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Stability criteria for Markovian jump neural networks with mode-dependent additive time-varying delays via quadratic convex combination

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ABSTRACT

This paper is mainly concerned on stability problem of Markovian jump neural networks with mode-dependent two additive time-varying delays based on quadratic convex combination approach. The jumping parameters are modeled as a continuous time, finite state Markov chain. By constructing a suitable augmented Lyapunov–Krasovskii functional, utilizing the Jensen's inequality, the idea of second order convex combination and the property of quadratic convex function, the sufficient conditions are derived to guarantee that the proposed neural networks are globally asymptotically stable. Moreover, these stability criteria are expressed in terms of linear matrix inequalities, which can be efficiently solved via the standard numerical packages. Finally, the numerical examples are given to validate the less conservatism and effectiveness of the theoretical results.

1. Introduction

During the past decades, Neural Networks (NNs) have been extensively studied and have also found many applications in various fields, such as image processing, pattern recognition, signal processing, combinatorial optimization, power systems, associative memory, and so on (for example [1–5]). All of these applications tediously depend on the dynamical characteristics. So the stability is an important property to many systems [6–11]; much effort has been done to study the stability problem of NNs with time delays because the existence of time delays may cause the system like instability and oscillation of NNs. So there exist several results on stability of NNs with either constant or time-varying delays [6–8,10,11,13–17,21–26].

Meanwhile, a new type of time-varying delay with two additive components in the state of NNs are introduced in [12]. Such a system may be encountered in many practical situations such as remote control and networked control system. For example, in networked controlled systems, signals transmitted from one point to another may experience a few segments of networks, which can possibly induce successive delays, one from the sensor to the controller and the other from the controller to the actuator, having different properties due to the variable network transmission conditions. This implies that the system with additive time-varying delays become more complicated and very interesting. Therefore, a great number of researchers investigated the system with additive time-varying delays (for example [13–17]). In [13], the authors investigated the synchronization of singular Markovian jumping complex dynamical networks with two additive time-varying delay components using the pinning control. In [14], the authors studied the problem of exponential synchronization of complex dynamical networks with two additive time-varying delay components and control packet loss using the stochastic sampled-data control. The authors in [17] analyzed the stability criteria for continuous time systems with additive time-varying delays.

In the real world, the NNs may exhibit the network mode jumping characteristic. Such jumping can be determined by the Markov chain. Recently, NNs with Markovian jump parameters have received much interest among researchers. This class of NNs is recognized as the best system to model the phenomenon of information latching and the abrupt phenomena, such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections, and so forth. To deal with this situation, the authors in [18–20] considered the model of NNs with Markovian jumping parameters, which are also called Markovian jump neural networks (MJNNs). Also these papers give the extensive applications of such models in manufacturing systems,

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power systems, actuator saturation, and communication systems and network-based control systems. Thus MJNNs is a hybrid system with two components $x(t)$ and $r(t)$. Here $x(t)$ is referred as the state, which is described by a differential equation and the $r(t)$ is referred as the mode. In its operation, this class of systems will switch from one mode to another mode in a random way and it is also governed by a continuous time Markov chain with a finite state space $S=\{1,2,\dots,N\}$. Therefore, it is important to study the dynamic behaviors of neural networks with Markovian jumping parameters and mode-dependent time-varying delays (for example [21–26]).

In [21], the authors discussed about the robust stochastic convergence for an uncertain Markovian jumping Cohen–Grossberg NNs with mode-dependent time-varying delays. The delay dependent stochastic stability criteria are studied in [23] for MJNNs with mode-dependent time-varying delays and partially known transition rates. In [25], the problem of asymptotic stability of MJNNs with randomly occurring nonlinearities is investigated in the mean square sense. In [26], the robust exponential stability of Markovian jumping stochastic Cohen–Grossberg NNs with mode-dependent probabilistic time-varying delays and continuously distributed delays are studied by using the impulsive perturbations.

