Neurocomputing 203 (2016) 121–128

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Finite-frequency model reduction of discrete-time T–S fuzzy state-delay systems

Da-Wei Ding [a], Xiangpeng Xie [b,*], Xin Du [c], Xiao-Jian Li [d]

[a] School of Automation and Electrical Engineering, University of Science and Technology Beijing, Beijing, 100083, PR China
[b] Institute of Advanced Technology, Nanjing University of Posts and Telecommunications, Nanjing 210003, PR China
[c] School of Mechatronic Engineering and Automation, Shanghai University, Shanghai 200072, PR China
[d] College of Information Science and Engineering, Northeastern University, Shenyang 110004, PR China

ARTICLE INFO

Article history:
Received 12 September 2015
Received in revised form
7 March 2016
Accepted 19 March 2016
Communicated by Guang Wu Zheng
Available online 12 May 2016

Keywords:
Model reduction
T–S fuzzy systems
Time delay
Finite frequency

ABSTRACT

This paper is concerned with the problem of model reduction for discrete-time Takagi–Sugeno (T–S) fuzzy state-delay systems with finite-frequency input signals. A new finite-frequency model reduction algorithm is proposed, which can get a better approximation performance than the existing full-frequency methods. Firstly, a finite-frequency H_∞ performance index is defined in the frequency domain. Then, a stability condition and a finite-frequency H_∞ performance analysis condition are developed by the aid of Jensen's inequality and Parseval's theorem, respectively. Based on these conditions, sufficient model reduction conditions are derived for discrete-time T–S fuzzy state-delay systems. An optimization algorithm is proposed to obtain a stable reduced-order model satisfying the finite-frequency performance specification. Finally, the effectiveness of the proposed method is illustrated by a numerical example.

1. Introduction

In recent years, there has been a growing interest in T–S fuzzy model [1] since it can approximate a wide class of nonlinear systems or even describe a certain class of nonlinear systems exactly. Stability analysis and control synthesis of T–S fuzzy systems has been fully investigated by various methods: quadratic Lyapunov functions [2–6], piecewise Lyapunov functions [7,8], fuzzy Lyapunov functions [9], homogenous polynomially parameter dependent Lyapunov functions [10–13], adaptive fuzzy approaches [14–16], and Type-2 fuzzy model [17–19], etc.

Model reduction has received considerable attention in the past decades since high or even infinite order mathematical models are frequently used to describe physical systems in many engineering applications. This poses serious difficulties in simulation, analysis and design of systems. Model reduction is an indispensable tool to generate simpler reduced-order model while sacrificing some accuracy. For linear systems, various effective approaches have been proposed, such as the balanced truncation method [20], the optimal Hankel-norm approximation method [21], the aggregation method [22], the Krylov subspace techniques [23], to mention a few. However, for T–S fuzzy systems, few results are available [24–26]. The H_∞ model reduction problem for continuous-time T–S fuzzy stochastic systems was studied in [24], where a convex linearization approach and a projection approach were proposed. The H_∞ model approximation problem for discrete-time T–S fuzzy time-delay systems was considered in [25], where a projection approach was developed to solve the problem.

It is worthy noting that many model reduction problems are inherently frequency dependent in practice, i.e., the requirement on the approximation accuracy over some frequency ranges is more important than others. For linear systems, the frequency weighted balanced truncation method [27] and the finite-frequency H_∞ method [28–30] based on the generalized KYP lemma [31,34] were proposed to solve such problems. However, for T–S fuzzy systems, this issue still remains challenging. Besides, time-delays [35–41] are inevitable in practice, which are the potential sources of instability. This motivates us to develop a novel finite-frequency model reduction method for T–S fuzzy time-delay systems.

This paper considers the model reduction problem of discrete-time T–S fuzzy state-delay systems with finite-frequency input signals. First, a new finite-frequency H_∞ performance index is defined in the frequency domain, which extends the traditional H_∞ performance index. Then, a finite-frequency H_∞ performance analysis condition is developed by the aid of Parseval's theorem. In addition, a stability condition is given by using Jensen's inequality.

* Corresponding author. Tel.: +86-15072304983.
E-mail addresses: dingdawei@ustb.edu.cn (D.-W. Ding), xiexiangpeng1953@sina.com (X. Xie), duxin@shu.edu.cn (X. Du), lixiaojian@ise.neu.edu.cn (X.-J. Li).

http://dx.doi.org/10.1016/j.neucom.2016.03.053
0925-2312/

Based on these conditions and Finsler's lemma, sufficient model reduction conditions are derived for discrete-time T–S fuzzy state-delay systems. An optimization algorithm is proposed to calculate a stable reduced-order model satisfying the finite-frequency performance specification. Finally, the effectiveness of the proposed method is illustrated by a numerical example.

