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Brief Papers

Time-varying group formation analysis and design for second-order multi-agent systems with directed topologies ☆

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ABSTRACT

Time-varying group formation control problems for second-order multi-agent systems with directed topologies are investigated. Firstly, a time-varying group formation control protocol is constructed using local relative positions and velocities of each agent and its neighbors. Then based on graph theory, nonsingular transformations are applied to the closed-loop multi-agent systems. Sufficient conditions for second-order multi-agent systems to achieve time-varying group formation are further presented together with the time-varying group formation feasibility constraints. Explicit expressions of the subgroup formation reference functions are derived to describe the macroscopic movement of the time-varying subgroup formations. Moreover, by solving an algebraic Riccati equation, an approach to design the time-varying group formation protocol is proposed. Finally, a numerical example with three subgroups is provided to demonstrate the effectiveness of the obtained results. In contrast to the traditional complete formation, where only one formation is realized by the multi-agent system, in the group formation discussed in the current paper, agents are classified into subgroups and each subgroup is required to form a specified time-varying sub-formation via inter-subgroup and intra-subgroup interactions.

1. Introduction

The research on formation control of multi-agent systems has attracted an increasing interest in recent years due to its broad potential applications in various fields, such as cooperative localization [1], load transportation [2], surveillance [3] and target enclosing [4]. Although in the past decades, several centralized and decentralized strategies have been proposed to deal with the formation control problems, how to realize the predefined formation via local neighboring interactions is still a current concern in both the control theory and robotics communities [5,6].

With the development of consensus theory, there is an emerging trend to study the formation control problem in the view of consensus control. Ren [7] proposed a general formation control framework for second-order multi-agent systems by extending the consensus protocols. Sufficient conditions for multi-agent systems with second-order dynamics to achieve time-invariant formations via local interactions were presented in [8]. Time-invariant formation control problems for second-order multi-agent systems with time delays were addressed in [9]. A distributed controller for first-order multi-agent systems to achieve rigid formations was constructed in [10]. Oh and Ahn [11] proposed a formation control strategy for first-order multi-agent systems using the distributed position estimation. Distributed formation control problems for first-order multi-agent systems were investigated in [12] by means of complex laplacian analysis. Sufficient conditions for first-order multi-agent systems to achieve circular formations in one-dimensional and three-dimensional spaces were presented in [13] and [14], respectively. Necessary and sufficient conditions for second-order multi-agent systems with fixed and switching topologies to achieve time-varying formations were proposed in [15] and [16], respectively.

It should be pointed out that in the aforementioned works [7–16], only complete formation control problems were considered, where all the agents form a single formation. However, in many practical circumstances, such as multi-target enclosing, obstacle avoidance and cooperative searching for multiple objects, agents in a multi-agent system may split into several subgroups to accomplish different distributed tasks. In such scenarios, group formation control problems arise, where there can be multiple different sub-formations in the multi-agent system and agents in the same subgroup achieve one predefined sub-formation. To the best of our knowledge, time-varying group formation control problems for second-order multi-agent systems have not been

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studied before. One relevant topic is group (or cluster) consensus control, where there exist multiple subgroups and agents in the same subgroup reach an agreement. Group consensus control problems for first-order multi-agent systems with switching topologies and communication delays were addressed in [17]. Necessary and sufficient conditions for first-order multi-agent systems to achieve the cluster or group consensus were provided in [18] and [19], respectively. Group consensus control problems for second-order multi-agent systems were studied in [20] through leader-following approach and pinning control. Necessary and sufficient conditions for second-order multi-agent systems to achieve group consensus were proposed in [21]. Although group/cluster consensus problems were studied in [17–21], the obtained results cannot be extended directly to deal with the time-varying group formation control problems for second-order multi-agent systems directly.

Motivated by the facts stated above, this paper investigates the time-varying group formation analysis and design problems for second-order multi-agent systems with directed topologies. Firstly, agents in the multi-agent systems are partitioned into subgroups. A time-varying sub-formation is predefined for each subgroup. Time-varying group formation control protocol is constructed based on the local inter-subgroup or intra-subgroup information. By assuming that the directed graph of the multi-agent system has an acyclic partition and each subgroup has a spanning tree, sufficient conditions for second-order multi-agent systems to achieve time-varying group formation are proposed. A time-varying group formation feasibility constraint is also given. Then explicit expressions of the subgroup formation reference functions are presented to describe the macroscopic movement of the time-varying group formation. Finally, an approach to design the time-varying group formation protocol is given by solving an algebraic Riccati equation.