Motivated by the above discussion, in this paper, we investigate the global asymptotic stability for MJNNs with mode-dependent two additive time-varying delays. At first, we construct a new augmented LKF terms like as $\int_{t-\overline{\tau}_1}^{t}\eta_1^T(t,s)H_1\eta_1(t,s)ds$, $\int_{t-\overline{\tau}_1}^{t}(\overline{\tau}_1-t+s)\eta_2^T(s)H_2\eta_2(s)ds$, $\int_{t-\overline{\tau}_1}^{t}(\overline{\tau}_1-t+s)^2\dot{x}^T(s)H_3\dot{x}(s)ds$. Here, the quadratic terms $\eta_2^T(s)H_2\eta_2(s)$ and $\dot{x}^T(s)H_3\dot{x}(s)ds$ are multiplied by the scalar function $(\overline{\tau}_1-t+s)$ and $(\overline{\tau}_1-t+s)^2$ respectively. By using Jensen's inequality and some new integral inequalities to solve these LKFs. Also, we will claim that the function

$$\zeta^T(t)[\Omega_0+\tau_{1i}(t)\Omega_1+\Omega_d]\zeta(t)<0,\quad \forall\tau_{1i}(t)\in[0,\ \overline{\tau}_1]$$

is a quadratic convex combination on $\tau_{1i}(t)$. Then the sufficient conditions are employed in terms of LMIs which ensuring the globally asymptotically stable of the proposed NNs. Finally, the effectiveness of theoretical results is validated by the numerical examples. However, to the best of our knowledge, until now there are no results on the stability problem of MJNNs with mode-dependent two additive time-varying delays based on the quadratic convex combination approach.

The outline of this paper is organized as follows: the NNs model is introduced and some necessary lemmas are given in Section 2. Section 3 includes the stability problem of MJNNs with mode-dependent two additive time-varying delays based on quadratic convex combination approach. Section 4 provides numerical examples to illustrate the effectiveness of the theoretical results. Finally, the conclusion is given in Section 5. To ease the analysis, let us provide the following notations.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. $Sym(M)$ is defined as $Sym(M)=M+M^T$. The superscript T denotes the transposition. The notation $M\geq 0$ (similarly, $M\leq 0$) denotes that M is a positive semi-definite matrix (similarly, negative semi-definite matrix). The identity and zero matrices of appropriate dimensions are denoted by I and 0, respectively. $x_t:=\{x(t+s): s\in[-\overline{\tau},0]\}$. The notation $*$ in a block matrix always represents the symmetric terms.

2. Problem formulation

Let $\{r(t),t\geq 0\}$ is a right-continuous Markov chain on a complete probability space $(\Omega,\mathcal{F},P)$ taking values in a finite space $S=\{1,2,\dots,N\}$ with operator $\Gamma=(\pi_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t)=j\,|\,r(t)=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j,\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j,\end{cases}$$

where $\Delta t>0$ and $\lim_{\Delta t\to 0}o(\Delta t)/\Delta t=0$, $\pi_{ij}\geq 0$ is the transition rate from i to j, if $i\neq j$ while $\pi_{ii}=-\sum_{j=1,j\neq i}^{N}\pi_{ij}$.

In this paper, we consider the following MJNNs with mode-dependent additive time-varying delays:

$$\dot{x}(t)=-A(r(t))x(t)+B(r(t))f(x(t))+C(r(t))f(x(t-\tau_1(t,r(t))-\tau_2(t,r(t)))).\tag{1}$$

