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Simulation and evaluation of interval-valued fuzzy linear Fredholm integral equations with interval-valued fuzzy neural network

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A B S T R A C T

In the present paper, using generalizations of the fuzzy integral equations for interval-valued fuzzy sets, we introduce and study new generalized interval-valued fuzzy linear Fredholm integral equation concepts. The work of this paper is an expansion of the research of real fuzzy linear Fredholm integral equations. In this paper interval-valued fuzzy neural network (IVFNN) can be trained with crisp and interval-valued fuzzy data. In this paper, a novel hybrid method based on IVFNN and Newton–Cotes methods with positive coefficient for the solution of interval-valued fuzzy linear Fredholm integral equations (IVFLFIEs) of the second kind is presented. Within this paper the fuzzy neural network model is used to obtain an estimate for the fuzzy parameters in a statistical sense. Then a simple algorithm from the cost function of the interval-valued fuzzy neural network is proposed, in order to find the approximate parameters. We propose a learning algorithm from the cost function for adjusting of interval-valued fuzzy weights. Here neural network is considered as a part of a larger field called neural computing or soft computing. Finally, we illustrate our approach by some numerical examples.

1. Introduction

Artificial neural networks are an exciting form of artificial intelligence which mimic the learning process of the human brain in order to extract patterns from historical data [67]. For many years this technology has been successfully applied to a wide variety of real-word applications [65]. Simple perceptrons need a teacher to tell the network what the desired output should be. These are supervised networks. In an unsupervised net, the network adapts purely in response to its inputs. Such networks can learn to pick out structure in their input. Fig. 4 shows typical three layers perceptron. Multi layered perceptrons with more than three layers, use more hidden layers [25,35]. Multi layered perceptrons correspond the input units to the output units by a specific nonlinear mapping [68]. The most important application of multi layered perceptrons is their ability in function approximation [11]. From Kolmogorov existence theorem we know that a three layered perceptron with $n(2n+1)$ nodes can compute any continuous function of n variables [45,28]. The accuracy of the approximation depends on the number of neurons in the hidden layer and does not depend on the number of the hidden layers [39]. Fuzzy neural network (FNN) systems are hybrid systems that combine the theories of fuzzy logic and neural networks. Designing the FNN system based on the input–output data is a very important problem [41,69]. FNN have been extensively studied [10,17]. Several authors investigated FNN, to compute crisp and even fuzzy informations with NN. An overview of different FNN architectures is given by Buckley and Hayashi in [13]. There existed only a few approaches to learning algorithms (see [24]) for FNN when Ishibuchi et al. presented two NNs which can be trained with interval vectors and with vectors of fuzzy numbers (see [29]). In both networks Ishibuchi et al. used crisp weights. For these networks they presented a backpropagation based learning algorithm. In a later paper Ishibuchi et al. [30] developed a FNN with symmetric triangular fuzzy numbers as weights. For this NN they evolved a learning algorithm in which the backpropagation algorithm is used to compute the new lower and upper limits of the support of the weights. The modal value of the new fuzzy weight is calculated as the average of the newly computed limits. Recently, FNN successfully used for solving fuzzy polynomial equation and systems of fuzzy polynomials [4,5], approximate fuzzy coefficients of fuzzy regression models [57–59,62], approximate solution of fuzzy linear systems and fully fuzzy linear systems [63,64] and fuzzy differential equations [55,56]. One of the major applications of FNN is treating control and synchronization of chaos [42–44,70,72,75,76]. Lagaris and Likas in [38] used multilayer perceptron to estimate the solution of differential equation. Their neural network model was trained over an interval (over which the differential equation must be solved), so the inputs of the neural network (NN) were the

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training points. Recently Effati and Pakdaman [20] used three layers NN to estimate the fuzzy solution of differential equation. They used parametric form of fuzzy numbers. The comparison of their method with the existing numerical method shows that their method was more accurate and the solution had also more generalizations.

The theory and application of integral equations (IEs) are important subjects within applied mathematics. Integral equations of various types appear in many fields of science and engineering. IE is encountered in a variety of applications in many fields including continuum mechanics, potential theory, geophysics, electricity and magnetism, kinetic theory of gases, hereditary phenomena in physics and biology, renewal theory, quantum mechanics, radiation, optimization, optimal control systems, communication theory, mathematical economics, population genetics, queuing theory, medicine, mathematical problems of radiative equilibrium, the particle transport problems of astrophysics and reactor theory, acoustics, fluid mechanics, steady state heat conduction, fracture mechanics, and radiative heat transfer problems. Fredholm integral equation (FIE) is one of the most important integral equations.

A computational approach to solving integral equation is an essential work in scientific research [66]. Some methods for solving second kind FIE are available in the open literature. The B-spline wavelet method, the method of moments based on B-spline wavelets by Maleknejad and Sahlan [50], and variational iteration method by He [26,27] have been applied to solve second kind Fredholm linear integral equations. The learned researchers Maleknejad et al. proposed some numerical methods for solving linear Fredholm integral equations system of second kind using Rationalized Haar functions method, Block-Pulse functions, and Taylor series expansion method [48,49,51]. Haar wavelet method with operational matrices of integration [40] has been applied to solve system of linear FIEs of second kind. B-spline wavelet method [37], wavelet Galerkin method [46], and also VIM [12] can be applied to solve nonlinear FIE of second kind. Mosleh and Otadi in [60] used the least squares approximation method for the solution of Hammerstein-Volterra delay integral equations. Some iterative methods like Homotopy perturbation method [21,33] and Adomian decomposition method [1,8,2] have been applied to solve nonlinear FIE of second kind.