Compared with the previous results on formation control and group/cluster consensus control, the novel features of the current paper are threefold. Firstly, the agents in the current paper can be partitioned into multiple subgroups and multiple sub-formations can be realized by the whole multi-agent system. In [7–16], only complete formation control problems were studied, where all agents interact as a whole group and only one formation is realized for all the agents. Group formation control problems are much complicated and challenging than the complete formation since there exist both inter-subgroup interactions and intra-subgroup interactions, and multiple sub-formations. Moreover, the time-varying group formation is more general and these complete formations discussed in [7–16] can all be regarded as special cases of the one in the current paper where only one group exists. Secondly, the multiple sub-formations can be time-varying. In [17–21], group/cluster consensus control problems for first-order and second-order multi-agent systems were addressed. Although the results on group/cluster consensus are potential to solve some constant group formation control problems, they cannot be extended to solve the time-varying group formation control problems in the current paper due to that the time-varying formation will bring the derivative of the formation information to both the analysis and design of the group formation control law. By choosing the time-varying formation vectors to be zero, the time-varying group formation control problems become group/cluster consensus problems investigated in [17–21]. Thirdly, explicit expressions for the time-varying formation reference functions of all the sub-formations are derived to describe the macroscopic movements of all the subgroups. Although the macroscopic movement of the whole multi-agent system can be described by the explicit expression of the formation reference function in [15] and [16], they only considered the complete formation case and the approaches proposed in [15] and [16] cannot be directly used to solve the problems in the current paper.

The remainder of this paper is organized as follows. In Section 2, basic definitions and notations on graph theory are given and the problem description is introduced. Main results which include sufficient criteria and protocol design approaches are proposed in Section 3. In Section 4, simulation results are given to demonstrate the effectiveness of the results in this paper. Finally, conclusions are given in Section 5.

Some notations used in this paper are given as follows. Let 0_n and $\mathbf{1}_n$ be zero matrices and column vectors of ones with dimension n. Let I_N represent an identity matrix with dimension N, and $\otimes$ denote the Kronecker product.

2. Preliminaries and problem description

In this section, basic concepts and notations on graph theory are introduced and the problem description is presented.

2.1. Preliminaries

A weighted directed graph of order N can be denoted by $G=\{V,E,W\}$, where $V=\{v_1,v_2,\dots,v_N\}$ is the node set, $E\subseteq\{(v_i,v_j):v_i,v_j\in V\}$ is the edge set and $W=[w_{ij}]\in\mathbb{R}^{N\times N}$ the weighted adjacency matrix. An edge in G is represented by $e_{ij}=(v_i,v_j)$ $(i\neq j)$. For any $i,j\in\{1,2,\dots,N\}$, the adjacency element w_{ij} in W satisfies that $w_{ji}\neq 0$ if and only if $e_{ij}\in E$, and $w_{ij}=0$ otherwise. Denote by $N_i=\{v_j\in V: e_{ji}\in E\}$ the neighbor set of node v_i. The in-degree of node v_i is defined as $\deg_{in}(v_i)=\sum_{j=1}^{N}w_{ij}$. Let $D=\mathrm{diag}\{\deg_{in}(v_i), i=1,2,\dots,N\}$ be the degree matrix of G. The Laplacian matrix $L\in\mathbb{R}^{N\times N}$ of G is defined as $L=D-W$. A directed path from v_i to v_j is a finite ordered sequence edges with the form of (v_i,v_{k_1}), $(v_{k_1},v_{k_2}),\dots,(v_{k_{l-1}},v_{k_l})$, (v_{k_l},v_j). A directed graph with no cycles is called a directed acyclic graph. A directed graph is said to contain a spanning tree if there exist at least a node which has a directed path to any other nodes.

2.2. Problem description

Consider a second-order multi-agent system with N agents. The dynamics of agent i $(i\in\{1,2,\dots,N\})$ is described by

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=\alpha_x x_i(t)+\alpha_v v_i(t)+u_i(t),\end{cases}\tag{1}$$

where $n\geq 1$ is the dimension of the space, $x_i(t)\in\mathbb{R}^n$, $v_i(t)\in\mathbb{R}^n$ and $u_i(t)\in\mathbb{R}^n$ denote, respectively, the position, velocity and control input vectors of agent i, and $\alpha_x\in\mathbb{R}$ and $\alpha_v\in\mathbb{R}$ are known damping constants. In the following, for simplicity of description, let $n=1$ if not otherwise specified. However, by applying the Kronecker product, all the results hereafter can be directly extended to the higher dimensional case. The interaction topology among the N agents can be modeled by the directed graph G with node i representing the agent i in multi-agent system (1) and e_{ij} denoting the interaction from agent i to agent j with the associated interaction strength w_{ji}.

Remark 1. From (1), one sees that in the case where $\alpha_x=0$ and $\alpha_v=0$, the second-order model in the current paper becomes the classic double-integrator one. Therefore, the second-order model described by (1) can be treated as an extension to the double-integrator one and has more generality.

Suppose that multi-agent system (1) consists of $g\in\mathbb{N}$ $(g\geq 1)$ subgroups, and the node set V can be partitioned into $V_1,V_2,\dots,V_g$, where $V_k\neq\varnothing$ $(k=1,2,\dots,g)$, $\cup_{k=1}^{g}V_k=V$ and $V_k\cap$