The rest of the paper is organized as follows. Section 2 gives the problem statement and preliminaries. Section 3 gives the main results, where a new finite-frequency model reduction method is proposed for discrete-time T–S fuzzy state-delay systems. Section 4 gives an example to illustrate the effectiveness of the proposed method. Finally, conclusions are given in Section 5.

Notations. For a matrix M, M^* and $M^{\perp}$ denote its conjugate transpose and orthogonal complement, respectively. $M>0(M<0)$ means that M is positive definite (negative definite). The symbol $\star$ will be used in some matrix expressions to induce a symmetric structure. The Hermitian part of a square matrix M is denoted by $\mathrm{He}(M) := M+M^*$.

2. Problem formulation and preliminaries

Consider a class of nonlinear systems described by the following T–S fuzzy time-delay model:

Plant rule i: IF $v_1(k)$ is $\mathrm{F}_1{}^i$, and …, and $v_s(k)$ is $F_s{}^i$, THEN

$$\begin{cases} x_{k+1} = A_i x_k + A_{di} x_{k-l} + B_i u_k \\ y_k = C_i x_k + C_{di} x_{k-l} + D_i u_k, i = 1, 2, \dots, r \\ x_k = 0, \text{ for } k = -l, -l+1, \dots, 0 \end{cases} \tag{1}$$

where $F_1^i, \dots, F_s^i$ are fuzzy sets, $x_k \in \mathbb{R}^n$ is the state, $y_k \in \mathbb{R}^p$ is the measured output, $u_k \in \mathbb{R}^m$ is the energy-bounded input with known finite-frequency range $[\vartheta_1, \vartheta_2]$. A_i, A_{di}, B_i, C_i, C_{di}, D_i are system matrices with compatible dimensions. l denotes the integer constant delay satisfying $0<l<\bar{l}$, where $\bar{l}$ is a positive integer representing the upper bound of the delay. r is the number of IF-THEN rules, and $v_1(k), \dots, v_s(k)$ are the premise variables and measurable.

The fuzzy basis functions are given by

$$h_i(v(k)) = \frac{\prod_{j=1}^{s} \mu_{ij}(v_j(k))}{\sum_{i=1}^{r} \prod_{j=1}^{s} \mu_{ij}(v_j(k))} \tag{2}$$

where $\mu_{ij}(v_j(k))$ is the grade of membership of $v_j(k)$ in F_j^i. In the following, we will drop the argument of $h_i(v(k))$ for brevity. By definition, the fuzzy basis functions satisfy

$$h_i \geq 0, \quad i = 1, 2, \dots, r, \quad \sum_{i=1}^{r} h_i = 1. \tag{3}$$

Let ρ be a set of basis functions satisfying (3). A more compact presentation of the T–S fuzzy delay model is given by

$$\begin{cases} x_{k+1} = A(h) x_k + A_d(h) x_{k-l} + B(h) u_k \\ y_k = C(h) x_k + C_d(h) x_{k-l} + D(h) u_k \end{cases} \tag{4}$$

where

$$A(h) = \sum_{i=1}^{r} h_i A_i, A_d(h) = \sum_{i=1}^{r} h_i A_{di}, B(h) = \sum_{i=1}^{r} h_i B_i$$

$$C(h) = \sum_{i=1}^{r} h_i C_i, C_d(h) = \sum_{i=1}^{r} h_i C_{di}, D(h) = \sum_{i=1}^{r} h_i D_i$$

with $h := (h_1, h_2, \dots, h_r) \in \rho$.

In this paper, we will approximate the T–S fuzzy delay system (4) by the following reduced-order T–S model:

Plant rule i: IF $v_1(k)$ is $\mathrm{F}_1{}^i$, and …, and $v_s(k)$ is $F_s{}^i$, THEN

$$\begin{cases} \hat{x}_{k+1} = \hat{A}_i \hat{x}_k + \hat{A}_{di} \hat{x}_{k-l} + \hat{B}_i u_k \\ \hat{y}_k = \hat{C}_i \hat{x}_k + \hat{C}_{di} \hat{x}_{k-l} + \hat{D}_i u_k, i = 1, 2, \dots, r \\ \hat{x}_k = 0, \text{ for } k = -l, -l+1, \dots, 0 \end{cases} \tag{5}$$

where $\hat{x}_k \in \mathbb{R}^{\hat{n}} (\hat{n} < n)$ is the state of the reduced-order model, $\hat{y}_k \in \mathbb{R}^p$ is the output of the reduced-order model, and $\hat{A}_i$, $\hat{A}_{di}$, $\hat{B}_i$, $\hat{C}_i$, $\hat{C}_{di}$, $\hat{D}_i$ are system matrices to be determined. The reduced-order model can be written in a compact form

