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Simultaneous optimization for robust correlation estimation in partially observed social network

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ABSTRACT

Correlation estimation plays a critical role in numerous applications of social network analysis. Traditionally, the numerical records of the interaction between users are used as quantitative metrics of correlation. The deficiencies are threefold. Firstly, a single source of interaction is far from sufficient to reveal the underlying correlation. Secondly, the data available are often partially observed result from the imperfection in data acquisition and storage techniques, thereby jeopardizing the reliability of estimation. Thirdly, the inference from the explicit features to the implicit correlation is far from straightforward. Simply taking interaction as correlation is neither theoretically nor practically plausible. The former issue can be addressed via matrix completion, whereas the latter is essentially a self-expressive matrix representation problem. Instead of solving the two problems separately, in this paper, we propose a simultaneous optimization algorithm for robust correlation estimation based on partially observed data. In this way, the global, rather than local, optima can be achieved in an effective manner. The experiments on both synthetic and real-world social network data demonstrate the advantage of the proposed method.

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1. Introduction

During the last few years, the emergence of social networks has introduced new ways of interpersonal communication among people all around the world [1–4]. Based on various interactions [5–7] in social networks, correlation between the users can be inferred. These interactions can be embodied as direct interpersonal activities, such as the status of friend or follower relationship, the frequency or intensity of online communications, and so on. There are also indirect interactions represented by the common friends or interests, the co-accessed contents, the resemblance in user profiles, etc. The interaction is an informative reflection of the correlation between users. Intuitively, closer interaction implies closer correlation, and vice versa. However, robust correlation estimation with high precision is no easy task. Apparently, friends of a given user are not equally crucial; likewise, users who have made more comments or forwarded more frequently are not necessarily more closely related to a given user. Robust correlation estimation is critical in social network analysis and has shown promising application prospect. As the old saying goes, birds of a feather flock together. The homophily in social network indicates that the highly correlated users are inclined to be grouped in the same community, and meanwhile share similar behavior and tastes [8]. Consequently, the users' interests can be accurately modeled and reliable personalized services are possible based on users' correlation.

Various interactions provide an insight into the correlation between users. Traditionally, the numerical records of a certain kind of interaction are directly used as quantitative metrics of correlation. Unfortunately, the reliability of the correlation is rather questionable. Firstly, a single source of interaction is far from sufficient to reveal the underlying correlation. Various interactions as well as the user attributes can be used to generate a comprehensive feature for correlation estimation. Secondly, both interactions and attributes available are often partially observed result from the imperfection in data acquisition and storage techniques. In other words, the input feature available for correlation estimation is only partially observed. As a result, the estimation performance is inevitably jeopardized. Last but not least, the inference from the explicit features to the implicit correlation is far from straightforward. Simply taking a certain kind of interaction as correlation is neither theoretically nor practically plausible.

Recent research work on subspace learning via matrix optimization has received great success and attracted intense attention [9–11]. As we can see, for robust correlation estimation based on partially observed social network, two key issues need to be addressed, i.e. data completion from the partially observed

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features and correlation estimation from the complete data. The former can be formulated as a matrix completion problem, which seeks the optimal low-rank approximation to fit the observed data. The latter can be interpreted as a self-expressive matrix representation problem, which searches for the optimal linear representation of the complete data. The two problems can be solved independently in a separate mode, which brings about two local optima. In this paper, we propose a simultaneous optimization algorithm for robust correlation estimation based on partially observed data, in which the two problems are incorporate into a unified process and the global optima can be achieved in an effective manner.

The rest of this paper is organized as follows. Section 2 gives a detailed description of robust correlation estimation based on partially observed data and formulates the proposed algorithm based on matrix optimization. In Section 3, we discuss the effective solution to the optimization problem via an alternating scheme. The experimental results are reported and discussed in Section 4. Finally, the conclusion is drawn in Section 5.

2. Robust correlation estimation based on partially observed data

2.1. Notation

Typically, a social network can be conventionally represented by a graph model $G=(V, E)$, where V ($|V|=n$) denotes the set of n users and E the set of links between the users [12–15]. The interaction between n users can be depicted by the adjacency matrix $\mathbf{A}\in\mathbb{R}^{n\times n}$. The element $a_{ij}\in\mathbf{A}$ indicates the interaction intensity between user v_i and v_j (v_i, v_j $\in V$), or equivalently the weight of edge $e_{ij}\in E$. There is a wide variety in the evaluation of the adjacency matrix $\mathbf{A}$. It can be attached with either binary values indicating the existence of direct connection between users, or numbers reflecting the interaction frequency. As a result, a set of multiple adjacency matrices, i.e. $\{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_k\}$, can be obtained from different ways of evaluation. Besides, attributes of all the users constitute another matrix $\mathbf{F}\in\mathbb{R}^{m\times n}$, where m is the dimension of the feature vector corresponding to a user's attributes. Based on the adjacency matrices and the attribute matrix, a feature matrix $\mathbf{M}\in\mathbb{R}^{(kn+m)\times n}$ can be obtained by stacking all the above matrices together.

$$\mathbf{M}=\left[\mathbf{A}_1^T,\mathbf{A}_2^T,\ldots,\mathbf{A}_k^T,\mathbf{F}^T\right]^T \tag{1}$$

As mentioned above, the partial observation scenario implies that not all the entries of $\mathbf{M}$ are available. Using Ω to indicate all the indices of observed entries, the available feature matrix is denotes as $\mathbf{M}_\Omega$. Based on the partially observed information derived from both the interactions and attributes, i.e. $\mathbf{M}_\Omega$, this paper aims at recovering the complete feature matrix with its approximation $\mathbf{N}\in\mathbb{R}^{(kn+m)\times n}$ and deducing the underlying correlation matrix $\mathbf{W}\in\mathbb{R}^{n\times n}$. The framework is illustrated in Fig. 1.

