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Elastic energy in locomotion: Spring-mass *vs.* poly-articulated models

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ABSTRACT

The human is often modeled as a Poly-Articulated Model (PAM) with rigid segments while some authors use a Spring Mass Model (SMM) for modeling locomotion. These two models are considered independent, and the objective of this study was to link them in order to enlighten the origin of the elasticity in locomotion.

Using the characteristics of the two models, a theoretical relationship demonstrates that the variation of elastic energy of the SMM equals the variation of the internal kinetic energy minus internal forces work of the PAM. This theoretical relationship was experimentally investigated among 19 healthy participants walking and running on a treadmill.

The results showed that the equality is verified except during the double support phase at 0.56 m s^{-1}, at high walking speeds (1.67 and 2.22 m s^{-1}) or during the aerial phase of running.

The formal relationship showed that the global stiffness of the SMM is directly related to the work of the internal forces of the PAM, and thus, to the characteristics of the musculoskeletal system. It also showed the relevance of taking into account the participation of each joint in the global stiffness. Finally, the coordination of internal forces work to produce a global stiffness may be considered as a new criterion of movement optimization for clinical purposes or motion planning for humanoid robots.

1. Introduction

The human locomotion can be modeled as a Spring Mass Model (SMM, Fig. 1) for both walking [1,2] and running [3]. The SMM is represented as a body mass at the center of mass (CoM) oscillating at the end of a massless spring (Fig. 1). This model has been originally promoted for running gait since it takes into account elastic energy, which seems to play an important role in the mechanical energy conservation [4]. Therefore, the use of this model to characterize walking suggests an equivalent role of elastic energy in this locomotion mode. Indeed, the SMM is a conservative system inducing no change in mechanical energy (${E_M}^{SMM}$, Eq. (1)) that can be calculated as in Eq. (2).

$$\Delta {E_M}^{SMM} = 0 \tag{1}$$

$${E_M}^{SMM} = {E_{Kext}}^{SMM} + {E_P}^{SMM} + {E_E}^{SMM} \tag{2}$$

With ${E_{Kext}}^{SMM}$ the kinetic energy of the CoM according to Duboy et al. [5], ${E_P}^{SMM}$ the potential energy due to gravity, and ${E_E}^{SMM}$ the elastic energy which is dependent of a global constant stiffness k. This constant stiffness k is computed from different ways [6–8]. The SMM predicts the displacement of the whole body CoM only and takes into account an elastic component. Although it highlights the basic mechanisms of the locomotion and reduces the mechanical parameters taken into account, the CoM trajectory is depending upon the segment masses and locations.

On the other hand, the human body is modeled as a Poly-Articular Model (PAM, Fig. 1), i.e. as a poly-articulated system of n rigid segments $S_i(i\varepsilon[1,n])$, with each a center of mass G_i and a mass m_i (Fig. 1). The PAM predicts the displacement of all the segments of the human body. Although the elastic component of the musculoskeletal system is involved and taken into account in the force and work production [9], the PAM does not take into account any elastic energy. The interest of this model is to simplify the model of the human body in order to measure relevant mechanical

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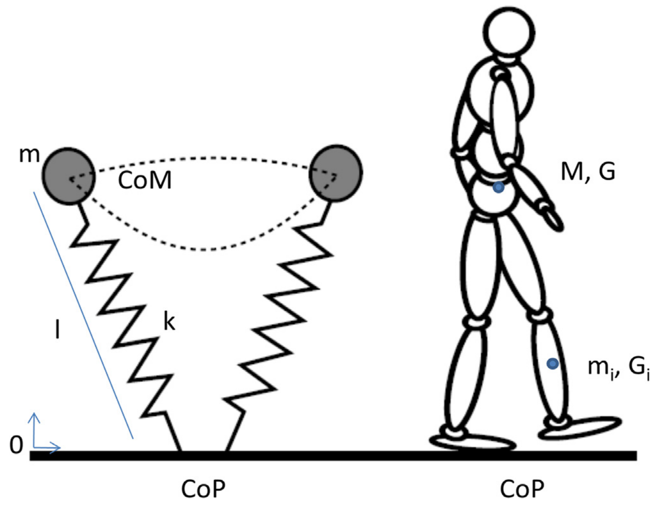

Fig. 1. Spring Mass Model (SMM, with m, the mass; CoP, the center of pressure; CoM, the center of mass; k, the spring stiffness; l, the distance between CoP and CoM) and Poly-Articular Model (PAM, with; CoP, the center of pressure, M, the total mass; G, the center of gravity; m_i and G_i the mass and the center of mass of the segment, respectively) including 16 segments (S_i) used for walking and running gaits.

