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Globally optimal distributed cooperative control for general linear multi-agent systems

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ABSTRACT

This paper aims to design distributed consensus protocols which satisfy two design requirements for identical general linear multi-agent systems on fixed, undirected graphs: meeting the global optimality and guaranteeing a prescribed convergence speed. By using inverse optimal approaches, the optimal partial stabilization is developed and the globally optimal distributed consensus problem for leader following and leaderless problems are solved. To obtain prescribed convergence speed of the multi-agent system, novel globally optimal distributed consensus design procedures are proposed. First, combining with the regional pole assignment, the optimal control can be found by solving a strict linear matrix inequality (LMI) problem. It turns out that the increasing number of the agent nodes will not increase the computational complexity. Then, a modified linear quadratic regulator (MLQR) design method is developed which leads to a model free design procedure by employing the adaptive dynamic programming (ADP) technique. Finally, a numerical example is given to illustrate the effectiveness of the proposed procedures.

1. Introduction

Cooperative distributed control of multi-agent systems has received increasing demands and been a priority research subject for a variety of military and civilian applications. All agents can communicate with each other and exchange information and use the data to develop distributed but coordinated control policies. Generally, the multi-agent systems can be categorized into the leader following consensus problem and the leaderless consensus problem. Fruitful results have been obtained, see [1–13] of the existing literature is dedicated to find optimal strategy subject to some given constraints, especially concerning some specific cost functions and the convergence speed [13].

In [10], Ren et al. proposed an open research problem, that is, how to design the optimal coordination control which not only guarantees the consensus of the multi-agent systems, but also minimizes some performance indexes. Optimality of the control protocols gives rise to desirable properties such as robust stability. The main difficulty is that the globally optimal problems generally require global information of agents which is difficult to obtain in most applications. In addition, for multi-agent systems, the graph topology interplays with the system dynamics, hence the globally optimal control problems are fairly complicated. A series of papers [14–22] addressed this issue. In the case of agents with identical linear time-invariant dynamics, a suboptimal design using local LQR design method was presented in [15]. The distributed games on graphs were studied by introducing the notion of iterative Nash equilibrium in [16], where each agent only minimizes its own performance index. In [17], consensus problem of multi-agent differential games of nonlinear systems based on the optimal coordination control was solved via fuzzy adaptive dynamic programming (ADP) approach. There are few papers working on globally optimal cooperative control for multi-agent systems. In [18], the optimal linear-consensus algorithm for multi-agent systems with single-integrator dynamics was proposed by defining two different global performance indexes. In [19], the LQR based inverse optimal design method was proposed to obtain the optimal distributed consensus protocols by constructing a global performance index, the lower bound of the coupling gain of the protocols was fairly complicated.

The convergence speed, which characterizes how fast consensus can be achieved therefore is desirable to optimize [13]. In the available studies, much attention has been paid to the cooperative control of single and double integrator systems. For multi-agent systems consisting of single-integrator kinematics, the smallest nonzero eigenvalue of the Laplacian matrix determines the worst-case convergence speed [2], hence the convergence speed has been maximized by choosing optimal weights associated with edges [23,24]. Another commonly way focused on the

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case when all agents converge to the average of the initial states [25] which makes the optimization problems challenging if it is unknown. In [30], it proved that the real part of the closed-loop poles of the general linear multi-agent can be assigned as negative as possible.

In our previous work [20], using the inverse optimal approach, the globally optimality and consensus speed issues were first considered together for identical linear multi-agent systems. However, the input matrix was assumed to be of full column rank and the resulting gain of the controller was very high, which would be unacceptable especially when the poles converge to the pre-specified objective slow dramatically. This restricts the application of the method. From practical view of point, to obtain satisfactory system performance, it generally just needs to assign the closed-loop poles to a specified region, not necessary to be precisely.

Adaptive dynamic programming (ADP) has been proved efficiently to solve optimal control problems for uncertain systems [31–40]. For identical linear multi-agent systems, the conventional LQR optimal solution can be obtained via ADP without the knowledge of the system dynamics [39]. However, the existing LQR methods [15,19,31,18] have encountered difficulties in addressing the convergence speed problems for certain and uncertain linear multi-agent systems. In [20], the prescribed convergence speed can be only obtain using partial pole assignment, but the dynamics of the agent system must be precisely known. Therefore, an alternative design method is required to consider such a practical issue, further, together with the optimality in performance.