2.2. Problem formulation

Given the partially observed feature matrix $\mathbf{M}_\Omega$, where Ω stands for the indices of observed entries, it is indispensable to infer the unobserved data. This is achieved via matrix completion, which seeks the optimal low-rank approximation $\mathbf{N}\in\mathbb{R}^{(kn+m)\times n}$ to fit the observed data.

$$\min_{\mathbf{N}}\|\mathbf{M}_\Omega-\mathbf{N}_\Omega\|_F^2+\lambda_1\operatorname{rank}(\mathbf{N}) \tag{2}$$

The optimization problem (2) is NP-hard because of the existence of matrix rank in the regularization term. Typically, convex relaxation is implemented by replacing the rank function with nuclear norm, i.e. the sum of singular values [16–19].

$$\min_{\mathbf{N}}\|\mathbf{M}_\Omega-\mathbf{N}_\Omega\|_F^2+\lambda_1\|\mathbf{N}\|_* \tag{3}$$

With the complete feature matrix $\mathbf{M}$ recovered, revealing the correlation matrix can be formulated as a self-expressive matrix representation problem, which searches for the optimal linear representation $\mathbf{W}$ of the complete data.

$$\min_{\mathbf{W}}\|\mathbf{N}-\mathbf{N}\mathbf{W}\|_F^2+\lambda_2\operatorname{rank}(\mathbf{W})\quad \text{s.t. } \operatorname{diag}(\mathbf{W})=\mathbf{0},\ \mathbf{W}\geq 0 \tag{4}$$

Similarly, convex relaxation can be applied to rank minimization as follows:

$$\min_{\mathbf{W}}\|\mathbf{N}-\mathbf{N}\mathbf{W}\|_F^2+\lambda_2\|\mathbf{W}\|_*\quad \text{s.t. } \operatorname{diag}(\mathbf{W})=\mathbf{0},\ \mathbf{W}\geq 0 \tag{5}$$

Traditionally, separate optimization is implemented on (3) and (5), respectively. It first recovers the complete feature matrix $\mathbf{N}$ by solving (3), and then estimates the underlying correlation $\mathbf{W}$ according to (5), using $\mathbf{N}$ as constant. The separate optimization method address the matrix completion and self-expressive matrix representation problems independently, yielding two local optima. In this paper, we incorporate (3) and (5) into a unified optimization function, so that the complete feature matrix $\mathbf{M}$ and the underlying correlation matrix $\mathbf{W}$ can be obtained simultaneously. We choose ℓ_1 norm, i.e. the sum of the absolute values of matrix entries, instead of Frobenius norm, so that the algorithm is less vulnerable to outlier influence. The formulation is as follows.

$$\min_{\mathbf{N},\mathbf{W}}\|\mathbf{M}_\Omega-\mathbf{N}_\Omega\|_F^2+\alpha\|\mathbf{N}\|_*+\beta\|\mathbf{N}-\mathbf{N}\mathbf{W}\|_1+\gamma\|\mathbf{W}\|_*\quad \text{s.t. } \operatorname{diag}(\mathbf{W})=\mathbf{0},\ \mathbf{W}\geq 0 \tag{6}$$

3. Algorithm optimization

3.1. Preliminary

Two basic and frequently confronted problems in robust matrix approximation are seeking for the optimal solution via sparse and low-rank representation. Fortunately, the close form solutions are available as given in (7) and (8) respectively [20].

$$\arg\min_{\mathbf{X}}\lambda\|\mathbf{X}\|_1+\frac{1}{2}\|\mathbf{X}-\mathbf{A}\|_F^2=S_\lambda(\mathbf{A}) \tag{7}$$

$$\arg\min_{\mathbf{X}}\lambda\|\mathbf{X}\|_*+\frac{1}{2}\|\mathbf{X}-\mathbf{A}\|_F^2=\mathcal{J}_\lambda(\mathbf{A}) \tag{8}$$

In the above equations, S_λ is the soft-thresholding or shrinkage operator defined as:

$$\mathcal{S}_\lambda(x)=\operatorname{sign}(x)\cdot\max(|x|-\lambda,0) \tag{9}$$

which can be extended to vectors and matrices by operating element wise; $\mathcal{I}_\lambda$ is defined as:

$$\mathcal{J}_\lambda(\mathbf{A})=\mathbf{U}_\mathbf{A}S_\lambda(\boldsymbol{\Sigma}_\mathbf{A})\mathbf{V}_\mathbf{A}^T \tag{10}$$

where $\mathbf{A}=\mathbf{U}_\mathbf{A}\boldsymbol{\Sigma}_\mathbf{A}\mathbf{V}_\mathbf{A}^T$ is the Singular Value Decomposition (SVD) of $\mathbf{A}$.

3.2. Alternating optimization

The optimization problem (6) involves minimizing a combination of ℓ_1 norm and nuclear norm, which is convex but nonsmooth. Directly solving the objective function as a whole is intractable. In order to deal with the problem separately, three slack variables $\mathbf{P}$, $\mathbf{E}$ and $\mathbf{X}$ are incorporated along with the

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