In the forthcoming, for simplicity, let $r(t)=i$. Then $A(r(t))=A_i$, $B(r(t))=B_i$, $C(r(t))=C_i$, $\tau_1(r(t))=\tau_{1i}$, $\tau_2(r(t))=\tau_{2i}$, with this system (1) can be rewritten as

$$\begin{cases}\dot{x}(t)=-A_ix(t)+B_if(x(t))+C_if(x(t-\tau_{1i}(t)-\tau_{2i}(t)))\\ x(t)=\phi(t)\quad t\in[-\overline{\tau},0],\end{cases}\tag{2}$$

where $x(t)=[x_1(t),\ x_2(t),\ \dots,x_n(t)]^T\in\mathbb{R}^n$ is the neural state vector, $A_i(i=1,2,\dots,N)$ is a positive diagonal matrix, B_i and C_i are the connection weight matrix and delayed connection weight matrix respectively. $f(x(t))=[f_1(x_1(t)),\ f_2(x_2(t)),\dots,f_n(x_n(t))]^T\in\mathbb{R}^n$ denotes the neuron activation functions. $\tau_{1i}(t)$, $\tau_{2i}(t)$ represents two different mode-dependent time-varying delays, which satisfies,

$$0\leq\tau_{1i}(t)\leq\tau_{1i},\quad 0\leq\tau_{2i}(t)\leq\tau_{2i},\quad \dot{\tau}_{1i}(t)\leq\mu_{1i},\quad \dot{\tau}_{2i}(t)\leq\mu_{2i}.$$

Let $\overline{\tau}_1=\max_{i\in S}\{\tau_{1i}\}$, $\overline{\tau}_2=\max_{i\in S}\{\tau_{2i}\}$. Here τ_{1i}, τ_{2i}, μ_{1i}, μ_{2i} are known constants. $\phi(t)$ is an initial condition with $t\in[-\overline{\tau},0]$, where $\overline{\tau}=\overline{\tau}_1+\overline{\tau}_2$. Throughout this paper, we make the following assumption:

Assumption 2.1. Each neuron activation function $f_l(\cdot)$, $l=1,2,\dots,n$, in system (2), satisfies the following condition:

$$\underline{k}_l\leq\frac{f_l(u)-f_l(v)}{u-v}\leq\overline{k}_l,$$

where $f_l(0)=0$, $\forall u,\ v\in\mathbb{R}$, $u\neq v$, and $\underline{k}_l$, $\overline{k}_l$ are real constants, $l=1,2,\dots,n$.

The following technical well-known propositions will be useful in the succeeding discussion.

Lemma 2.2 (Rakkiyappan et al. [26] (Schur complement)). *Given constant matrices A, B and C with appropriate dimensions, where $A^T=A$ and $B^T=B>0$ then $A+C^TB^{-1}C<0$ if and only if*

$$\begin{bmatrix}A & C^T\\ * & -B\end{bmatrix}<0,\quad\text{(or)}\quad\begin{bmatrix}-B & C\\ * & A\end{bmatrix}<0.$$

Lemma 2.3 (Dharani et al. [30]). *For any constant matrix $X\in R^{n\times n}$, $X=X^T>0$, two scalars a and b, $a<b$ such that the integrations concerned are well defined, then the following inequalities holds:*

$$\frac{-(b-a)^2}{2}\int_a^b\int_\theta^b x^T(s)Xx(s)dsd\theta\leq-\left(\int_a^b\int_\theta^b x(s)dsd\theta\right)^T X\left(\int_a^b\int_\theta^b x(s)dsd\theta\right),\ -(b-a)\int_a^b x^T(s)Xx(s)ds\leq-\left(\int_a^b x(s)ds\right)^T X\left(\int_a^b x(s)ds\right).$$

Lemma 2.4 (Zhang et al. [35]). *Let $W>0$, and $\omega(s)$ be an appropriate dimensional vector. Then, we have the following facts for any scalar function $\beta(s)\geq 0$, $\forall s\in[t_1,t_2]$:*

$$(1)\ -\int_{t_1}^{t_2}\omega^T(s)W\omega(s)ds\leq(t_2-t_1)\zeta^TF_1^TW^{-1}F_1\zeta+2\zeta^TF_1^T\int_{t_1}^{t_2}\omega(s)ds;$$