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Stability analysis of recurrent neural networks with interval time-varying delay via free-matrix-based integral inequality ☆

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ABSTRACT

This paper is concerned with the stability analysis of recurrent neural networks with an interval time-varying delay. A new Lyapunov–Krasovskii functional (LKF) containing some augmented double integral and triple integral terms is constructed, in which the information of the activation function and the lower bound of the delay are both fully considered. Then, a free-matrix-based integral inequality is employed to deal with the derivative of the LKF such that an improved stability criterion is derived. Finally, two numerical examples are provided to illustrate the effectiveness and the benefit of the proposed stability criterion.

1. Introduction

In recent years, recurrent neural networks (RNNs) have been widely used in various fields due to their extensive applications such as pattern recognition, signal processing, associative memories and other scientific areas [1,2]. And the key of these applications with RNNs is that the equilibrium points of the designed network are stable. As a result, stability analysis of RNNs plays an important role. On the other hand, as we all know, a time delay which inevitably exists in many RNNs is usually a cause of oscillation and instability. Therefore, the stability problem of the delayed neural networks (DNNs) is of a great deal of importance in both theory and practice, and also has attracted much attention [4–22]. In the field of stability analysis, many results on this topic can be classified into the delay-dependent one and delay-independent one. Since the former considers more information of the delay and is usually less conservative, much attention has been put into employing some less conservative delay-dependent stability conditions.

In delay-dependent stability analysis of system with interval time-varying delay, we assume that there exists an upper bound of the delay. When the delay is in the interval from the given lower bound to the upper bound of delay, the delay system is asymptotic stable. Based on Lyapunov theory, constructing a suitable Lyapunov–Krasovskii functional (LKF) and estimating its derivative are two key points to enhance the feasible regions of stability criteria and reduce the conservatism. For the construction of the LKF, the simple LKF was firstly employed in DNNs, and rich results are gotten by that [4–6]. However, as we all know, the stability criteria are conservative by using the simple LKF since the delay information was not taken fully into account. To deal with this problem, the augmented LKF method was proposed [3] and widely used to the stability analysis problem of DNNs [7–21]. Furthermore, to reduce the conservatism, using the delay-decomposition idea [7], considering more information of the activation functions [8–10], augmenting the double integral terms [10,11], and introducing the triple integral terms [11] were employed to construct the LKF.

For estimating the derivative of the LKF, the free weighting matrix (FWM) approach [12,13], its improved forms [14,15] and the integral inequality method such as Jensen's inequality [13,15], Wirtinger-based integral inequality [16,17] are the most popular methods reported in the literature. The FWM method is once the best one because of neither the model transformation nor the cross-term bounding being required. However, with the proposal of the convex combination approach, which successfully avoids replacing the delay by its lower or upper bound directly, integral

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inequality was widely applied due to small amount of calculation [18,19]. Moreover, since there proposed a newly free-matrix-based integral inequality, which included the well-known Wirtinger-based integral inequality as a special case, the less conservative stability conditions were obtained [20,21].

For RNNs, the problem of delay-dependent stability was investigated in [6]. Noted that some negative semi-definite terms were ignored and the lower bound of time delay was restricted to be 0, improved stability conditions were derived following the FWM approach in [12]. By constructing an augmented LKF that contained some triple integral terms and using Jensen's inequality combined with the convex combination method, new conditions were established in [13]. In order to take more information about the lower bound of delay into account, a new LKF containing some new double integral terms and triple integral terms were introduced in [15]. In [16], a matrix-based quadratic convex approach was proposed to investigate the stability of neural networks (NNs) with interval time-varying delay and some improved conditions were obtained. Very recently, by using Wirtinger-based integral inequality combined with the convex combination method, new conditions were obtained in [17]. As the free-matrix-based integral inequality contains the Wirtinger-based integral inequality, it can be desired to derive the improved criteria by using this inequality to estimate the derivative of the LKFs for the RNNs with interval time-varying delay.

Motivated by the above statement, the problem of stability analysis for RNNs with time-varying delay is further studied. The main contributions of this paper are summarized as follows:

(1) Compared with the literature, an augmented LKF with more general form, not only considering much information of the activation function but also containing some augmented double integral and triple integral terms, is constructed.
(2) The free-matrix-based integral inequality, recently developed in the previous work [28], is used to estimate the derivative of the LKF such that a less conservative stability criterion is derived.

The contribution of the above techniques to reduce the conservatism of the criterion is demonstrated through two numerical examples.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices; N^T and N^{-1} stand for the transpose and the inverse of the matrix N, respectively; $P>0$ (≥ 0) means that P is a real symmetric and positive-definite (semi-positive-definite) matrix; diag$\{\cdots\}$ denotes a block-diagonal matrix; I represent the identity matrix; symmetric term in a symmetric matrix is denoted by $*$; and $\mathrm{Sym}\{X\}=X+X^T$.

2. Problem formulation

Consider the following RNN with a time-varying delay:

$$\begin{cases}\dot z(t)=-Az(t)+f(W(z(t-d(t)))+J)\\ z(t)=\phi(t),\quad -d_2\leq t\leq 0\end{cases}\tag{1}$$

where $z(t)=[z_1(t),z_2(t),\ldots,z_n(t)]^T\in\mathbb{R}^n$ is the neuron state vector; $J=[J_1,J_2,\ldots,J_n]^T\in\mathbb{R}^n$ is the constant input vector; $f(\cdot)=[f_1(\cdot),f_2(\cdot),\cdots,f_n(\cdot)]^T\in\mathbb{R}^n$ represents the neuron activation function; $\phi(t)$ is the initial condition; $A=diag\{a_1,a_2,\ldots,a_n\}>0$ and $W=\left[W_1^T,W_2^T,\ldots,W_n^T\right]^T$ are the known interconnection weight matrices; the time delay, $d(t)$, is a continuous differentiable function satisfying

$$d_1\leq d(t)\leq d_2\tag{2}$$

$$|\dot d(t)|\leq\mu\tag{3}$$

where d_1,d_2 and $0<\mu<1$ are constants. The neuron activation function $f(\cdot)$ is assumed to satisfy the following assumption.

