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Brief Papers

# Mean-square exponential input-to-state stability for neutral stochastic neural networks with mixed delays

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ABSTRACT

This paper is concerned with the input-to-state stability problem of a class of neutral stochastic neural networks. The stochastic neural networks that we consider contain both neutral terms and mixed delays. By utilizing the Lyapunov–Krasovskii functional method, stochastic analysis techniques and It ô's formula, some sufficient conditions are derived to ensure the mean-square exponential input-to-state stability of the addressed system. Two numerical examples and their simulations are given to illustrate the effectiveness of the derived results.



## 1. Introduction

In the past decades, neural networks have been extensively investigated owing to their promising applications in many areas such as associative memory, image processing, pattern recognition, signal processing, and combinatorial optimization [1–6]. These attained applications depend on the stability of equilibrium points of neural networks since stability is the precondition that a system can work normally. In the real world, noise disturbance is ubiquitous. Just as pointed out by Haykin [7], the synaptic transmission can be viewed as a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes in nervous systems. In general, Gaussian noise has been employed to describe the noise disturbance arising in neural networks, and its existence can cause instability and poor performances. Consequently, it is of great significance to investigate the stability of stochastic neural networks with Gaussian noise.

On the other hand, time delays are frequently occurring in hardware implementations because of the finite switching speed of amplifiers or information processing, which can cause complex dynamic behaviors such as oscillations, divergence and even instability in designing neural network systems. Hence, the stability analysis of stochastic neural networks with time delays has become an interesting research topic [8–16]. As we know, the existing works on delayed neural networks can be generally classified into four categories: constant delays, time-varying delays, distributed delays and mixed delays. It has been recognized that the variable time delays and distributed delays have more practical significance than the constant delays. Besides, due to the complicated dynamic properties of the neural cells, there exist many neural network models such as distributed networks, chemical reactors, and heat exchanges that cannot characterize the properties of a neural reaction process precisely [17]. It is natural and important that the systems will contain some information about the derivative of the past state. In order to describe the dynamics for complex responses, such systems have been referred as neutral-type systems, and they have the state derivative with delays which are called neutral delays. Under this circumstance, a class of stochastic neural networks with neutral delays has been introduced. In recent years, the stability analysis problem of stochastic time-varying neural networks with neutral terms has received considerable attention [18–22].

In addition, dynamical behaviors of neural networks are often affected by control inputs. In order to check robust stability, the input-to-state stability (ISS) was first introduced by Sontag [23,24], which was more general than the traditional stability since the ISS properties imply not only that the unperturbed system is asymptotically stable in the Lyapunov sense but also that its behavior

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remains bounded when its inputs are bounded. During the last two decades, ISS has become a central foundation of modern nonlinear feedback and design due to its usefulness. It is also considered as a key tool in systems with recursive design, coprime factorizations, small gain theory, and a connection between the input–output (or external) stability and state (or internal) stability [25]. Meanwhile, the ISS analysis also opens a new path for application of dynamic neural networks to nonlinear control. Recently, there have been a large number of works on the input-to-state stability analysis of neural networks. By using Lyapunov functional method, Sanchez and Perez [26] obtained nonlinear feedback matrix norm conditions to guarantee ISS which also ensured global asymptotic stability. Guo [27] further gave two new results on input-to-state convergence of recurrent neural networks with variable inputs. According to a tuning algorithm, Yu and Li [28] examined some stability properties like asymptotic stability, input-to-state stability and bounded input-bounded stability of neural networks. For switched Hopfield neural networks with time delays under parametric uncertainty, Ahn [29] proposed a new passive weight learning law and the input-to-state stability was also considered. Based on the results in [26], Zhu and Shen [30] established two algebraic criteria for exponential input-to-state stability of recurrent neural networks. More recently, with the help of the Lyapunov–Krasovskii functional, stochastic analysis theory and It ô's formula, Zhu and Cao [31,32] introduced and studied a new stability criterion: the mean-square exponential input-to-state stability of stochastic recurrent neural networks and Cohen–Grossberg neural networks. Lou and Ye [33] investigated the input-to-state stability problem of a class of memristor-based neural networks with stochastic effects and time-varying delays via non-smooth analysis and stochastic techniques. Furthermore, by means of Razumikhin technique and new Halanay differential inequalities, Zhou et al. [34] considered the mean-square exponential input-to-state stability of a class of non-autonomous stochastic Cohen–Grossberg neural networks. Although the traditional stability of neutral stochastic systems is examined, up to now, there has been no results on the input-to-state stability analysis for neutral stochastic neural networks with mixed delays.

