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Delay-dependent exponential stabilization of nonlinear fuzzy impulsive systems with time-varying delay

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ABSTRACT

This paper investigates the problem of exponential stabilization for a class of nonlinear fuzzy impulsive systems with time-varying delay. Firstly, the systems are expressed by the extended Takagi–Sugeno (T–S) fuzzy model. Secondly, the combination of Lyapunov–Krasovskii type functionals and the parallel distributed compensation (PDC) idea is employed to design a state feedback controller such that the closed-loop systems are globally exponentially stable. The corresponding sufficient delay-dependent conditions are derived in terms of linear matrix inequalities (LMIs). Finally, two examples are presented to demonstrate the effectiveness of the theoretical contribution.

1. Introduction

In the past two decades, issues related to fuzzy-logic control of nonlinear systems have received considerable attention since fuzzy-logic control methods have capability of designing and analyzing nonlinear systems effectively. Among various fuzzy methods, fuzzy-model-based control is widely investigated and employed in that the stability analysis and controller synthesis problems of the overall system can be conducted systematically by using well-established classical linear systems theory [1–5]. More recently, the stability analysis and controller synthesis problems of Takagi–Sugeno (T–S) fuzzy systems have received particular attention [2–17].

It is evident that time delay is a commonly encountered source of instability and poor performance of systems [18–24]. Stability criteria can be categorized: delay-dependent stability criteria [20,21] and delay-independent stability criteria [18,19]. Many researchers have investigated the delay-related stability of fuzzy systems [15–21,25–28]. Generally speaking, the latter is more conservative than the former when time delay is trivial.

In practice, there exist natural phenomena that systems states might be changed abruptly at certain moments. It is assumed that these perturbations act instantaneously, i.e., in the form of impulses [29,30]. Impulsive disturbances can severely degrade closed-loop system performance and even make a stable system unstable [31,32]. Over the past few decades, the qualitative properties of impulsive differential equations have been intensively studied [29,30,33–35]. The development of impulsive fuzzy differential equations was initiated by [34], and was extended to impulsive functional differential inclusions in [33]. Meanwhile, the stability analysis and controller synthesis problems of impulsive systems have received considerable attention by many researchers, e.g., [36–39]. For instance, the authors investigated the problem of robust decentralized stabilization for a class of large-scale, time delay, and uncertain impulsive dynamical systems [37]. Several criteria were established for robust stability, robust asymptotic stability and robust exponential stability of uncertain impulsive dynamical systems in [38].

Recently, the stability analysis and controller synthesis problems of T–S fuzzy impulsive systems have been paid considerable attention [40–43]. In [41], a class of nonlinear fuzzy impulsive systems was defined by extending the ordinary T–S fuzzy model and sufficient conditions were derived for global exponential stability of closed-loop systems. Several criteria for uniform stability and uniform asymptotic stability of T–S fuzzy time-delay systems with impulse were proposed in [40]. The problem of robust fuzzy control for a class of nonlinear fuzzy impulsive systems with time-varying delay was investigated and sufficient conditions for global exponential stability of the closed-loop system were proposed in [42]. In [43], the authors investigated the problem of robust fuzzy

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control for a class of nonlinear fuzzy impulsive stochastic systems with time-varying delays. Moreover, there are some interesting applications of impulsive control or synchronization of chaotic systems based on the T–S fuzzy model [32,44,45]. To the best of our knowledge, however, there are few results on delay-dependent exponential stabilization of fuzzy impulsive systems with time-varying delay, which motivates this study. In the design of controller systems, one is interested in both global stability and other performance properties. In particular, it is expected that the closed-loop systems converge quickly and guarantee the require performance. This drives researchers to study the exponential stability analysis problem of neural networks [46,47], stochastic systems [48], and others. The authors in [49] investigated the global exponential stability and global asymptotic stability of neural networks with impulsive effects and time-varying delays.

In this paper, we investigate the problem of exponential stabilization for a class of nonlinear fuzzy impulsive systems with time-varying delay by employing Lyapunov–Krasovskii type functionals. The main contributions of our paper include: (1) Both nominal and uncertain fuzzy impulsive systems with time-varying delay are considered by extended the ordinary T–S fuzzy model; (2) The controllers for the fuzzy impulsive systems with time-varying delay are proposed by employing the parallel distributed compensation (PDC) idea; (3) Some conserver delay-dependent conditions in terms of linear matrix inequalities are derived to guarantee the global exponential stability of the closed-loop system.

The remainder of the paper is arranged as follows. In Section 2, a class of nonlinear fuzzy impulsive systems with time-varying delay is defined by extending the ordinary T–S fuzzy model. The parallel distributed compensation (PDC) idea is employed to design a state feedback controller. In Section 3, sufficient delay-dependent conditions for global exponential stability of the closed-loop system are derived in terms of linear matrix inequalities (LMIs). The design of a controller for fuzzy impulsive systems with time-varying delay is also proposed. Then the stability analysis and controller synthesis results are extended to uncertain fuzzy impulsive systems with time-varying delay. In Section 4, two examples are presented to show the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notation: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}_+ = [0, \infty)$, $\mathbb{N} = \{0, 1, 2, \ldots\}$, $\mathbb{N}_+ = \{1, 2, \ldots\}$; $\mathbb{R}^n$ denotes n-dimensional Euclidean space; The vector norm of $x \in \mathbb{R}^n$ is Euclidean, i.e., $x = \sqrt{x^T x}$; $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices; I is an appropriately dimensioned identity matrix; For $P \in \mathbb{R}^{n \times n}$, $\lambda_{\min}(P)$ ($\lambda_{\max}(P)$) denotes the smallest (largest) eigenvalue of P. For real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix X–Y is positive semi-definite (respectively, positive definite).