$$\begin{cases} \hat{x}_{k+1} = \hat{A}(h) \hat{x}_k + \hat{A}_d(h) \hat{x}_{k-l} + \hat{B}(h) u_k \\ \hat{y}_k = \hat{C}(h) \hat{x}_k + \hat{C}_d(h) \hat{x}_{k-l} + \hat{D}(h) u_k \end{cases} \tag{6}$$

where

$$\hat{A}(h) = \sum_{i=1}^{r} h_i \hat{A}_i, \hat{A}_d(h) = \sum_{i=1}^{r} h_i \hat{A}_{di}, \hat{B}(h) = \sum_{i=1}^{r} h_i \hat{B}_i$$

$$\hat{C}(h) = \sum_{i=1}^{r} h_i \hat{C}_i, \hat{C}_d(h) = \sum_{i=1}^{r} h_i \hat{C}_{di}, \hat{D}(h) = \sum_{i=1}^{r} h_i \hat{D}_i.$$

By combining (4) and (6), we can obtain the following approximation error system

$$\begin{cases} \xi_{k+1} = \overline{A}(h) \xi_k + \overline{A}_d(h) \xi_{k-l} + \overline{B}(h) u_k \\ e_k = \overline{C}(h) \xi_k + \overline{C}_d(h) \xi_{k-l} + \overline{D}(h) u_k \end{cases} \tag{7}$$

where $\xi_k := [x_k^* \;\; \hat{x}_k^*]^*$, $e_k := y_k - \hat{y}_k$, and

$$\overline{A}(h) = \begin{bmatrix} A(h) & 0 \\ 0 & \hat{A}(h) \end{bmatrix}, \quad \overline{A}_d(h) = \begin{bmatrix} A_d(h) & 0 \\ 0 & \hat{A}_d(h) \end{bmatrix}, \quad \overline{B}(h) = \begin{bmatrix} B(h) \\ \hat{B}(h) \end{bmatrix}$$

$$\overline{C}(h) = \begin{bmatrix} C(h) & -\hat{C}(h) \end{bmatrix}, \quad \overline{C}_d(h) = \begin{bmatrix} C_d(h) & -\hat{C}_d(h) \end{bmatrix}, \quad \overline{D}(h) = D(h) - \hat{D}(h).$$

To formulate the finite-frequency model reduction problem, we introduce the following definition.

Definition 1. Let $\gamma > 0$ be a given scalar, then system (7) is said to be with a finite-frequency H_∞ performance bound no larger than γ, if the following inequality

$$\int_{\vartheta_1}^{\vartheta_2} E^*(\theta) E(\theta) d\theta \leq \gamma^2 \int_{\vartheta_1}^{\vartheta_2} U^*(\theta) U(\theta) d\theta \tag{8}$$

holds, where $E(\theta), U(\theta)$ are the Fourier transform of the error signal e_k and the input signal u_k, respectively, and ϑ_1, ϑ_2 are the lower and upper frequency bounds of the input signal, respectively.

Remark 1. When $\vartheta_1 = -\pi$, $\vartheta_2 = \pi$, (8) becomes

$$\int_{-\pi}^{\pi} E^*(\theta) E(\theta) d\theta \leq \gamma^2 \int_{-\pi}^{\pi} U^*(\theta) U(\theta) d\theta$$

By Parseval's theorem [42], we have

$$\sum_{k=0}^{\infty} e_k^* e_k \leq \gamma^2 \sum_{k=0}^{\infty} u_k^* u_k$$

which is the full-frequency H_∞ performance index in [25].

With the help of Definition 1, the finite-frequency model reduction problem can be formulated as follows: For a given stable T–S fuzzy state-delay system (4), frequency bounds $\vartheta_2 \geq \vartheta_1$, and a scalar $\gamma > 0$, find a stable reduced-order model (6) such that the approximation error system (7) satisfies a finite-frequency H_∞ performance (8).

To conclude the section, two lemmas are given, which will be used in the sequel.

Lemma 1 (*[32]: Finsler's lemma*). *For a given vector $\eta \in \mathbb{C}^n$ and matrices $\mathcal{G} \in \mathbb{C}^{n \times n}$, $\mathcal{H} \in \mathbb{C}^{n \times n}$, if the rank of matrix $\mathcal{H}$ is less than n, then the following two statements are equivalent:*

(*i*) $\eta^T \mathcal{P} \eta < 0$ holds for any $\mathcal{H}\eta = 0, \forall \eta \neq 0$.