Neurocomputing 205 (2016) 498–506

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief Papers

Master–slave synchronization of heterogeneous dimensional delayed neural networks

Tao Li [a,*], Ting Wang [b], Guobao Zhang [c], Shumin Fei [c]

[a] School of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[b] School of Information Science and Technology, Nanjing Forestry University, Nanjing 210042, PR China
[c] School of Automation, Southeast University, Nanjing 210096, PR China

ARTICLE INFO

Article history:
Received 11 December 2015
Received in revised form
20 March 2016
Accepted 6 April 2016
Communicated by Hongyi Li
Available online 13 May 2016

Keywords:
Delayed neural networks (DNNs)
Master–slave synchronization
Reduced-order observer
Lyapunov–Krasovskii functional (LKF)
LMI approach

ABSTRACT

In this paper, the problem on synchronization is studied for a class of master–slave delayed neural networks (DNNs) with heterogeneous dimensions. Through designing a reduced-order observer and choosing an augmented Lyapunov–Krasovskii functional, a delay-dependent stability criterion on the error system is presented and the synchronization one is formulated in terms of linear matrix inequalities (LMIs) via parameters' transformation. Especially, since some novel inequality techniques are used, those previously ignored information can be reconsidered during the discussion and the conservatism can be effectively reduced. Furthermore, the proposed condition can be conveniently presented and the controller gain can be checked by solving the proposed LMIs. Finally, two numerical examples are presented to illustrate the presented results.

1. Introduction

In the past decades, chaotic synchronization has been extensively discussed due to its potential applications in many engineering fields, such as teleoperation control, secure communication, image processing, and so on. Chaos is a complex nonlinear behavior that can be observed in many dynamic systems, in which different initial states or random disturbances often lead to different dynamic behaviors. Thus, since the pioneering works of Pecora and Carroll were reported [1,2], the synchronization of chaotic systems has received considerable attention. Especially, artificial neural network models can exhibit some chaotic behaviors and therefore, the synchronization of chaotic NNs has become an important issue of scientific research, see the references [9–31] and therein.

Since neural networks were widely utilized [3–5], then as a special dynamical systems, the DNNs have been found to exhibit complex and unpredictable behaviors, such as stable equilibria, periodic oscillations, bifurcation, and chaotic attractors [6–11]. Then many works on synchronization have appeared and a large number of elegant results have been proposed [10–30]. Especially recently, in [12], by utilizing M-matrix, the adaptive exponential synchronization in p-th moment was considered for neutral-type DNNs with Markovian switching. In [13], the exponential synchronization was studied by virtue of intermittent control and mathematical induction technique. In [14–16], as for memristor-based DNNs, the exponential synchronization and non-fragile one have been studied via using fuzzy theory and observer-based one. The fixed-time synchronization of Cohen–Grossberg DNNs was investigated [17], and its convergence time relied on the initial synchronization errors. However the above results in [12–17] cannot be easily checked. Considering fuzzy DNNs with Markovian jumping, the synchronization criteria were obtained in terms of LMIs through dividing delay interval into two parts [18]. The robust dissipativity-based synchronization was fully investigated with actuator failures, and a desired fault-tolerant controller was designed [19]. In [20], based on LMI technique, a non-fragile procedure was introduced to study master–slave case and the controller gain fluctuation appeared in a random way. In [21], as for memristor-based BAM DNNs, some LMI-based conditions were presented to guarantee the synchronization with the random impulse. Meanwhile, in order to implement continuous-time DNNs for simulation or computational purposes, it is important to formulate discrete-time DNNs from the continuous-time ones by using a discretization technique. Ideally, the discrete-time analogue should inherit the dynamical behaviors of the continuous-time networks and maintain the functional similarity to the continuous-time ones. Unfortunately, they cannot preserve the dynamics of the continuous-time counterpart even for a small sampling period [22,23]. Thus in view of discrete-time case, through using saturation control [24] and stochastic dropouts [25], some delay-dependent synchronization criteria were obtained in the form of LMIs. Meanwhile, in many

* Corresponding author. Tel.: +86 25 84892301.
E-mail address: autolitao@nuaa.edu.cn (T. Li).

http://dx.doi.org/10.1016/j.neucom.2016.04.051
0925-2312/

practical cases, the digital controllers were more efficient than the continuous ones owing to that they can achieved the synchronization only by using the sampled data of the states in both the master system and slave one at discrete time instants. Thus in [26–31], the controllers based sampled-data has been presented to study the master–slave synchronization in DNNs. In [26], a sampled-data synchronization scheme was developed for distributed parameter DNNs. The variable sampling control was deeply studied and some LMI results have been reported [27–29], in which they heavily depended upon the maximum value of sampling interval. Most recently, since the event-triggering idea was proven to be more effective to extend the application area [31], the work [30] has studied the decentralized event-triggered scheme and an integrated error model was built to couple the scheme and time-varying delay in a unified framework, in which the co-design of the controllers was given.

