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LMI-based global exponential stability of equilibrium point for neutral delayed BAM neural networks with delays in leakage terms via new inequality technique ☆

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ABSTRACT

The paper is concerned with a class of neutral BAM neural networks with delays in leakage terms. The existence of equilibrium point dependent on time delays of system (1.1) is obtained by establishing and applying two new inequalities. Sufficient condition is established to ensure the global exponential stability of the above neutral networks by establishing and employing above two new inequalities and new LMI methods. The results of this paper are new and complementary to the previously known results.

1. Introduction

The bidirectional associative memory (BAM) neural networks first introduced by Kosko [1–3] are composed of neurons arranged in two layers, the U-layer and V-layer. The neurons in one layer are fully interconnected to the neurons in the other layer, while there are no interconnections among neurons in the same layer. Through iterations of forward and backward information flows between the two layers, it serves as a two-way associative search for stored bipolar vector pairs and generalizes the single-layer auto-associative Hebbian correlation to a two-layer pattern-matched hetero-associative circuits. Hence, BAM neural networks possess good application prospects in signal processing, image processing, pattern recognition, and associative memories. So far, the global stability and periodic solutions of BAM neural networks with various types of delays have been widely investigated by many authors [4–23,40,54]. So far, some authors have studied the stability of neural networks involving time-delay in the leakage term, for example, see [13,14,24–31,42]. Up to now, there are some papers that have taken neutral-type phenomenon into account in delayed neural networks [8,18,32–39,44–46].

Recently, there are only a few papers [31,43,45] considered the periodic solution or almost periodic solution for neutral-type delayed neural networks with delays in the leakage terms. In [31], the existence and global exponential stability of almost periodic solutions for a class of neutral delay BAM neural networks with the delays in the leakage terms were considered. In [43], the existence and stability of pseudo almost periodic solutions for shunting inhibitory cellular neural networks with neutral type delays and time-varying leakage delays were considered. In [45], the almost periodic solutions for neutral type BAM neural networks with distributed leakage delays on time scales were studied.

To the best of our knowledge, up to now, there are no papers published on the existence and global stability of equilibrium point to neutral delay BAM neural networks with delays in the leakage terms. Motivated by the discussion above, in this paper, we are concerned with the following neutral delayed BAM neural networks with time delays in leakage terms:

$$\begin{cases} x_i'(t)-r_ix_i'(t-c)=-a_ix_i(t-\alpha)+\sum\limits_{j=1}^{m}p_{ij}f_j(y_j(t-\tau))+I_i, & i=1,2,\dots,n,\\ y_j'(t)-r_j^*y_j'(t-d)=-b_jy_j(t-\beta)+\sum\limits_{i=1}^{n}q_{ji}g_i(x_i(t-\sigma))+J_j, & j=1,2,\dots,m, \end{cases} \tag{1.1}$$

where $t\in R$, n, m are the number of neurons in layers, $x_i(t)$ and $y_j(t)$ denote the activations of the ith neuron and the jth neuron at time t, a_i and b_j represent the rates with which ith neuron and jth

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neuron will reset their potential to resting state in isolation when they are disconnected from the network and the external inputs at time t; f_j, g_j are the input–output functions (the activation functions); α, β, τ, σ, c, d are transmission delays at time t, p_{ij}, r_i are elements of feedback templates at time t and q_{ji}, r_j^* are elements of feed-forward templates at time t, I_i, J_j denote biases of the ith neuron and jth neuron at time t, $i=1,2,\ldots,n$, $j=1,2,\ldots,m$; a_i, b_j, τ, σ, c, d, α, β are positive constants, p_{ij}, q_{ji} are constants.

The initial values of system (1.1) are

$$\begin{cases} x_i(s)=\phi_{xi}(s), x_i'(s)=\psi_{xi}(s), & -\theta\le s\le 0,\\ y_j(s)=\phi_{yj}(s), y_j'(s)=\psi_{yj}(s), & -\theta\le s\le 0,\end{cases}\tag{1.2}$$

where $\theta=\max\{c,d,\tau,\sigma,\alpha,\beta\}$, $\phi_{xi}(s),\psi_{xi}(s),\phi_{yj}(s),\psi_{yj}(s)$ are bounded and continuous functions.

