Neurocomputing 201 (2016) 104–111

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Towards characterization of driver nodes in complex network with actuator saturation

Mahaveer Singh [a], Ram Niwash Mahia [b], Deepak M. Fulwani [b,*]

[a] Centre for System Science, Indian Institute of Technology Jodhpur, Jodhpur, Rajasthan-342011, India
[b] Department of Electrical Engineering, Indian Institute of Technology Jodhpur, Jodhpur, Rajasthan-342011, India

ARTICLE INFO

Article history:
Received 29 November 2015
Received in revised form
19 January 2016
Accepted 7 March 2016
Available online 28 March 2016

Keywords:
Control of complex network
Driver nodes
Region of attraction

ABSTRACT

The paper proposes a theory and an algorithm to characterize driver node (control node) of a complex network. The proposed algorithm identifies an appropriate driver node when multiple options are available to select a driver node. The method is based on concept of maximization of stability regions. A realistic situation where driver node has limited actuating capability is considered. The proposed control law considers actuator saturation *a priori* and also ensures a specified convergence rate. Formation control in robotic network and numeric examples are used to verify the theoretical developments.

1. Introduction

A complex network can be defined as an entity (system) with interlinked nodes. A node can represent: a switch in a communication network, a document in citation network, a disease gene in the human disease network to mention a few examples. Two nodes are said to be interlinked when they share or exchange some kind of information. For example, two switches exchange voice/data signals, a document cites other documents, or genes associated with similar disorders. State of the network is given by the current state values of each node that can change over time. The state value of a node can represent: a magnetic or electric field of the inductor or capacitor in an electrical circuit, compression/expansion of the spring in the spring-mass system, and so forth.

The recent control theoretic developments of complex networks provide a detailed behavioral analysis of the complex networks [1–10]. Controllability property is investigated by modeling a complex network as a linear time invariant system; adjacency matrix is used as the system matrix. Authors in [11] have shown the relation between the network structure and its controllability index for the directed networks and also presented an optimized control design. Recently, Wen et al. [12] have shown that the global synchronization control problem of switching complex networks can be solved by using topology dependent multiple Lyapunov functions. Liu et al. [1] developed a minimum input theory to characterize the structural controllability of directed networks using minimum set of driver nodes to control the network. There are many interesting work on dynamics of complex cellular network see [13–17] and the references therein. Authors in [5] proposed trade-off between the driver nodes and the control energy as a function of the network dynamics using the smallest eigenvalue of the Controllability Gramian. In recent years there have been quite a few work devoted to this problem [1,12,18–21] and see the references therein.

The complex network can be controlled by driver node(s). Minimum number of driver nodes required to control a network is fixed [1]. However, these nodes are not unique. For a network, in general, multiple options exist to choose a driver node. The existing work available in literature do not characterize driver nodes. Furthermore, for any real network driver node can have only limited actuation capacity. While considering the problem of controlling a complex network, it is important to consider practical limitations of the driver nodes. Driver nodes do not have infinite (unlimited) actuation capability, i.e. maximum input a driver node can provide is limited. When actuator has limited capacity, control objectives may not be achieved, if this limitation is not considered *a priori*. This problem becomes more complex when the adjacency matrix has unstable eigenvalues; even the stability cannot be guaranteed in this situation. With unstable eigenvalues of the adjacency matrix of a complex network and limited actuation capabilities, the region in state space, where stability is guaranteed, is finite. This Region Of Attraction (ROA) depends upon the number of open-loop unstable eigenvalues of the system [22] as well as choice of driver node and control law. This work characterizes driver nodes and proposes a theory and an algorithm such that a right driver node, among many, can be identified to maximize the ROA for a given linear feedback control

* Corresponding author.
E-mail address: df@iitj.ac.in (D.M. Fulwani).

http://dx.doi.org/10.1016/j.neucom.2016.03.011
0925-2312/

law. Furthermore, it ensures a specified rate of convergence of states. The ROA is described by contractively invariant ellipsoid in n-dimensional state space. In the context of linear systems with actuator saturation, many excellent work exists [22–31] and see the references therein.

In this work, we have addressed two important issues, first is regarding selection of a driver node and in the second, we consider driver node limitation. We propose a theory and an algorithm to select a driver node for a given feedback control law such that the ROA corresponding to this driver node maximizes while ensuring the specified rate of convergence. The proposed work uses Linear Quadratic (LQ) control law with a fixed convergence rate. In [32,33], we have proposed algorithms to select the right driver nodes of complex networks and multi-agent systems which maximizes the corresponding ROA, however, a fixed convergence rate was not considered *a priori*. The maximization of ROA as well as the fixed convergence rate of states are important requirements in the control problem of complex networks. Application domain of the proposed theory and algorithm is as diverse as social networks to biological networks. The proposed algorithm can be used to select the right driver node of robotic networks, mobile sensor networks, gene regulatory networks, power grid networks, social interactions networks, etc., which maximizes the ROA of the network while ensuring the specified rate of convergence of states. To the best of authors knowledge, this problem has not been addressed so far.