2. Problem formulation and preliminaries

Consider the following nonlinear system with time-varying delay represented by a T–S fuzzy model, which can be called a nonlinear fuzzy impulsive system with time-varying delay.

Plant Rule i

$$\begin{aligned}
&\text{IF } \theta_1(t) \text{ is } M_1^i \text{ and}, \ldots, \text{and } \theta_g(t) \text{ is } M_g^i \\
&\quad \text{THEN} \quad \dot{x}(t) = A_{i1}x(t) + A_{i2}x(t-\tau(t)) + B_i u(t), \quad t \neq t_k, \\
&\quad \Delta x(t_k) = G_{ki}x(t_k), \quad k \in \mathbb{N}_+, \\
&\quad x(t) = \phi(t), \quad t \in [t_0 - \tau_0,\ t_0], \\
&\quad i = 1, 2, \ldots, q,
\end{aligned} \tag{1}$$

where M_j^i is the jth fuzzy set in ith rule, q is the number of rules, $\theta(t) = [\theta_1(t), \theta_2(t), \ldots, \theta_g(t)]^T$ is the premise variable, $x(t) \in \mathbb{R}^n$ is the state vector, $u(t)$ is the control input, $A_{i1} \in \mathbb{R}^{n \times n}$, $A_{i2} \in \mathbb{R}^{n \times n}$ and $B_i \in \mathbb{R}^{n \times m}$ are constant matrices. $\tau(t)$ is the unknown bounded time-varying delay in the state and there exist two real numbers τ_0 and τ_1 such that $0 \leq \tau(t) \leq \tau_0$, $\dot{\tau}(t) \leq \tau_1 < 1$; this constraint is also imposed in [18] for standard fuzzy time-delay systems. Define $\Delta x(t_k) = x(t_k^+) - x(t_k^-)$, $x(t_k^+) = \lim_{t \to t_k^+} x(t)$ and $x(t_k^-) = \lim_{t \to t_k^-} x(t)$. Without loss of generality, it is assumed that $\lim_{t \to t_k^-} x(t) = x(t_k)$, which means that the solution $x(t)$ is left continuous at time t_k. The impulsive matrices $G_{ki} \in \mathbb{R}^{n \times n}$ are constant matrices. The impulsive time instants t_k satisfy $0 \leq t_0 < t_1 < t_2 < \cdots < t_{k-1} < t_k < \cdots$ and $\lim_{k \to \infty} t_k = +\infty$. We assume that there exists a constant $L > 1$, such that $t_k - t_{k-1} \geq L\tau_0$; this constraint was also considered in [37] for large-scale uncertain impulsive dynamical systems with time delay.

In (1), if $u(t) = 0$, then the nonlinear system reduces to an unforced fuzzy impulsive system with time-varying delay. In (1), if $G_{ki} = 0$, $k \in \mathbb{N}_+$, $i = 1, 2, \ldots, q$, then the nonlinear system reduces to a typical continuous T–S time-delay fuzzy model. The stability of this T–S model has been intensively investigated [18–21,25]. And if $\tau(t) = 0$, then the nonlinear system reduces to a fuzzy impulsive system. The stability of this T–S model was investigated in [41]. In (1), if $\tau(t) = 0$, $G_{ki} = 0$, $k \in \mathbb{N}_+$, $i = 1, 2, \ldots, q$, then the nonlinear system reduces to a typical continuous T–S fuzzy model. The stability of this T–S model has been intensively investigated [2,5].

Using the fuzzy inference method with singleton fuzzification, product inference, and center average defuzzification, the overall fuzzy model has the form as below,

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{q} h_i(\theta(t))[A_{i1}x(t) + A_{i2}x(t-\tau(t)) + B_i u(t)], \quad t \neq t_k, \\
\Delta x(t_k) &= \sum_{i=1}^{q} h_i(\theta(t_k)) G_{ki} x(t_k), \quad k \in \mathbb{N}_+,
\end{aligned} \tag{2}$$

where

$$h_i(\theta(t)) = \frac{w_i(\theta(t))}{\sum_{i=1}^{q} w_i(\theta(t))}, \quad w_i(\theta(t)) = \prod_{j=1}^{g} M_j^i(\theta_j(t)).$$

We assume that $w_i(\theta(t)) \geq 0$ and $\sum_{i=1}^{q} w_i(\theta(t)) > 0$. It is clear that

$$h_i(\theta(t)) \geq 0, \sum_{i=1}^{q} h_i(\theta(t)) = 1.$$

The control objective is to design a state feedback fuzzy controller such that the closed-loop system is globally exponentially stable, that is to say, there exists M, $\gamma > 0$ such that

$$\| x(t) \| \leq M\phi_0 e^{-\gamma(t-t_0)} \to 0, t \to +\infty, \tag{3}$$

where $\phi_0 = \sup_{t_0 - \tau_0 \leq t \leq t_0} \| \phi(t) \|$.

Following the PDC idea, the state feedback fuzzy controller is designed as follows,

Plant Rule i

$$\begin{aligned}
&\text{IF} \quad \theta_1(t) \text{ is } M_1^i \text{ and}, \ldots, \text{and } \theta_g(t) \text{ is } M_g^i \\
&\quad \text{THEN} \quad u(t) = -K_i x(t), \\
&\quad i = 1, 2, \ldots, q,
\end{aligned} \tag{4}$$

where $K_i \in \mathbb{R}^{m \times n}, i = 1, 2, \ldots, q$, are constant control gains to be determined later.

By using the fuzzy inference method with singleton fuzzification, product inference, and center average defuzzification, the overall fuzzy regulator is represented by

$$u(t) = -\sum_{i=1}^{q} h_i(\theta(t)) K_i x(t). \tag{5}$$