote the neuron activation functions; $\tau(k)$ denotes the time-varying delay, $J = (J_1, J_2, \ldots, J_n)^T$ is an external input vector. $\tilde{\sigma} : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a continuous function, $\omega(k)$ is a scalar Wiener process on a probability space $(\Omega, \mathscr{F}, \mathcal{P})$ with

$$\mathbb{E}\{\omega(k)\} = 0, \qquad \mathbb{E}\{\omega^2(k)\} = 1, \qquad \mathbb{E}\{\omega(i)\omega(j)\} = 0 \quad (i \neq j).$$

Throughout this paper, the neuron activation functions and time-delay are assumed to satisfy the following assumptions:

Assumption 1 (*Liu, Wang, & Liu, 2006; Wang, Shu, Liu, Ho, & Liu, 2006*)**.** For $i \in \{1, 2, \ldots, n\}$, the neuron activation functions $f_i(\cdot)$ and $g_i(\cdot)$ are continuous and bounded, and satisfy the following conditions

$$\begin{aligned} &l_i \leq \frac{\tilde{f}_i(s_1) - \tilde{f}_i(s_2)}{s_1 - s_2} \leq L_i \\ &\omega_i \leq \frac{\tilde{g}_i(s_1) - \tilde{g}_i(s_2)}{s_1 - s_2} \leq W_i, \\ &\quad \forall s_1, s_2 \in \mathbb{R}(s_1 \neq s_2)\ i = 1, 2, \ldots, n \\ &\tilde{f}_i(0) = \tilde{g}_i(0) = 0, \quad i = 1, 2, \ldots, n. \end{aligned} \qquad (2)$$

Remark 1. This assumption was first proposed in Liu et al. (2006) and Wang, Shu et al. (2006), and has been subsequently studied in many recent neural networks papers. The constants l_i, L_i, ω_i, W_i in Assumption 1 are allowed to be positive, negative, or zero, so the conditions in Assumption 1 are more general than the usual sigmoid functions and Lipschitz conditions. Such a description is very precise in quantifying the lower and upper bounds of the activation functions, therefore it is very effective for employing LMI method to reduce the possible conservatism.

Assumption 2. $\tilde{\sigma}(k, x(k))$ is the continuous function satisfying $\tilde{\sigma}(0, 0) = 0$,

$$[\tilde{\sigma}(k, x(k)) - \tilde{\sigma}(k, y(k))]^T[\tilde{\sigma}(k, x(k)) - \tilde{\sigma}(k, y(k))] \leq \rho(x(k) - y(k))^T(x(k) - y(k))$$

where $\rho > 0$ is a known constant scalar.

Assumption 3. The time delay $\tau(k)$ is bounded, $0 \leq \tau_m < \tau(k) \leq \tau_M$, and its probability distribution can be observed, i.e., suppose $\tau(k)$ takes values in $[\tau_m : \tau_0]$ or $(\tau_0 : \tau_M]$ and $\text{Prob}\{\tau(k) \in [\tau_m : \tau_0]\} = \delta_0$, where $\tau_m \leq \tau_0 < \tau_M$, and $0 \leq \delta_0 \leq 1$.