It is well known that the existence of equilibrium point for neural networks in existing papers is independent of time delay, on the other hand, in many papers, LMI inequalities were established by using some conventional LMI inequalities. So, our main purpose of this paper is to obtain the existence of equilibrium point dependent on time delay and by establishing and using two new inequalities to obtain LMI based condition for the existence and global exponential stability of equilibrium point for system (1.1). Hence, the main contributions of this paper include two aspects: (1) the existence result of equilibrium point dependent on time delay of system (1.1) is established; (2) two new inequalities are established and applied to obtain the LMI for global exponential stability of equilibrium point of system (1.1).

This paper is organized as follows. Some preliminaries and lemmas are given in Section 2. In Section 3, the sufficient condition is derived for the existence and uniqueness of equilibrium point for (1.1). In Section 4, the sufficient condition is derived for the global exponential stability of equilibrium point for the system (1.1). In Section 5, two illustrative examples are given to show the effectiveness of the proposed theory.

2. Preliminaries

For arbitrary matrix A, A^T stands for the transpose of A, A^{-1} denotes the inverse of A. If A is a symmetric matrix, $A>0$ $(A\ge 0)$ means that A is positive definite (positive semidefinite). Similarly, $A<0$ $(A\le 0)$ means that A is negative definite (negative semidefinite), $\lambda_m(A),\lambda_M(A)$ denote the minimum and maximum eigenvalues of a square matrix A respectively. For any $A=(a_{ij})_{m\times m}\in R^{m\times m}$, we define $\|A\|=\sqrt{\lambda_M(A^TA)}$. Let R^m be an m-dimensional Euclidean space, which is endowed with a norm $\|\cdot\|$ and inner product $(\cdot,\cdot)$, respectively. Given column vector $x=(x_1,x_2,\ldots,x_m)^T\in R^m$, the norm is the Euclidean vector norm, i.e., $\|x\|=(\sum_{i=1}^m x_i^2)^{\frac12}$. $|\cdot|$ denotes the Euclidean norm in R. $|x|=(|x_1|,|x_2|,\ldots,|x_m|)$.

In this paper, we make the following assumptions:

(H_1) There exist positive constants l_j, k_i $(i=1,2,\ldots,n;j=1,2,\ldots,m)$ such that for $\forall x,y\in R$,

$$|f_j(x)-f_j(y)|\le l_j|x-y|;$$

$$|g_i(x)-g_i(y)|\le k_i|x-y|.$$

(H_2) For $i=1,2,\ldots,n;j=1,2,\ldots,m$,

$$\max\{1-a_i-a_i\alpha, a_i\alpha-1-a_i\}\le|r_i|\le 1+a_i+a_i\alpha;$$

$$\max\{1-b_j-b_j\beta, b_j\beta-1-b_j\}\le|r_j^*|\le 1+b_j+b_j\beta.$$

Lemma 1 (Forti and Tesi [47]). *Let $H:R^n\to R^n$ be continuous. Assume that the H satisfies the following conditions.*

1. $H(u)$ is injective on R^n; 2. $\|H(u)\|\to\infty$ as $\|u\|\to\infty$.

Then H is a homeomorphism.

Lemma 2. *If $a>0$, $4ab\ge d^2$, $(de-2af)^2\le(4ab-d^2)(4ac-e^2)$, then $ax^2+by^2+cz^2+dxy+exz+fyz\ge 0$, $\forall x,y,z\in R$.*

Proof. Since $(de-2af)^2\le(4ab-d^2)(4ac-e^2), 4ab\ge d^2$, then $(2de-4af)^2z^2-4(d^2-4ab)(e^2-4ac)z^2\le 0$. Hence, $(d^2-4ab)y^2+(e^2-4ac)z^2+(2de-4af)yz\le 0$. Thus $(dy+ez)^2-4a(by^2+cz^2+fyz)\le 0$. Since $a>0$ then $ax^2+by^2+cz^2+dxy+exz+fyz\ge 0$, $\forall x,y,z\in R$.□

Lemma 3. *If $a_1\ge a_2\ge a_3$, $b_1\ge b_2\ge b_3$, then $2(a_1b_1+a_2b_2+a_3b_3)\ge a_1b_2+a_1b_3+a_2b_1+a_2b_3+a_3b_1+a_3b_2$, $\forall a_i,b_i$ $(i=1,2,3)\in R$.*