Though the methods mentioned above are elegant, there still exist several points waiting for improvements. Firstly, when the LKF method was used to tackle the delay-dependence, many effective techniques have been proposed, such as free-weighting matrix, Moon's inequality, and convex combination. Yet in [18–30], when the integral term $\int_{t-\tau}^{t}\dot{x}^T(s)Q\dot{x}(s)ds$ in the derivative of LKF was dealt with, many useful information still has been ignored. Recently, the works [37,38] have presented a Wirtinger-based inequality to tackle the issue on time-delay, which can be more efficient than those present ones. However, no matter how they effectively dealt with the derivative of double- or triple-integral LKF, the time-delay involved was still constant one. As we know, time-delay is always variable. Thus some novel and improved techniques need to be put forward to consider this point. Secondly, as for the master–slave synchronization [10–30], the common assumptions were that the dimensions of master system and slave one should be the same or even the forms of two systems were identical except for the state and initial conditions. Yet from a practical viewpoint, it is favorable and more meaningful to allow the dimensions and forms of master and slave systems to be different, i.e., the dimensions of two systems are heterogeneous.

Motivated by the discussions above, in this work, the problem on master–slave synchronization for DNNs with heterogeneous dimensions will be deeply investigated. Firstly, an improved Lyapunov–Krasovskii functional will be constructed and some effective techniques will be utilized to estimate the LKF derivative more tightly. Furthermore, the derived criteria are presented in terms of LMIs and their feasibility can be easily tested by Matlab LMI Toolbox. Finally, two numerical examples will be presented to illustrate the efficiency of the derived results.

Notations: Throughout this paper, the term $\mathbf{N}$ stands for the set of positive integers, $\mathbf{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbf{R}^{m\times n}$ is the set of $m\times n$ constant matrices; $\mathbf{sym}\{X\}$ means the sum of X and its symmetric matrix, i.e., $\mathbf{sym}\{X\}=X+X^T$. The notation $X>0$ (respectively, $x\geq 0$) means that the matrix X is a real symmetric positive-definite (positive semi-definite).

2. Problem formulations

Consider a master–slave system with heterogeneous dimensions where the model of the master system is described by the following DNNs:

$$\begin{cases}\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau(t)))+L,\\ y(t)=Dx(t),\end{cases}\tag{1}$$

where $x(t)\in\mathbf{R}^s$ and $y(t)\in\mathbf{R}^{m_n}$ respectively represent the state vector and output one of the master system. Here $x(t)=[x_1^T(t)\ x_2^T(t)\ \cdots\ x_n^T(t)]^T$ with $x_i(t)\in\mathbf{R}^{m_i}$ ($m_i\in\mathbf{N}$), $\sum_{i=1}^{n}m_i=s$; $f(x(\cdot))=[f_1^T(x_1(\cdot))\ f_2^T(x_2(\cdot))\ \cdots\ f_n^T(x_n(\cdot))]^T\in\mathbf{R}^s$ denotes the neuron activation function with $f_i(x_i(\cdot))\in\mathbf{R}^{m_i}$ ($i=1,\ldots,n$); $L=[L_1^T\ L_2^T\ \cdots\ L_n^T]^T\in\mathbf{R}^s$ is a constant input vector with $L_i\in\mathbf{R}^{m_i}$ for $i=1,\ldots,n$. Furthermore, $y(t)=x_n(t)$, i.e., $D=[0_{m_n\times(s-m_n)}, I_{m_n}]\in\mathbf{R}^{m_n\times s}$; $C=\mathrm{diag}(C_1,C_2,\ldots,C_n)>0$ with $C_i=\mathrm{diag}(c_{i1},c_{i2},\ldots,c_{im_i})\in\mathbf{R}^{m_i\times m_i}$, and A,B are the constant matrices of the proper dimensions.

The dynamics of the slave system are given as the following DNNs:

$$\dot{z}(t)=-\overline{C}z(t)+\overline{A}g(z(t))+\overline{B}g(z(t-\tau(t)))+u(t)+L_n,\tag{2}$$

where $z(t)\in\mathbf{R}^{m_n}$ represents the state vector. Here $\overline{C}=\mathrm{diag}(\overline{c}_1,\overline{c}_2,\ldots,\overline{c}_{m_n})>0$ and $\overline{A},\overline{B}\in\mathbf{R}^{m_n\times m_n}$; $g(z(\cdot))\in\mathbf{R}^{m_n}$ denotes the activation function and $u(t)$ is the control input that will be designed later.

