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Pinning outer synchronization between two delayed complex networks with nonlinear coupling via adaptive periodically intermittent control

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ABSTRACT

This paper is concerned with the problem of pinning outer synchronization between two delayed complex networks with nonlinear coupling. Both the internal delay and coupling delay are included in coupled network model. By designing appropriate adaptive intermittent controllers and using the Lyapunov stability theory, some pinning outer synchronization criteria are derived, which can guarantee the response network asymptotically synchronizes to drive network. Furthermore, the restrictions about delay and control width are removed. Simultaneously, two simple adaptive intermittent pinning outer synchronization conditions are obtained based on the proposed criteria. A numerical example is provided to demonstrate the effectiveness of the theoretical results.

1. Introduction

It is well known that complex network is a useful modeling tool of real-world systems. In reality, many practical and natural systems, such as Internet, World Wide Web, food webs, electric power grids, traffic network, biological network, scientific citation network and social network, can be described by the models of complex networks. Among the dynamical behaviors of complex networks, synchronization is an interesting phenomenon and has become a significant topic in the study field of complex networks [1–8]. The main reason for that is network synchronization not only can well explain many natural phenomena but also has many potential applications, such as secure communication, synchronous information exchange in the internet, and the synchronous transfer of digital signals in communication network. Considering that coupled networks cannot achieve synchronization by themselves, a great many control methods are employed to accomplish this goal, such as adaptive feedback [9,10], impulsive control [11,12], passive method [13,14], sampled-data control [15,16], and so forth. In addition, the synchronization problems of coupled networks under communication constraints have drawn researcher's interest [17–20].

Generally speaking, a complex dynamical network is usually composed of a large number of interconnected nodes, and it is difficult to all controllers to all nodes. To reduce the number of controlled nodes, pinning control, in which some local feedback controllers are only applied to a small fraction of network nodes, has been introduced in the study of synchronization for dynamical networks [21–25]. On the other hand, periodically intermittent control, as a discontinuous control strategy in engineering field, has attracted much research interest. In this kind of control strategy, each period usually contains two types of time, one is work time, and the other one is rest time. The controller is activated in each work time and is off in the rest time [26]. To better reduce the control cost and amount of transmitted information, combining pinning control and periodically intermittent control, has been extensively utilized in realizing synchronization of complex networks [27–31].

It is worth noting that the works mentioned above only focused on synchronization phenomenon that all nodes in a network achieve a collective behavior, which can be regarded as inner synchronization [32]. Except the aforementioned inner synchronization behavior, there exists another synchronization phenomenon named outer synchronization, which occurs between two or more coupled networks [32]. In reality, many practical systems can be used to illustrate the outer synchronization phenomenon between two networks, such as the infectious disease spreads between different communities, the avian influenza spreads among domestic and wild birds, and the different species development in balance [33]. With the rapid development of theories and applications of complex networks, outer synchronization has been widely and deeply

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investigated in various fields [34–41]. By designing effective adaptive controllers, the outer synchronization between two dynamical networks with nonidentical topological structures was realized in [34]. The projective synchronization between two complex networks with time-varying coupling delay was studied by the adaptive control method in [35]. By using the stability theory of fractional-order differential system and the adaptive control technique, Yang and Jiang in [36] investigated the adaptive synchronization in the drive-response fractional-order dynamical networks with uncertain parameters. In [37], the lag synchronization problem between two different complex networks was achieved based on state observer. Based on the adaptive impulsive pinning control, Wu et al. in [38] investigated the outer synchronization between drive-response networks. The mixed outer synchronization between two different networks without and with time-varying delayed couplings was investigated in [39]. The finite-time outer synchronization problem between two coupled dynamical networks has been discussed in [40,41].

Although research on outer synchronization between two coupled networks via pinning control or periodically intermittent control has gained so much attention, little of that has been devoted to outer synchronization via intermittent pinning control. As illustrated in previous, intermittent pinning control is an effective synchronization technique in practical applications, thus it is meaningful to consider the outer synchronization problem using intermittent pinning control. In addition, to give a more precise and realistic description of dynamical networks, it is necessary and important to take into account nonlinear coupling and time-delayed coupling in many practical problems [22]. Nevertheless, the authors in [42] only considered network models with linear coupling but neglected the effect of time-delayed coupling. Therefore it is still an interesting but very difficult task to investigate outer synchronization between two coupled delayed dynamical networks with nonlinear coupling by using the periodically intermittent pinning control. This motivates us to further study in this paper.

Inspired by the above discussions, in this paper, the problem of pinning outer synchronization between two delayed complex dynamical networks with nonlinear coupling is investigated. Not only internal delay but also coupling delay is considered in our network model. By adding adaptive periodically intermittent controllers to a small fraction of response network nodes, some pinning outer synchronization conditions are achieved, which have released the restrictive conditions in [42]. Furthermore, how to choose the least number of pinned nodes are provided. Also, two pinning intermittent outer synchronization conditions are obtained for simple cases. A numerical example is presented to verify the theoretical analysis in this paper.