Inspired by the above discussion, we will investigate the mean-square exponential input-to-state stability of neutral stochastic neural networks with mixed delays in this paper. The contributions of our work lie in three aspects. First, the structure of the proposed model is more general than the ones in [30–34] since many factors such as neutral terms, mixed delays and stochastic perturbations are considered. Second, the existing works mainly focused on the asymptotic stability in the mean square, exponential stability in the mean-square and almost surely exponential stability of neutral stochastic systems with mixed delays, but the mean-square input-to-state stability has not been investigated, which differentiates our work from the previous works. Third, by utilizing the Lyapunov–Krasovskii functional method, stochastic analysis techniques and It ô's formula, some novel criteria on mean-square exponential input-to-state stability are established. The Lyapunov–Krasovskii functional in the paper is more complex comparing with the ones in [30–34] since it covers neutral terms and double integrals. Meanwhile our results are computationally efficient as the derived sufficient conditions can be easily checked.

The remainder of this paper is organized as follows. In Section 2, the model of neutral stochastic neural networks with mixed delays is introduced, and some assumptions and definitions needed in this paper are presented. By utilizing the Lyapunov–Karasovskii functional approach and stochastic analysis techniques, some sufficient conditions are derived to ensure the mean-square exponential input-to-state stability of the addressed system in Section 3. In Section 4, two numerical examples are given to demonstrate the effectiveness of the obtained results. Finally, conclusions are drawn in Section 5.

## 2. Model description and problem formulation

Throughout this paper, unless otherwise specified, we let $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\ge 0}, \mathbb{P})$ be a complete probability space with a filtration $\{\mathcal{F}\}_{t\ge 0}$. Let $\omega(t)$ be an $n$-dimensional Brownian motion defined on the probability space satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). $\mathbb{R}$ denotes the set of real number, and $\mathbb{R}_{\ge 0}$ denotes the set of positive real number. $\mathbb{R}^n$ represents the $n$-dimensional space. The superscript "T" denotes the transpose of a matrix or vector. Let $\overline{\tau} > 0$ and $C([-\overline{\tau}, 0], \mathbb{R}^n)$ denote the family of continue functions $\varphi$ from $[-\overline{\tau}, 0]$ to $\mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{-\overline{\tau}\le s\le 0}|\varphi(s)|$, where $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. $L^2_{\mathcal{F}_0}([-\overline{\tau}, 0], \mathbb{R}^n)$ denotes the family of all $\mathcal{F}_0$-measurable $C([-\overline{\tau}, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(\theta) : -\overline{\tau} \le \theta \le 0\}$ such that $\sup_{-\overline{\tau}\le\theta\le 0}\mathbb{E}|\xi(\theta)|^2 < \infty$, where $\mathbb{E}[\cdot]$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$. We also let $l_\infty$ denote the class of essentially bounded function $u$ from $[0, \infty)$ to $\mathbb{R}^n$ with $\|u\|_\infty = \mathrm{esssup}_{t\ge 0}\{|u(t)|, t \ge 0\} < \infty$. The shorthand $\mathrm{diag}\{\cdots\}$ denotes the block diagonal matrix.

Consider the following class of stochastic neutral recurrent neural networks with mixed delays:

$$
\begin{cases}
d[x_i(t) - \sum_{j=1}^{n} p_{ij} x_j(t-\tau_1(t))] = [-d_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau_1(t))) + \sum_{j=1}^{n} c_{ij} \int_{t-\tau_2(t)}^{t} h_j(x_j(s)\, ds \\
\qquad + u_i(t)]\, dt + \sum_{j=1}^{n} \sigma_{ij}(x_j(t), x_j(t-\tau_1(t)), x_j(t-\tau_2(t)))\, d\omega_j(t) \\
x_i(t) = \xi_i(t), \quad -\overline{\tau} \le t \le 0,
\end{cases}
\tag{1}
$$

for all $t \ge 0, i = 1, 2, \ldots, n$, where $x_i(t)$ is the state variable of the $i$th neuron at time $t$, and the constant $d_i$ denotes the self-feedback connection weight coefficient of the $i$th unit. The constants $a_{ij}, b_{ij}$ and $c_{ij}$ are the weight coefficients of the neurons, and $f_j(x_j(t)), g_j(x_j(t))$ and $h_j(x_j(t))$ are the neuron activation functions. $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T$ is an external input vector to neurons. The time-varying delay $\tau_1(t)$ and distributed delay $\tau_2(t)$ satisfy $0 \le \tau_1(t) \le \tau_1, 0 \le \tau_2(t) \le \tau_2$. Let $\overline{\tau} = \max\{\tau_1, \tau_2\}$. The noise perturbation $\sigma_{ij}(t) : \mathbb{R} \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is a Borel measurable function, and $\{\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_n(t)), t \ge 0\}$ is an $n$-dimensional standard