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Brief Papers

Finite-time lag synchronization of time-varying delayed complex networks via periodically intermittent control and sliding mode control ☆

Taiyan Jing [a,b,*], Fangqi Chen [c], Xiaohua Zhang [d]

[a] Department of Mechanics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[b] School of Mathematics and Informatics, Henan Polytechnic University, Jiaozuo 454000, China
[c] Department of Mathematics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China
[d] Department of Information Technology, Jiangsu Institute of Commerce, Nanjing 211168, PR China

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ABSTRACT

In this paper, finite-time lag synchronization of time-varying delayed complex networks is investigated. By designing a periodically intermittent feedback controller and adjusting periodically intermittent control strengths with the updated laws in two parts respectively, we achieve finite-time lag synchronization between two time-varying delayed complex networks. In addition, based on the same finite-time stability theory and the same sliding mode control, we ensure that the trajectory of error system converges to a chosen sliding surface within finite time and remains on it forever. Finally, two examples are given to demonstrate the effectiveness and correctness of the theoretical results obtained here.

1. Introduction

Over the past decade, chaos synchronization has its potential applications in various fields [1–4]. Later, some controllers can be added to complex networks and then force the state variables of complex networks to follow the dynamical behavior we desire. Since the pioneering work was made by Pecora and Carroll [5], the problem of synchronization and control has been extensively studied in its potential engineering applications from secure communication to information processing [6–8]. Accordingly, various control methods are proposed to solve some problems of synchronization between complex networks, such as adaptive control [9–12], nonlinear feedback control [13,14], sliding mode control [15,16], and descriptor model transformation method [17].

In real-world complex networks, time-varying delays unavoidably exist in neural processing in implementation of complex networks, thus, increasing the difficulties to prove the theorem. Therefore, many existing works of time-varying delays have been investigated [18–21]. It has also been discovered that time delay coupling has great influence on the behavior of dynamical systems [22–24]. Therefore, it is important to study the linkage between the nodes in the network which is composed of non-delay and delay coupling.

Sliding mode control is more robust in the process of lag synchronization, and, on the basis of the controllers and the coupling strengths with update laws, the robust sliding mode control is designed to guarantee the existence of the sliding motion. Furthermore, compared with continuous control methods, intermittent control is more efficient because the system output is measured intermittently rather than continuously. In view of those merits, most of existing complex networks and chaotic nonlinear systems are investigated by means of intermittent control or sliding mode control [25,26]. In [27], a class of Cohen–Grossberg complex networks with time-varying delays is studied through designing a periodically intermittent controller. Cai et al. [28] investigate the problem of synchronization in complex dynamical networks with time-varying delays. In [29], exponential lag synchronization for delayed fuzzy cellular neural networks via periodically intermittent control is investigated. In [30], a robust adaptive sliding mode controller (RASMC) is proposed to realize chaos synchronization between two different chaotic systems with

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* Corresponding author at: Department of Mechanics, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, PR China.
E-mail address: taiyan825@163.com (T. Jing).

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uncertainties, external disturbances and fully unknown parameters. From the above analysis, finite-time control technique [31–33] are imposed in the previous existing papers concerning periodically intermittent control, which is useful in real applications. As is known to all, there are few papers with regard to the finite-time lag synchronization for complex networks with time-varying delay and time-varying delay coupling by applying periodically intermittent control and sliding mode control.

Motivated by the above discussion, in our paper, a robust sliding mode controller is designed to synchronize two chaotic systems, in the process of lag synchronization, together with periodically intermittent feedback controller and periodically intermittent control strengths with the updated laws, respectively, we can achieve finite-time lag synchronization.

The rest of this paper is organized as follows. In Section 2, a general time-varying delayed dynamical system is introduced and some mathematical preliminaries used in this paper are given. In Section 3, finite-time lag synchronization of time-varying delayed complex networks is studied by using the periodically intermittent feedback controller and periodically intermittent control strengths with the updated laws. In Section 3.2, two examples are given to illustrate the analytical results obtained here. Finally, some conclusions are drawn in Section 4.

2. Problem description and preliminaries

In this paper, we consider a class of time-varying delayed complex networks each consisting of N nonlinearly coupled identical nodes, with each being an n-dimensional dynamical system, respectively.

The drive networks are characterized by

$$\dot{x}_i = f_i(t, x_i, x_i(t-\tau_1(t))) + \sum_{j=1}^{N} b_{ij} h_j(x_j) + \sum_{j=1}^{N} c_{ij} g_j(x_j(t-\tau_2(t))),$$

$$i = 1, 2, \ldots, N, \tag{1}$$

or in a compact form:

$$\dot{x} = f(t, x, x(t-\tau_1(t))) + Bh(x) + Cg(x(t-\tau_2(t))), \tag{2}$$

where $x_i(t) = (x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t))^T \in R^n$ are the state vectors of the ith node, $x(t) = (x_1(t), x_2(t), \ldots, x_N(t))^T \in R^{nN}$ denotes the state vector, $f(t, x, x(t-\tau_1(t))) = (f_1(t, x_1, x_1(t-\tau_1(t))), f_2(t, x_2, x_2(t-\tau_1(t))), \ldots, f_N(t, x_N, x_N(t-\tau_1(t))))^T : R \times R^{nN} \times R^{nN} \to R^{nN}$ is a smooth nonlinear function, $\tau_1(t), \tau_2(t)$ are the time delays. $h(x) = (h_1(x_1), h_2(x_2), \ldots, h_N(x_N))^T \in R^{nN}$ and $g(x) = \left(g_1(x_1), g_2(x_2), \ldots, g_N(x_N)\right)^T \in R^{nN}$ are the inner connecting functions in each node. While $B, C \in R^{nN \times nN}$ are the weight configuration matrices. If there is a connection from node i to node $j (j \neq i)$, then the coupling $b_{ij} \neq 0$, $c_{ij} \neq 0$; otherwise, $b_{ij} = c_{ij} = 0 (j = i)$, and $b_{ii} = -\sum_{j=1, j\neq i}^{N} b_{ij}$, $c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}$.