Fuzzy Linear integral equations arise frequently in physical problems as a result of the possibility of super-imposing the effects due to several reasons. The most important contribution of the theory of fuzzy integral equations consists in the solution of fuzzy initial and boundary value problems. The fuzzy boundary value problems for equations of elliptic type can be reduced to fuzzy Fredholm integral equations while the study of parabolic and hyperbolic fuzzy differential equations leads to fuzzy Volterra integral equations such as longitudinal vibrations. The appropriate interval-valued fuzzy differential equation for modeling the above problem is $\frac{d^2\varphi}{dt^2} = F(t)$ where $F(t)$ is a known continuous interval-valued fuzzy function. When a physical problem is transformed into a deterministic linear Fredholm integral equations, we usually cannot be sure that this modeling is perfect. If the nature of errors is random, then instead of a deterministic problem linear Fredholm integral equations we will get a random integral equation. But if the underlying structure is not probabilistic, e.g. because of subjective choice, then it may be appropriate to use fuzzy numbers instead of real random variables. Fuzzy integral equations (FIEs) have been rapidly growing in recent years. The fuzzy mapping function was introduced by Chang and Zadeh [14]. Later, Dubois and Prade [18] presented an elementary fuzzy calculus based on the extension principle also the concept of integration of fuzzy functions was first introduced by Dubois and Prade [19]. Alternative approaches were later suggested by Goetschel and Voxman [22], Kaleva [34], Matloka [52], Nanda [61] and others. While Goetschel and Voxman [22] and later Matloka [52] preferred a Riemann integral type approach, Kaleva [34] chose to define the integral of fuzzy function, using the Lebesgue type concept for integration. Babolian et al. and Abbasbandy et al. in [3,9] obtained a numerical solution of linear Fredholm FIEs of the second kind with Adomian decomposition. Mosleh [53,54] used FNN for approximate solution of fuzzy linear and nonlinear Fredholm integro-differential equation.

In the present paper, using generalizations of the fuzzy integral equations for interval-valued fuzzy sets, we introduce and study new generalized interval-valued fuzzy linear Fredholm integral equations concepts. In this paper interval-valued fuzzy neural network (IVFNN) can be trained with crisp and interval-valued fuzzy data. In this work we concentrate on numerical procedures for solving IVFLFIEs using innovative mathematical tools and neural-like systems of computation. In this proposed method, IVFNN is applied as a universal approximator. The rest of this paper is organized as follows: In Section 2, basic concepts of fuzzy numbers are presented. In Section 3, we are introduced linear Fredholm integral equations. In Section 4, we use numerical experiments to illustrate the efficiency of our algorithm.

2. Basic concepts of fuzzy numbers

2.1. Generalized fuzzy numbers

In this section, we briefly review basic concepts of generalized fuzzy numbers. Chen and Wei [15,71] represented a generalized triangular fuzzy number represented a generalized triangular fuzzy number $\tilde{A}$ as $\tilde{A} = (a_1, a_2, a_3; w)$ where a_1,a_2 and a_3 are real values and $0 < w \le 1$, as shown in Fig. 1. The membership function $\mu_{\tilde{A}}$ of a generalized fuzzy number $\tilde{A}$ satisfies the following conditions:

(1) $\mu_{\tilde{A}}$ is a continuous mapping from the universe of discourse $\mathbb{R}$ to the closed interval in [0, 1];
(2) $\mu_{\tilde{A}} = 0$, where $-\infty < x \le a_1$;
(3) $\mu_{\tilde{A}}$ is monotonical increasing in $[a_1, a_2]$;
(4) $\mu_{\tilde{A}} = w$, where $x = a_2$;
(5) $\mu_{\tilde{A}}$ is monotonical decreasing in $[a_2, a_3]$;
(6) $\mu_{\tilde{A}} = 0$, where $a_3 \le x < +\infty$.

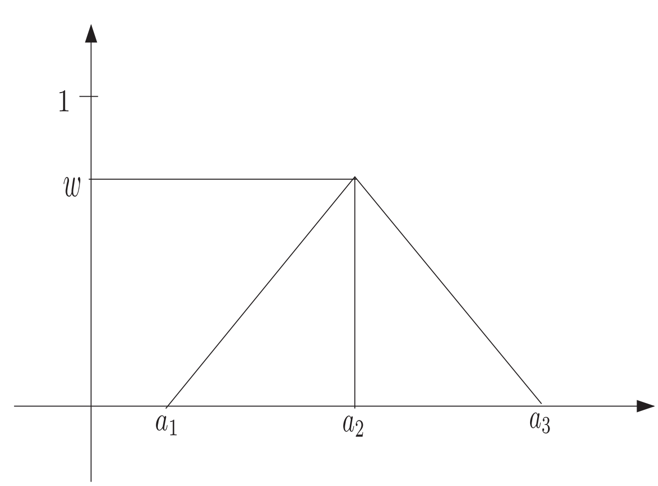

Fig. 1. A generalized triangular fuzzy number.