Proof. Since

$$\begin{aligned}&3(a_1b_1+a_2b_2+a_3b_3)-(a_1+a_2+a_3)(b_1+b_2+b_3)\\&\quad=\frac12\big[(a_1-a_2)(b_1-b_2)+(a_1-a_3)(b_1-b_3)\\&\qquad+(a_2-a_1)(b_2-b_1)+(a_3-a_1)(b_3-b_1)\\&\qquad+(a_3-a_2)(b_3-b_2)+(a_2-a_3)(b_2-b_3)\big]\ge 0,\end{aligned}$$

thus

$$3(a_1b_1+a_2b_2+a_3b_3)\ge(a_1+a_2+a_3)(b_1+b_2+b_3).$$

Hence

$$2(a_1b_1+a_2b_2+a_3b_3)\ge a_1b_2+a_1b_3+a_2b_1+a_2b_3+a_3b_1+a_3b_2.$$

We rewrite system (1.1) as the following form:

$$\begin{cases} x_i'(t)=-a_ix_i(t)+a_i\int_{t-\alpha}^t x_i'(s)\,ds+r_ix_i'(t-c)+\sum\limits_{j=1}^m p_{ij}f_j(y_j(t-\tau))+I_i,\\ y_j'(t)=-b_jy_j(t)+b_j\int_{t-\beta}^t y_j'(s)\,ds+r_j^*y_j'(t-d)+\sum\limits_{i=1}^n q_{ji}g_i(x_i(t-\sigma))+J_j.\end{cases}\tag{2.1}$$

Hence, we only need to prove the existence and global exponential stability of equilibrium point of system (2.1).□

3. Existence and uniqueness of equilibrium point

In this section, we will establish LMI-based condition for existence and uniqueness of equilibrium point of system (1.1) or (2.1) by applying Homeomorphism theory, LMI method and new inequality techniques.

Theorem 3.1. *Assume that (H_1) and (H_2) hold. Then system (1.1) has a unique equilibrium point if there exist diagonal matrices $U=diag(u_1,u_2,\ldots,u_n)$, $V=diag(v_1,v_2,\ldots,v_n)$, $C=diag(c_1,c_2,\ldots,c_n)$, $D=diag(d_1,d_2,\ldots,d_n)$, $E=diag(e_1,e_2,\ldots,e_n)$, $R_1=diag(r_{11},r_{12},\ldots,r_{1n})$, $\mu_1=diag(\mu_{11},\mu_{12},\ldots,\mu_{1n})$, $\mu_2=diag(\mu_{21},\mu_{22},\ldots,\mu_{2m})$, $U^*=diag(u_1^*,u_2^*,\ldots,u_m^*)$, $V^*=diag(v_1^*,v_2^*,\ldots,v_m^*)$, $C^*=diag(c_1^*,c_2^*,\ldots,c_m^*)$, $D^*=diag(d_1^*,d_2^*,\ldots,d_m^*)$, $E^*=diag(e_1^*,e_2^*,\ldots,e_m^*)$, $R_2=diag(r_{21},r_{22},\ldots,r_{2m})$ with $u_i>0$, $4u_iv_i\ge d_i^2$, $(d_ie_i-2u_ir_{1i})^2\le(4u_iv_i-d_i^2)(4u_ic_i-e_i^2)$, $u_j^*>0$, $4u_j^*v_j^*\ge(d_j^*)^2$, $(d_j^*e_j^*-2u_j^*r_{2j})^2\le[4u_j^*v_j^*-(d_j^*)^2][4u_j^*c_j^*-(e_j^*)^2]$ $(i=1,2,\ldots,n;j=1,2,\ldots,m)$, positive diagonal matrices $P=diag(p_1,p_2,\ldots,p_n)$, $Q=diag(q_1,q_2,\ldots,q_m)$, n order positive diagonal matrices Y_1, K, m order positive diagonal matrices Y_2, L, positive diagonal matrices $K_i=diag(k_{i1},k_{i2},\ldots,k_{in})(i=3,4,5,6,7)$, $L_j=diag(l_{j1},l_{j2},\ldots,l_{jm})(j=3,4,5,6,7)$, $K_i^*=diag(k_{i1},k_{i2},\ldots,k_{in})(i=3,4)$, $L_j^*=diag(l_{j1},l_{j2},\ldots,l_{jm})(j=3,4)$ such that*

$$\Omega_1=\begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15}\\ * & T_{22} & T_{23} & T_{24} & T_{25}\\ * & * & T_{33} & T_{34} & T_{35}\\ * & * & * & T_{44} & T_{45}\\ * & * & * & * & T_{55}\end{pmatrix}<0,$$