The rest of this paper is organized as follows. In Section 2, the model of coupled complex dynamical network is presented and some preliminaries are also provided. Some criteria for ensuring outer synchronization in drive-response dynamical networks via adaptive intermittent pinning control are derived in Section 3. Numerical simulation is given in Section 4. Finally, a conclusion is presented in Section 5.

2. Network model and preliminaries

Consider a nonlinearly coupled complex delayed dynamical network consisting of N nodes, in which each node is an n-dimensional dynamical system

$$\dot{x}_i(t)=F(t,x_i(t),x_i(t-\tau_1))+c_0\sum_{j=1}^{N}a_{ij}\Gamma x_j(t)+c_1\sum_{j=1}^{N}b_{ij}\Gamma h(x_j(t-\tau_2)),\quad i=1,2,\dots,N \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\dots,x_{in}(t))^T\in R^n$ is the state variable of the ith node, $F(t,x_i(t),x_i(t-\tau_1)) =(f_1(t,x_i(t),x_i(t-\tau_1)),f_2(t,x_i(t),x_i(t-\tau_1)),\dots,f_n(t,x_i(t),x_i(t-\tau_1))^T\in R^n$ is a nonlinear vector valued function describing the dynamics of nodes. $\tau_1>0$ and $\tau_2>0$ are the delay of node dynamics and coupling delay, respectively. The positive constants c_0 and c_1 are the strengths for the constant and delayed coupling, respectively. The nonlinear coupling function $h(x_j(t-\tau_2))$ is continuous and has the form $h(x_j(t-\tau_2))=(h(x_{j1}(t-\tau_2)),h(x_{j2}(t-\tau_2)),\dots,h(x_{jn}(t-\tau_2)))^T$. $\Gamma\in R^{n\times n}$ is the inner-coupling matrix between nodes. $A=(a_{ij})\in R^{N\times N}$ and $B=(b_{ij})\in R^{N\times N}$ are the coupling configuration matrices. If there is a connection between node i and node j ($i\neq j$), then $a_{ij}>0$ and $b_{ij}>0$; otherwise, $a_{ij}=0$ and $b_{ij}=0$. The diagonal elements of matrices A and B are defined as $a_{ii}=-\sum_{j=1,j\neq i}^{N}a_{ij}$ and $b_{ii}=-\sum_{j=1,j\neq i}^{N}b_{ij}$, respectively. Clearly, in this paper, the coupling configuration matrices A and B may be different from each other. Furthermore, A and B are not assumed to be symmetric.

To realize the outer synchronization between two coupled complex networks with node delay and coupling delay, we refer to model (1) as the drive network, and the response network is characterized by

$$y^{\cdot}_i(t)=F(t,y_i(t),y_i(t-\tau_1))+c_0\sum_{j=1}^{N}a_{ij}\Gamma y_j(t)+c_1\sum_{j=1}^{N}b_{ij}\Gamma h(y_j(t-\tau_2))+u_i(t),\quad i=1,2,\dots,N \tag{2}$$

where $y_i(t)=(y_{i1}(t),y_{i2}(t),\dots,y_{in}(t))^T\in R^n$ and $u_i(t)\in R^n$ are, respectively, the state variable and the control input of the ith node in the response network. Moreover, other parameters involved in the system (2) all have the same meanings with the corresponding parameters in system (1).

Remark 1. Compared with the model in [42], there are some advantages in our drive-response network model. Firstly, the model include not only the non-delayed coupling but also delayed coupling in this paper, but the authors in [42] only considered the non-delayed coupling. Secondly, the nonlinear coupling between nodes is introduced in our model. Finally, both the internal delay τ_1 and coupling delay τ_2 coexist in our model, which may lead to a more complicated case. Moreover, there is no relationship between τ_1 and τ_2. Thus the network model considered here is more general than [42].

In this paper, our main aim is to control the response network (2) such that the states of response network $y_i(t)$ can synchronize to the states of drive network $x_i(t)$. We are now in a position to introduce the notion of outer synchronization between networks (1) and (2).

Definition 1. The drive network (1) and response network (2) are said to achieve outer synchronization, if $x_i(t)-y_i(t)\to 0$, $t\to\infty$, $i=1,\dots,N$.

To reduce the number of controlled nodes and save control cost, we adopt the intermittent pinning control approach to achieve outer synchronization between drive network (1) and response network (2), which means that the control actions are only added to partial nodes of the response network and most of nodes in response network are not needed to be controlled. Let $e_i(t)=x_i(t)-y_i(t)$ be the outer synchronization error. Without loss of generality, assume that the first l nodes are selected and pinned with the adaptive periodically intermittent controllers, which are defined as follows:

$$u_i(t)=\begin{cases}-k_i(t)e_i(t), & t\in[mT,mT+T_1),\ 1\le i\le l\\ 0, & t\in[mT,mT+T_1),\ l+1\le i\le N\\ 0, & t\in[mT+T_1,(m+1)T),\ 1\le i\le N\end{cases} \tag{3}$$