Throughout this paper, we have the following assumptions, lemmas and definition.

Assumption 1. For the vector valued function $f(t, x(t), x(t-\tau_1(t)))$, assume that there exist positive constants $\alpha > 0$, $\beta > 0$ such that f satisfies the semi-Lipschitz condition

$$(y(t)-x(t))^T (f(t, y(t), y(t-\tau_1(t))) - f(t, x(t), x(t-\tau_1(t))))$$
$$\leq \alpha (y(t)-x(t))^T (y(t)-x(t)) + \beta (y(t-\tau_1(t)) - x(t-\tau_1(t)))^T (y(t-\tau_1(t)) - x(t-\tau_1(t))),$$

for all $x, y \in R^{nN}$ and $t \geq 0$.

Assumption 2. The time-varying delays $\tau_k(t) (k = 1, 2)$ are differential functions with

$$0 \leq \dot{\tau}_k(t) \leq \tau_k \leq 1,$$

where $\tau_k (k = 1, 2)$ are constants.

Assumption 3. Functions $h(\cdot)$ and $g(\cdot)$ are Lipschitz, that is, there exist non-negative constants l_h, l_g for all $x, y \in R^{nN}$ such that

$$\| h(x) - h(y) \| \leq l_h \| x - y \|, \ \| g(x) - g(y) \| \leq l_g \| x - y \|.$$

Lemma 1 (*Tang 34*). *Assume that a derivable, positive-definite function $V(t)$ satisfies the following inequality:*

$$\dot{V}(t) \leq -\beta V^{\eta}(t), \quad \forall t \geq t_0, \quad V(t_0) \geq 0,$$

where $\beta > 0, 0 < \eta < 1$ are two constants. Then, for any given $t_0, V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \leq V^{1-\eta}(t_0) - \beta(1-\eta)(t - t_0), \quad t_0 \leq t \leq t_1,$$

and

$$V(t) \equiv 0, \ \forall t \geq t_1,$$

with t_1 given by

$$t_1 = t_0 + \frac{V^{1-\eta}(t_0)}{\beta(1-\eta)}.$$

Lemma 2 (*Mei et al. 35*). *Suppose that function $V(t)$ is continuous and non-negative when $t \in [0, +\infty)$ and satisfies the following conditions:*

$$\begin{cases} \dot{V}(t) \leq -\lambda V^{\eta}(t), t \in [lT, lT+h], \\ \dot{V}(t) \leq 0, t \in [lT+h, lT+T) \end{cases},$$

where $\lambda > 0$, $T > 0, 0 < \eta$, $h < 1, l \in N = \{0, 1, \ldots\}$, then the following inequality holds:

$$V^{1-\eta}(t) \leq V^{1-\eta}(0) - \lambda h(1-\eta)t, 0 \leq t \leq T^*,$$

for the constant T^* is the setting time.

Lemma 3 (*Boyd et al. 36*). *Given any real matrices Σ_1, Σ_2, Σ_3 of appropriate dimensions and a scalar $\varepsilon > 0$, such that $0 < \Sigma_3 = \Sigma_3^T$. Then the following inequality holds:*

$$\Sigma_1^T \Sigma_2 + \Sigma_2^T \Sigma_1 \leq \varepsilon \Sigma_1^T \Sigma_3 \Sigma_2 + \varepsilon^{-1} \Sigma_2^T \Sigma_3^{-1} \Sigma_1.$$

Lemma 4 (*Jing et al. 37*). *For x_1, x_2, x_3, $x_4 \in \mathrm{R}^{nN}$ and $0 < q < 2$, $0 < p \leq 1$, the following inequalities hold:*

$$\| x_1 \|^q + \| x_2 \|^q \geq \left(\| x_1 \|^2 + \| x_2 \|^2 \right)^{q/2},$$

$$\| x_1 \|^p + \| x_2 \|^p + \| x_3 \|^p + \| x_4 \|^p \geq \left(\| x_1 \|^2 + \| x_2 \|^2 + \| x_3 \|^2 + \| x_4 \|^2 \right)^p.$$

Definition 1. The master system and the slave system are said to be lag synchronization in finite time if there exists a constant $T > 0$ such that

$$\lim_{t \to T} \| e(t) \| = \lim_{t \to T} \| y(t) - x(t-\theta) \| = 0 \quad \text{and} \quad \| e(t) \| = 0 \text{ if } t > T,$$

where θ is a time-delayed positive constant.

3. Finite-time lag synchronization

3.1. Time-varying delayed networks via periodically intermittent feedback control and sliding mode control

In this section, we design a periodically intermittent feedback controller and a robust sliding mode controller which are capable of making the trajectory of error system fall on the designated