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Exponentially stable guaranteed cost control for continuous and discrete-time Takagi–Sugeno fuzzy systems

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ABSTRACT

This paper investigates exponentially stable guaranteed cost control (GCC) for a class of nonlinear systems which is represented by Takagi–Sugeno (T–S) fuzzy systems. State feedback controllers of parallel distributed compensation (PDC) structure are designed by the means of GCC for continuous and discrete-time T–S fuzzy systems respectively. GCC methods in this paper adopt quadratic performance functions, which take effects of control efforts, regulation errors and convergence rates into consideration simultaneously, to provide desirable performance and fast response for closed-loop systems. Sufficient design conditions that guarantee exponential stabilities of resulting closed-loop systems with predefined convergence rates are presented. By setting different convergence rates, response speed of the closed-loop nonlinear system can be adjusted. The proposed design procedures are eventually converted into linear matrix inequalities (LMIs) problems of minimizing upper bounds of the guaranteed cost functions. Finally, a well-known nonlinear benchmark control example and a truck–trailer example demonstrate the effectiveness and feasibility of all the proposed methods.

1. Introduction

In real world, it is not an easy task to design satisfactory controllers for complex systems due to serious inherent nonlinearities and parameter uncertainties in these systems. Although approximate nonlinear mathematical models could be established to describe parts of the nonlinearities and uncertainties, sometimes these models are too complicated to be tackled directly for controller design with existing mathematic tools. Moreover, in some cases limited knowledge and measurement to the system's mechanism even prevent people from obtaining appropriate nonlinear mathematical models. At this point, fuzzy technique provides one of the powerful ways to reduce the difficulties, which has been successful in modeling and control of a wide range of various nonlinear systems, for example, non-adaptive fuzzy control [1,2], adaptive fuzzy control [3,4], etc. Among them, Takagi–Sugeno (T–S) fuzzy model which is firstly introduced in [5] has found popularity both in literature and industry since many kinds of nonlinear systems can be approximated by the model succinctly and human's experience could be incorporated in. T–S fuzzy model uses linear combinations of system inputs to form linear subsystems as their inference consequences and then nonlinearly synthesizes these subsystems to reconstruct the original system outputs. In this indirect way, T–S fuzzy model provides a powerful bridge connecting the actual nonlinear systems and classical linear systems so that the abundant theories and mature techniques in linear systems are possible to be applied, which mitigates the pain of designing controllers for nonlinear systems directly.

Many studies have been done in terms of applying linear control theories into T–S fuzzy models. However, most of existing works concern only the ultimate control performance, for example, stability, tracking, robustness and so on [2,6–9]. Little attention has been paid to transient control performance, such as response speed, settling time, overshoot, etc. In general, there are infinite controllers which guarantee stabilities of corresponding systems. It is necessary to select one controller which satisfies predefined transient performance from them. So it is interesting to figure out how to obtain a satisfactory controller under some specific transient performance for T–S fuzzy model.

One natural idea is to extend optimal control results of linear system to the T–S fuzzy model. By using the concept of PDC [1], locally optimal T–S fuzzy controllers were designed in [10]

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through solving standard LQR problem for each subsystem. If the synthesized controller satisfied the sufficient stability conditions for T–S models proposed in [2], a suboptimal controller was obtained, otherwise the design procedure repeated. Obviously this method is trial and error. By relaxing the Riccati equation constraint into inequality, guaranteed cost control (GCC) approach was achieved via LMIs through minimizing an upper bound of a quadratic performance measure in [11,12]. Based on relaxed stability conditions [13], a globally suboptimal fuzzy controller was designed by minimizing the upper bound of a given global quadratic performance function [14]. In [15] a globally optimal and stable fuzzy controller design method for finite and infinite horizons was claimed to be found. However the controller may only be considered as suboptimal according to comments [16,17].

In reality, in addition to stability and optimality, fast response is also a desired property for control systems. The time optimal control problem is formulated for this objective. Although it is well known that for some classes of nonlinear systems the time optimal control is a bang-bang control [18], this control strategy is rarely implemented in real systems due to the difficulties of characterizing the switching surface. Another concept related to fast response control is the convergence rate [19], which could be explained in the definition of exponential stability, a special case of asymptotic stability [20]. If the system state variable $x(t)$ satisfies

$$\|x(t)\| \le k\|x(0)\|e^{-\alpha t}, \quad \forall x(0)\in D_0, \quad \forall t>0$$

where $D_0=\|x\|<r_0$ is a domain containing $x=0$, α, r_0 and k are positive constants, then the system is said to be exponentially stable and α is the convergence rate. Tanaka et al. proposed a fuzzy controller design method which can guarantee the predefined convergence rate of closed-loop system in [21]. By using LMI regions relevant to pole location constraints in [22], Ref. [23] presents a number of controller design methods for T–S fuzzy systems via H^∞ optimization with pole assignment specifications.