Motivated by the above facts, in this paper, we develop a new inverse optimal scheme to solve the globally optimal cooperative problem for the general linear multi-agent systems. Firstly, we extend the inverse optimal approaches to solve the optimal partial stabilization for the general linear systems, where the input matrix can be not full column rank. Then the globally optimal distributed protocols are designed for the leader following consensus and leaderless consensus problem. The results indicate that if the feedback gain is optimal for the linear node with respect to some cost functions, then the resulting distributed protocol using it becomes a candidate with respect to some global cost functions. Such global cost functions are constructed and proved to be the interaction-related [18]. The feedback gain can be found by solving a LMI problem under the poles assign demands, i.e., all closed-loop poles can be assigned in specified region to achieve desired convergence speed. Inspired by this, a modified linear quadratic regulator (MLQR) optimal design method is developed. The MLQR can be transformed into a standard LQR, thus a completely model free design procedure is proposed using adaptive dynamic programming.

The contributions of the paper include:

- The inverse optimal approaches are developed for the general linear systems. Compared with [20], the input matrix B is not necessary to be full of column rank and the performance index to be minimized can be constructed directly.
- The globally optimal cooperative problem for the general linear leaderless and leader following multi-agent systems are well solved by using the inverse optimal approach. It turns out that the resulting linear distributed protocols naturally are globally optimal. The developed methods are also valid for a class of digraphs.
- By using the regional pole assignment technique, the globally optimal feedback gain can be design such that all closed-loop eigenvalues are assigned in specified region to achieve desired convergence speed rather than assigned precisely in [20]. The tremendous benefit brought by this is that the resulting gain is much lower thus of more practical value. It is also worth pointing out that the increasing number of the agent nodes of the multi-agent system will have no affection on the computational complexity of the design procedures.
- A modified linear quadratic regulator (MLQR) design method is developed which can guarantee prescribed convergence speed for multi-agent systems. Then a model free design procedure is given by employing an online adaptive dynamic programming (ADP) algorithm.

The rest of the paper is organized as follows. In Section 2, we show some concepts of the graph theory and the inverse optimal approaches for general linear systems. The main results of this paper are given in Sections 3 and 4, which solves the globally optimal distributed consensus problem and proposes the design procedures to obtain prescribed convergence speed, respectively. In Section 5, a numerical example is given to illustrate the effectiveness of the proposed methods. Conclusions are given in Section 6.

2. Preliminaries

Notations: $A>0(>0)>$ means matrix A is positive (negative) definite, $A\geq 0(\leq 0)$ means matrix A is positive (negative) semi-definite. The Kronecker product is denoted by $\otimes$. The transposition of matrix A is denoted by A^T. I_n denotes the n dimensional identity matrix in $\mathbb{R}^{n\times n}$. $1_n \in \mathbb{R}^n$ is the vector with all elements 1. $\ker(A)$ denotes the null space of matrix A.

2.1. Graph theory

Consider a weighted digraph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ with a nonempty finite set of N nodes $\mathcal{V}=\{v_1,v_2,\ldots,v_N\}$, a set of edges $\mathcal{E}\subset\mathcal{V}\times\mathcal{V}$ and the associated adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$. An edge rooted at node j and ended at node i is denoted by (v_j,v_i), which means that the information flows from node j to node i. The weight a_{ij} of edge (v_j,v_i) is positive, i.e., $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, otherwise, $a_{ij}=0$. In this paper, we assume that there are no repeated edges and no self-loops, i.e., $a_{ii}=0$, $\forall i\in\mathcal{N}$, where $\mathcal{N}=\{1,2,\ldots,N\}$. If $(v_j,v_i)\in\mathcal{E}$, then node j is called a neighbor of node i. The set of neighbors of node i is denoted as $\mathcal{N}_i=\{j\,|\,(v_j,v_i)\in\mathcal{E}\}$. Define the in-degree matrix as $D=\mathrm{diag}\{d_i\}\in\mathbb{R}^{N\times N}$ with $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$ and the Laplacian matrix as $L=D-\mathcal{A}$. Hence, $L1_N=0$. A graph is said to be connected if every two vertices can be joined by a path. A graph is said to be strongly connected if every two vertices can be joined by a directed path. If $\mathcal{G}$ is strongly connected, then the zero eigenvalue of is simple, and $\ker L=\mathrm{span}\{1_N\}$. If there is a node i_r, such that there is a directed path from the node i_r to every other nodes in a digraph, then it is said to have a spanning tree.

2.2. Inverse optimality of general linear systems

In this subsection, we aim to develop the optimal partial stabilization for the general system by using the inverse optimal approach [26,27]. Partial stabilization plays an important role in addressing the leaderless consensus problem by the Lyapunov approaches [21], since the consensus space that all state trajectories converging to may not be the zero-dimensional space, especially when the system dynamics of the nodes are open-loop unstable. There, the specified space is generally the consensus subspace, say, the null space of the Laplacian matrix of an undirected, strongly connected graph.