Neurocomputing 205 (2016) 421–429

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Globally exponential stability and dissipativity for nonautonomous neural networks with mixed time-varying delays

Minghui Jiang*, Juan Mu, Dasong Huang

Institute of Nonlinear Complex Systems, China Three Gorges University, YiChang, Hubei 443000, China

ARTICLE INFO

Article history:
Received 3 November 2015
Received in revised form
1 March 2016
Accepted 14 April 2016
Communicated by S. Arik
Available online 12 May 2016

Keywords:
Existence
Dissipativity
Halanay inequality
Matrix measure
Stability
Nonautonomous neural networks

ABSTRACT

In this paper, the problems of globally exponential stability, dissipativity and solutions' existence are investigated for nonautonomous neural networks with mixed time-varying delays as well as general activation functions. The mixed time-varying delays consist of both discrete and distributed delays. First, we give a Halanay inequality and combine matrix measure function inequality, sufficient conditions are established to ensure the dissipativity and globally exponential stability of the solutions of the considered neural networks in the end, then a criterion are obtained to guarantee the existence of the solutions of system. Finally, numerical examples are given to show the effectiveness of our theoretical results.

1. Introduction

In recent years, autonomous neural networks have been extensively studied and applied in many different fields such as signal processing,automatic control engineering, associative memory and some optimization problems. However, in these applications they mainly discussed the dynamic behaviors of the neural networks. For example,the stability, dissipativity and the existence have been attracted increasing attention, and have appeared many results. To our sadness, in practical application, neural networks are always affected by time. Such as time delays can be caused by the finite switching speed of the neuron amplifiers; and the finite speed of signal propagation [1–3]. Beside, in hardware implementation, time delays may lead to oscillation; divergence, or instability, which may be harmful to a system [4,5]. Therefore, the study of dynamic behaviors for delayed neural networks has received considerable attention in recent years [see [6–16]].

The authors investigated the dynamic behaviors of various of nonautonomous neural networks by applying the properties of M-matrix, the techniques of inequality analysis and constructing appropriate Lyapunov–Krasovskii functionals in [17–25]. They obtained some useful results, but some of these papers are only limited to discrete and distributed delays and their results are not easy to test. In addition, the activation functions of the neural network were usually required to be bounded to ensure the existence and globally stability of an equilibrium point [18,19]. However, in some engineering problems, for example, when neural networks were designed to solve optimization problems in the presence of linear or quadratic constraints, the activation functions modeled by diode-like exponential-type functions are unbounded [26]. Therefore, boundedness of the activation functions is canceled in the paper.

Furthermore, various forms of generalized Halanay inequalities were proposed [27–31] to study dissipativity of more complex and realistic delay systems. For example, in [34] the authors discuss some various of generalized Halanay inequalities and investigated dissipativity of delayed neural networks. Based on a large amount of papers. However, so far, globally exponential stability, dissipativity and existence for nonautonomous neural networks with mixed time-varying delays which have received little research attention, mainly due to the mathematical difficulties in dealing with discrete and distributed delays simultaneously. Hence, it's significant to solve such an important problem for us in this paper.

Motivated by the above discussion, our paper is concerned with the stability and dissipativity for neural networks with discrete and distributed delays by using a Halanay inequality even an extend one. Besides, comparing with the results in [35], we get the less conservative conditions for guaranteeing the dissipativity and stability of neural networks after we use Halanay inequality and the good properties of Matrix measure in [32,33,38–40] which may be negative or positive, in comparison to the matrix norm

* Corresponding author.
E-mail address: jiangminghui@ctgu.edu.cn (M. Jiang).

http://dx.doi.org/10.1016/j.neucom.2016.04.025
0925-2312/

that always be nonnegative. The last, three examples are presented and analyzed to demonstrate our results.

The rest of this paper is organized as follows: In Section 2, preliminaries including some necessary definitions are presented. In Section 3, we give a general Halanay-type inequality and its proof. In Section 4, we investigate the existence of the solution of nonautonomous neural networks with mixed time-varying delays, and main results about globally exponential stability and dissipativity are derived with Halanay inequality. In Section 5, three examples with simulations are presented to illustrate our main results. In the end, the study in this paper is concluded in Section 6.