parameters for locomotion studies, like the variation of mechanical energies of each body segment [10], the mechanical cost of movement [11], the joint torques with inverse dynamics [12–14] and the kinematic parameters (e.g. 3D body segment orientation and translation). From a gait cycle to another, the mechanical energy is the same whereas it varies throughout the movement because of internal forces work [10]. According to the mechanical energy theorem (Eq. (3)), the mechanical energy variation of the PAM results from the work of non conservative forces ($W_{Fnc}{}^{PAM}$). Assuming that the liaison between the foot and the ground is not dissipative and the weight is conservative, the $W_{Fnc}{}^{PAM}$ represents the internal forces work ($W_{Fint}{}^{PAM}$).

$$\Delta E_M{}^{PAM} = W_{Fnc}{}^{PAM} = W_{Fint}{}^{PAM} \tag{3}$$

The mechanical energy of the PAM can be computed as in Eq. (4).

$$E_M{}^{PAM} = E_{Kext}{}^{PAM} + E_P{}^{PAM} + E_{Kint}{}^{PAM} \tag{4}$$

With $E_{Kint}{}^{PAM}$ the kinetic energy of the body segments in the barycentric coordinate system according to Duboy et al. [5], decomposed from the 2nd König's theorem.

Both of these models have their own advantages (Fig. 1): (i) the SMM predicts the displacement of the body CoM and considers elastic energy without taking into consideration the translational and rotational energies of the body segments, and (ii) the more complex PAM takes into account the body segment energies without considering elastic energy. However, as explained, both are aware to model gait [3,10].

These two models are generally presented as independent in the literature,and the objective of this paper is to establish a link between them. By applying the theorem of the mechanical energy to the SMM and the PAM for the same movement, we obtain:

$$\begin{gathered}\Delta\left(E_{Kext}{}^{SMM} + E_P{}^{SMM} + E_E{}^{SMM}\right) = 0\\ \Delta\left(E_{Kext}{}^{PAM} + E_P{}^{PAM} + E_{Kint}{}^{PAM}\right) = W_{Fint}{}^{PAM}\end{gathered} \tag{5}$$

Two terms are common to both the equations: ΔE_{Kext} and ΔE_P. By assuming their equality, the link between the SMM and the PAM can be established as follows:

$$\Delta E_E{}^{SMM} = \Delta E_{Kint}{}^{PAM} - W_{Fint}{}^{PAM} \tag{6}$$

Finally, the variation of the elastic energy ($\Delta E_E{}^{SMM}$) represents the variation of the kinetic energy of the body segment in the barycentric coordinate system ($\Delta E_{Kint}{}^{PAM}$) minus the work of the internal forces ($W_{Fint}{}^{PAM}$). The energy balance being considered during one gait cycle, the comparison of SMM and PAM supposes that the mechanical work of the internal forces balances the dissipation/storage and generation/restitution of the energy to zero over the whole gait cycle.

By this way, we should experimentally verify Eq. (6) and then determine the link between SMM & PAM, which are generally used separately in the literature. The goal of the study is therefore to experimentally investigate both side of Eq. (6) and verify the equality over a gait cycle.

2. Methods

2.1. Population

Nineteen healthy men volunteered (23 ± 5 y; 1.79 ± 0.07 m; 80.7 ± 11 kg) for this experimentation. They were equipped with 42 reflective markers recorded by twelve optoelectronic cameras sampled at 200 Hz (VICON, Oxford's metrics, Oxford, UK). The participants performed barefoot walking and running tests on a treadmill (PF 500 CX, PRO FORM, Villepreux, France) embed on a large force platform recording at 1 kHz (AMTI, Watertown, MA, USA). The kinematic and kinetic data were filtered with 4th order zero lag Butterworth filters with a cut off frequency of 6 Hz and 10 Hz [15], respectively

2.2. Experimentation

To induce dynamic similarity between the participants, the speed and frequency were determined according to Froude ($Fr = v^2/gl$; with v the speed, g the gravity and l the CoM height) and Strouhal ($Str = fl/v$; with f the step frequency) combination as suggested by Villeger et al. [2] for walking and Villeger et al. [16] for running.

Firstly, the participants were asked to walk and run with their preferred step frequency at 0.56, 1.11, 1.67, 2.22 m.s^{-1} and 1.67, 2.22, 2.78, 3.33, 3.89, 4.44 m.s^{-1}, respectively. From these tests, a mean of Fr ($\overline{Fr}$) and Str ($\overline{Str}$) was computed for each speed stage.

Secondly, similar speed and similar step frequency were imposed to each subject j at each speed stage (Eq. (7) and Eq. (8)).

$$v_{sim_j} = \sqrt{\overline{Fr} \cdot g \cdot l_j} \tag{7}$$

$$f_{sim_j} = \left(\overline{Str} \cdot v\right)/l_j \tag{8}$$

Only these similar conditions (v_{sim}, f_{sim}) were treated in the present study.

2.3. Assessed parameters

In this study, the human body was considered as a whole of 16 rigid body segments [17]. The functional centers of rotation of the hips and the shoulders were determined with the SCoRE method [18].

The walking and running trials were performed on a treadmill. While the belt induces a foot translation during the stance phase, we have to consider that the horizontal component of the GRF is