The following assumptions on the systems (1)–(2) are made throughout this paper.

H1. Here, $\tau(t)$ denotes the interval time-varying delay satisfying

$$0\leq\tau(t)\leq\tau_m,\quad \mu_0\leq\dot{\tau}(t)\leq\mu_m,\tag{3}$$

and we denote $\overline{\mu}_m=\mu_m-\mu_0$.

H2. There exist the constants $\sigma_j^+,\sigma_j^-,\epsilon_i^-,\epsilon_i^+$, the activation functions $f_j(\cdot),g_i(\cdot)$ in (1)–(2) satisfy the conditions

$$\sigma_j^-\leq\frac{f_j(\alpha)-f_j(\beta)}{\alpha-\beta}\leq\sigma_j^+,\quad\forall\alpha,\beta\in\mathbf{R},\alpha\neq\beta,j=1,2,\ldots,s;$$

$$\epsilon_i^-\leq\frac{g_i(\alpha)-g_i(\beta)}{\alpha-\beta}\leq\epsilon_i^+,\quad\forall\alpha,\beta\in\mathbf{R},\alpha\neq\beta,i=1,2,\ldots,m_n.$$

Here we also introduce the denotations $\overline{\Sigma}=\mathrm{diag}(\sigma_1^+,\cdots,\sigma_s^+)$, $\Sigma=\mathrm{diag}(\sigma_1^-,\cdots,\sigma_s^-)$, and

$$\Sigma_1=\mathrm{diag}\left(\sigma_1^+\sigma_1^-,\ldots,\sigma_n^+\sigma_s^-\right),\quad \Sigma_2=\mathrm{diag}\left(\frac{\sigma_1^++\sigma_1^-}{2},\ldots,\frac{\sigma_s^++\sigma_s^-}{2}\right).\tag{4}$$

Remark 1. In assumption **H1**, together with the values of both the lower and upper bounds on the derivative of $\tau(t)$, the value of μ_0 is always less than 0 and the one of μ_m is always greater than 0, which can make $\tau(t)$ to be bounded and changing in $[0,\tau_m]$.

To facilitate the analysis, it is assumed that the state matrices $\overline{C},\overline{A},\overline{B}$ and the function $g(\cdot)$ of (2) are available to controller design. The goal of this paper is to design the controller $u(t)$ such that the slave system (2) can asymptotically track the output of the master system (1), i.e., $\lim_{t\to+\infty}\|z(t)-y(t)\|=0$ for any initial conditions $x(t)\in\mathbf{R}^s, z(t)\in\mathbf{R}^{m_n}$ with $t\in[-\tau_m,0]$.

Now based on the methods in [32–36], an observer with the order $s-m_n$ is first designed for the slave system to estimate the unavailable states of the master one. Then an observer-based controller will be given for the slave system. Let $\xi(t)=\left[\xi_1^T(t)\ \xi_2^T(t)\ \cdots\ \xi_{n-1}^T(t)\right]^T\in\mathbf{R}^{s-m_n}$ be the state vector of the reduced-order observer with $\xi_i(t)\in\mathbf{R}^{m_i}$ ($i=1,\ldots,n-1$). For the convenience, we can divide the matrices C,A,B into the following forms,

$$C=\begin{bmatrix}C_1&0&\cdots&0\\0&C_2&\cdots&0\\\vdots&\vdots&\ddots&\vdots\\0&0&\cdots&C_n\end{bmatrix},\quad A=\begin{bmatrix}A_{11}&A_{12}&\cdots&A_{1n}\\A_{21}&A_{22}&\cdots&A_{2n}\\\vdots&\vdots&\ddots&\vdots\\A_{n1}&A_{n2}&\cdots&A_{nn}\end{bmatrix},$$

$$B=\begin{bmatrix}B_{11}&B_{12}&\cdots&B_{1n}\\B_{21}&B_{22}&\cdots&B_{2n}\\\vdots&\vdots&\ddots&\vdots\\B_{n1}&B_{n2}&\cdots&B_{nn}\end{bmatrix},\quad \begin{matrix}C_i\in\mathbf{R}^{m_i\times m_i},\\A_{ij}\in\mathbf{R}^{m_i\times m_j},\\B_{ij}\in\mathbf{R}^{m_i\times m_j},\\i,j=1,2,\ldots,n.\end{matrix}\tag{5}$$

Now denoting $\zeta(t)=[\xi^T(t)\ z^T(t)]^T$ and letting $\varepsilon(t)=\zeta(t)-x(t)$, one can easily check that as for DNNs (1) and (2), the master–slave synchronization can be achieved when $\lim_{t\to+\infty}\|\zeta(t)-x(t)\|=\lim_{t\to+\infty}\|\varepsilon(t)\|=0$.