2. Preliminaries and model description

The notations are quite standard. Throughout this paper, if not explicitly stated, A^T denotes the transpose of a square matrix A, $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T\in\Re^n$, and $\dot{x}(t)$ means $\frac{dx}{dt}$. Assume $\|A\|_p$ $(p=1,2,\infty)$ denote the Euclidean p-norm in $R^{n\times n}$ and the p-norm $\|x(t)\|_p$ $(p=1,2,\infty)$ of the column vectors are written as $\|x(t)\|_1=\sum_{i=1}^n|x_i(t)|$, $\|x(t)\|_2=\sqrt{\sum_{i=1}^n|x_i(t)|^2}$, $\|x(t)\|_\infty=\max_{1\le i\le n}|x_i(t)|$.

In this paper, we will investigate the following delayed dynamic systems about nonautonomous neural networks with mixed time-varying delays.

$$\dot{x}_i(t)=-c_i(t)x_i(t)+\sum_{j=1}^n a_{ij}(t)g_j(x_j(t))+\sum_{j=1}^n b_{ij}(t)g_j(x_j(t-\tau_j(t))) \\ +\sum_{j=1}^n d_{ij}(t)\int_{t-\sigma(t)}^t g_j(x_j(s))ds+u_i(t),\ i=1,2,\dots,n, \tag{1}$$

where the variables $x_i(t)(i=1,\dots,n)$ denotes the state variables; $c_i(t)>0$ represents the self-inhibition of the ith neuron at time t; $a_{ij}(t)$ stand for the weights of neuron interconnections; $b_{ij}(t)$ and $d_{ij}(t)$ denote the connection weights of the jth unit on the ith unit related to the neurons with discrete delay and distributed delay, respectively; $g_j:\Re\longrightarrow\Re, i=1,\dots,n$ are activation functions; $u_i(t), i=1,\dots,n$ are external continuous bounded inputs. $\tau_i(t)(i=1,2,\dots,n)$ denotes the discrete delays which is assumed to be bounded and continuously differentiable function; i.e., there exist a positive number τ and a constant $\overline{\tau}_i$, such that $0\le\tau_i(t)\le\tau, \dot{\tau}_i(t)\le\overline{\tau}_i<1.\sigma(t)$ is distributed delay. In order to establish the conditions of main results for neural networks (1), we have the following assumptions:

(1) The set of activation functions which satisfy the growth condition is defined as

$$|g_i(x_i)|\le k_i|x_i|+n_i, k_i\ge 0, i=1,\dots,n$$

(2) $c_i(t), a_{ij}(t), b_{ij}(t)$ and $d_{ij}(t)\in C(R,R)$. Moreover, there exist constants c_i^M, c_i^m, a_{ij}^M, a_{ij}^m, b_{ij}^M, b_{ij}^m, d_{ij}^M, d_{ij}^m such that

$$c_i^M\ge c_i(t)\ge c_i^m>0\ a_{ij}^M\ge a_{ij}(t)\ge a_{ij}^m\ b_{ij}^M\ge b_{ij}(t)\ge b_{ij}^m \quad \text{and} \quad d_{ij}^M\ge d_{ij}(t)\ge d_{ij}^m.$$

(3) Let $\psi_i(t)=t-\tau_i(t)$,

$$0\le\tau_i(t)\le\tau, \dot{\tau}_i(t)\le\overline{\tau}_i<1, \tag{2}$$

in which $\tau=\sup_{1\le i\le n}\tau_i(t), \tau$ and $\overline{\tau}_i$ are constants.

In this paper, the initial functions of the non-autonomous neural networks (1) satisfy the following:

$$x_i(s)=\varphi_i(s), s\in[-\tau,0], i=1,\dots,n, \tag{3}$$

which $\varphi_i(s)$ is a continuous function from $[-\tau,0]$ to R. Rewritten the neural networks (1) in the vector form as :

$$\dot{x}(t)=-C(t)x(t)+A(t)g(x(t))+B(t)g(x(t-\tau(t))) \\ +D(t)\int_{t-\sigma(t)}^t g(x(s))ds+u(t), \tag{4}$$

where $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$; $C(t)=diag(c_1(t),c_2(t),\dots,c_n(t))$; $A(t)=(a_{ij}(t))_{n\times n}$ and $B(t)=(b_{ij}(t))_{n\times n}, D(t)=(d_{ij}(t))_{n\times n}$; $g(x(t))=(g_1(x_1(t)), g_2(x_2(t)),\dots,g_n(x_n(t)))^T$; $g(x(t-\tau(t)))=(g_1(x_1(t-\tau_1(t))), g_2(x_2(t-\tau_2(t))),\dots,g_n(x_n(t-\tau_n(t))))^T\in R^n$; $u(t)=(u_1(t),u_2(t),\dots,u_n(t))^T$. Now, we will give some definitions which can be used in the following.