Assumption 1. The function $f_i(\cdot)$ in RNN (1) is continuous and satisfies [23,24]

$$F_i^-\leq\frac{f_i(\alpha_1)-f_i(\alpha_2)}{\alpha_1-\alpha_2}\leq F_i^+,\quad i=1,2,\ldots,n\tag{4}$$

where $\alpha_1,\alpha_2\in\mathbb{R}$, $\alpha_1\neq\alpha_2$, and F_i^- and F_i^+ are known real scalars.

Based on Assumption 1, we assume that there exists an equilibrium point z^* for RNN (1). By defining $x(\cdot)=z(\cdot)-z^*$, RNN (1) can be transformed as

$$\begin{cases}\dot x(t)=-Ax(t)+g(W(x(t-d(t)))\\ x(t)=\varphi(t),\quad -d_2\leq t\leq 0\end{cases}\tag{5}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$ is the state vector; $\varphi(t)=\phi(t)-z^*$ is the initial condition; the transformed activation function $g(Wx(\cdot))=f(Wx(\cdot)+Wz^*+J)-f(Wz^*+J)$ satisfies

$$F_i^-\leq\frac{g_i(\alpha_1)-g_i(\alpha_2)}{\alpha_1-\alpha_2}\leq F_i^+,\quad g_i(0)=0,\quad i=1,2,\ldots,n\tag{6}$$

where $\alpha_1,\alpha_2\in\mathbb{R}$, $\alpha_1\neq\alpha_2$.

If $\alpha_2=0$, then we have

$$F_i^-\leq\frac{g_i(\alpha)}{\alpha}\leq F_i^+,\quad i=1,2,\ldots,n\tag{7}$$

To deal with the quadratic single integral and double integral terms, we introduce Lemmas 1 and 2:

Lemma 1 (*Jensen's inequality Gu et al. [25], Sun et al. [26]*). *Let ω be a differentiable signal in $[\alpha,\beta]\to\mathbb{R}^n$, for positive definite matric $R\in\mathbb{R}^{n\times n}$, the following inequalities hold:*

$$(\alpha-\beta)\int_\beta^\alpha\omega^T(s)R\omega(s)ds\geq\left(\int_\beta^\alpha\omega(s)ds\right)^TR\left(\int_\beta^\alpha\omega(s)ds\right)\tag{8}$$

$$\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_s^\alpha\omega^T(s)R\omega(s)dsd\theta\geq\left(\int_\beta^\alpha\int_s^\alpha\omega(s)dsd\theta\right)^TR\left(\int_\beta^\alpha\int_s^\alpha\omega(s)dsd\theta\right)\tag{9}$$

$$\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_\beta^s\omega^T(s)R\omega(s)dsd\theta\geq\left(\int_\beta^\alpha\int_\beta^s\omega(s)dsd\theta\right)^TR\left(\int_\beta^\alpha\int_\beta^s\omega(s)dsd\theta\right)\tag{10}$$

Lemma 2 (*Free-matrix-based integral inequality Zeng et al. [28]*). *Let x be a differentiable signal in $[\alpha,\beta]\to\mathbb{R}^n$, for symmetric matrices $R\in\mathbb{R}^{n\times n}$, X, $Z\in\mathbb{R}^{3n\times 3n}$, and any matrices $Y\in\mathbb{R}^{3n\times 3n}$, N_1, $N_2\in\mathbb{R}^{3n\times n}$ satisfied*

$$\begin{bmatrix}X&Y&N_1\\ *&Z&N_2\\ *&*&R\end{bmatrix}\geq 0$$

the following inequality holds:

$$-\int_\alpha^\beta\dot x^T(s)R\dot x(s)ds\leq\varpi^T\hat\Omega\varpi\tag{11}$$

where $\hat\Omega=(\beta-\alpha)(X+\frac{1}{3}Z)+\mathrm{Sym}\{N_1G_1+N_2G_2\}$, $G_1=\bar e_1-\bar e_2$, $G_2=2\bar e_3-\bar e_1-\bar e_2$, $\bar e_1=[I\ 0\ 0]$, $\bar e_2=[0\ I\ 0]$, $\bar e_3=[0\ 0\ I]$, $\varpi=[x^T(\beta)x^T(\alpha)\frac{1}{\beta-\alpha}\int_\alpha^\beta x^T(s)ds]^T$.

Remark 1. It is worth mentioning that if we let $X=N_1R^{-1}N_1^T$, $Y=N_1R^{-1}N_2^T$, $Z=N_2R^{-1}N_2^T$, $N_1=\frac{1}{\beta-\alpha}[-R\ R\ 0]^T$ and $N_2=\frac{3}{\beta-\alpha}[R\ R\ -2R]^T$, then the free-matrix-based integral inequality is written as the Wirtinger-based integral inequality [27]. That is to say, the Wirtinger-based integral inequality shown to be more tighter than Jensen's inequality is a special case of the