However, all the aforementioned studies aim at either minimizing the standard quadratic performance function or achieving fast response. To author's best knowledge, few results which consider both the optimality and response speed at the same time are available for T–S fuzzy systems. Motivated by the above concerns, in this paper, we try to figure out an effective approach of GCC design with desirable convergence rate for a class of nonlinear systems represented by linear T–S fuzzy model. Firstly, the continuous-time modified quadratic cost function and its discrete counterpart are borrowed from classic linear system theories. Secondly, by utilizing the PDC concept, sufficient conditions are presented respectively for continuous- and discrete-time systems such that the designed controllers are able to exponentially stabilize the closed-loop system with predefined convergence rate and simultaneously minimize the upper bound of the cost functions. Thirdly, we show that the controller design problems can be transformed into LMI problems solved efficiently by available convex optimization tools. Finally, simulation results of two numerical examples are presented to illustrate the effectiveness and advantages of the proposed design methods. Compared with the previous literature, the main contributions of this paper are as follows.

1. By using the modified quadratic cost functions which not only represent the control effort and the regulation error but also take response speed into account [24], the proposed control method can provide closed-loop systems with ability of fast response while makes the cost functions minimized.
2. The relationship between response speed and convergence rate of the proposed control method is more intuitive than that between response speed and weighted matrices of classical GCC. By setting different convergence rates, the response speeds of closed-loop systems can be adjusted efficiently.

Notation: Throughout this paper, if not explicitly stated, matrices are assumed to have compatible dimensions. "Continuous-time fuzzy systems" and "discrete-time fuzzy systems" are abbreviated as CFS and DFS, respectively. $A>0$ means that the matrix A is positive definite; the notation $A>B$ means that $A-B>0$ is positive definite; I is the identity matrix of appropriate dimension; 0 is the zero matrix of appropriate dimension; In the square matrix, the symbol '$*$' stands for the transposed elements in the symmetric positions. In addition, $\sum_{i<j}^{r}$ and $\sum_{i\ne j}^{r}$ mean, for instance,

$$\sum_{i<j}^{3} x_{ij} = x_{12}+x_{23}+x_{13}$$

$$\sum_{i\ne j}^{3} x_{ij} = x_{12}+x_{13}+x_{21}+x_{23}+x_{31}+x_{32}$$

2. Problem formulation and preliminaries

The T–S fuzzy models compose of a set of fuzzy "IF–THEN" rules with fuzzy sets in the antecedents and dynamic LTI systems in the consequents [5]. For a plant to be controlled, we can use the following sets of rules to describe it:

ith *Plant Rule: IF* $z_1(t)$ *is* M_{i1}, $z_2(t)$ *is* M_{i2},…,$z_p(t)$ *is* M_{ip}, *THEN* $\nabla x(t)=A_i x(t)+B_i u(t)$, $y(t)=C_i x(t)$. $i=1,2,\dots,r$.

Where for expression simplicity, t is used to represent either continuous or discrete time without causing ambiguity, ∇ represents an operator. For CFS cases $\nabla x(t)$ means $\dot{x}(t)$ and for DFS cases $\nabla x(t)$ represents $x(t+1)$ [25]. $x(t)\in R^n$ is the state vector, $u(t)\in R^m$ is the input vector, $y(t)\in R^l$ is the output vector, $A_i\in R^{n\times n}$, $B_i\in R^{n\times m}$ and $C_i\in R^{l\times n}$. $z(t)\in R^p$ is the premise variable vector, which is assumed to be independent of the input vector $u(t)$.

Each linear state equation in the consequent parts is called a "subsystem". By aggregating all the subsystems, the synthesized model is obtained as:

$$\nabla x(t)=\frac{\sum_{i=1}^{r} w_i(z(t))(A_i x(t)+B_i u(t))}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))(A_i x(t)+B_i u(t)) \tag{1}$$

$$y(t)=\frac{\sum_{i=1}^{r} w_i(z(t))C_i x(t)}{\sum_{i=1}^{r} w_i(z(t))} = \sum_{i=1}^{r} h_i(z(t))C_i x(t) \tag{2}$$

where

$$w_i(z(t))=\prod_{j=1}^{p} M_{ij}(z_j(t))\ge 0$$
$$Z(t)=[z_1(t)\ \ z_2(t)\ \ \cdots\ \ z_p(t)]$$
$$h_i(z(t))=\frac{w_i(z(t))}{\prod_{i=1}^{r} w_i(z(t))} \tag{3}$$

$M_{ij}(z_j(t))$ is the grade of membership of $z_j(t)$ in the jth fuzzy set M_{ij} for all t. From (1) to (3), there exist

$$\begin{cases} 1\ge h_i(z(t))\ge 0 \\ \sum_{i=1}^{r} h_i(z(t))=1 \end{cases} \quad i=1,2,\dots,r \tag{4}$$

The structure of the PDC controller for above fuzzy model is

ith *Controller Rule: IF* $z_1(t)$ *is* M_{i1}, $z_2(t)$ *is* M_{i2},…,$z_p(t)$ *is* M_{ip}, *THEN* $u_i(t)=-K_i x(t)$. $i=1,2,\dots,r$.