Definition 2.1 (*Song [27]*). The response system (1) is said to be globally dissipative system, if there exists a compact set $\Omega\subset\Re^n$ such that for any $\varphi(s)\in\Re^n$, $s\in[-\tau,0]$, and there exists $T>0$, when $t\ge t_0+T, x(t,t_0,\varphi)\subseteq\Omega$, where $x(t,t_0,\varphi)$denotes the solution of system (1) from initial time t_0. In this case, Ω is called a globally attractive set. A set Ω is called positive invariant, if for any $\varphi\in\Omega$ implies $x(t,t_0,\varphi)\subseteq\Omega$for $t\ge t_0$. Clearly, the globally attractive set is positive invariant.

Definition 2.2. The system (1) is said to be globally exponentially stable, if there exist positive constants N and α satisfy

$$\|x(t,\varphi)-y(t,\varphi)\|_p\le N\sup_{-\tau\le s\le 0}\{\|\varphi(s)-\phi(s)\|_p\}e^{-\alpha t}, p=1,2,\infty,\ t\ge 0,$$

in which $\varphi(s)=(\varphi_1(s),\varphi_2(s),\dots,\varphi_n(s))^T$ and $\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T\in C([-\tau,0],\Re^n)$ are initial functions of $x(t)$ and $y(t)$ of the solutions of the neural networks (1).

Definition 2.3 (*Baker [28]*). The matrix measure of a real square matrix $A=(a_{ij})_{n\times n}$ is as follows:

$$\mu_p(A)=\lim_{\varepsilon\to 0^+}\frac{\|I+\varepsilon A\|_p-1}{\varepsilon},$$

in which $\|\bullet\|_p$ is an induced matrix norm on $\Re^{n\times n}$, I is the identity matrix, and $p=1,2,\infty$.

From the matrix norm $\|A\|_1=\max_j\sum_{i=1}^n|a_{ij}|$, $\|A\|_2=\sqrt{\lambda_{max}(A^TA)}$, $\|A\|_\infty=\max_i\sum_{j=1}^n|a_{ij}|$, the matrix measure $\mu_1(A)=\max_j\left\{a_{jj}+\sum_{i=1,i\ne j}^n|a_{ij}|\right\}, \mu_2(A)=\frac{1}{2}\lambda_{max}(A^T+A), \mu_\infty(A)=\max_i\left\{a_{ii}+\sum_{j=1,j\ne i}^n|a_{ij}|\right\}$. are derived.

Remark 2.1. The value of matrix measure can be positive or negative, so the results which we get are less restrictive than those by applying the matrix norm that is always nonnegative.

3. A generalized halanay inequality

In this section, we will give the proof of a generalized Halanay-type inequality. Then we can use it to discuss the following dissipativity and stability of neural networks (4).

Theorem 3.1 (*(Halanay-type Inequality)*). *If $y(t)$ is continuous on $[-\tau,+\infty)$, and*

$$D^+y(t)\le-\beta(t)y(t)+\eta(t)\sup_{t-\tau_1(t)\le s\le t}\{y(s)\}+\vartheta(t)\int_{t-\tau_2(t)}^t y(s)ds, \tag{5}$$

for $t\ge t_0$, where $\tau=\sup_{\tau_1(t)>0,\tau_2(t)>0}\{\tau_1(t),\tau_2(t)\}$ and continuous functions $\beta(t),\eta(t)\ge 0,\vartheta(t)\ge 0$, and if there exists a positive constant θ such that

$$-\beta(t)+\eta(t)+\vartheta(t)\tau_2(t)\le-\theta,\quad \text{for } t\ge t_0, \tag{6}$$