The outline of this paper is as follows. In Section 2, modeling of a complex network and existing results are briefly summarized. In Section 3, we have used a motivational example of unstable network with actuator saturation to explain the concept. Main results for identifying the driver node of complex network with maximum region of attraction are discussed in Section 4 and two numerical examples are also simulated to verify the theoretical developments in Section 5, followed by an application of robot formation control in Section 6. Conclusion of the work is summarized in Section 7.

2. Mathematical modeling and preliminaries

This section is divided into two parts. In the first part, we have presented mathematical model of a complex network with actuator saturation and in the second part, some existing results of LTI systems with actuator saturation are recalled.

2.1. Modeling of complex network with saturated actuator

A complex network is a set of nodes (n) with some rules of interactions between them and can be represented by a graph $G{:=}(V,E)$, where $V{:=}\{v_1,v_2,..,v_n\}$ and $E\subseteq V\times V$ are the sets of vertices and edges, respectively. Interconnections of graph $G{:=}(V,E)$ with n nodes is mathematically represented by adjacency matrix (A) with n rows and n columns, where $a_{ij}\in\mathbb{R}$, $i=j=1,2,..,n$ is the weight of the edge e_{ij}. Let us assume that the network (G), subjected to actuator saturation, is independently controllable by each of the inputs (nodes) u_i from the set $U=\{u_1,u_2,\dots,u_m\}, m\le n$.

Consider a complex network (G) with input u_i, let us designate corresponding input matrix as B_i, with this, dynamics of the network (G) with actuator saturation can be written as

$$\dot{x}(t)=Ax(t)+B_i sat(u_i(t)) \tag{1}$$

where $A\in\mathbb{R}^{n\times n}$ is the adjacency matrix, $x\in\mathbb{R}^n$ state vector and $B_i\in\mathbb{R}^{n\times 1}$ input matrix corresponding to control input $u_i\in\mathbb{R}$, for $i=1,2,\dots,m$, of the network. The saturation function 'sat' is defined as $sat:\mathbb{R}\to\mathbb{R}$, i.e. $sat(u_i(t))=sign(u_i(t))\min\{u_{i,max},|u_i(t)|\}$, where $u_{i,max}>0$, $\forall i=1,2,\dots,m$ is saturation limit of i^{th} control input of the network and $|u_i(t)|\le u_{i,max}$. Without loss of generality let us assume $u_{i,max}=1$, $\forall i=1,2,\dots,m$.

2.2. Existing results of LTI systems with saturated actuator

Consider a single input LTI system subject to actuator saturation

$$\dot{x}=Ax+Bu, x\in\mathbb{R}^n, u\in\mathbb{R}, \quad |u|_\infty\le 1 \tag{2}$$

Zhou and Duan [30] considered the following objective function for the linear system (2) in the absence of actuator saturation:

$$J(u)=\int_0^\infty e^{\gamma t}u^2(t)\,dt \tag{3}$$

and found that $J(u)$ is minimized if and only if there exists a $P(\gamma)>0$, which satisfies (5) and the corresponding optimal control is given by (4) with the convergence rate of the closed-loop system is no less than $e^{\frac{-\gamma t}{2}}$.

A control law to minimize $J(u)$ is given as

$$u=-B^TP(\gamma)x \tag{4}$$

here $P(\gamma)>0$, a positive definite matrix, is the solution of the following Algebraic Riccati Equation (ARE):

$$A^TP(\gamma)+P(\gamma)A-P(\gamma)BR^{-1}B^TP(\gamma)+Q(\gamma)=0 \tag{5}$$

where $Q(\gamma)=\gamma P(\gamma)>0$ is a positive definite matrix, $\gamma>0$ and $R=1$.

Assume that the linear system (2) is controllable and let $V(x)=x^TP(\gamma)x$ be the Lyapunov candidate, where $P(\gamma)>0$ is a positive definite matrix obtained from (5). Then the level sets associated with $V(x)$ are the solid ellipsoids $\varepsilon(P(\gamma),\rho(\gamma))=\{x:x^TP(\gamma)x\le\rho(\gamma)\}$, where $\rho(\gamma)$ is a positive number. The ellipsoids $\varepsilon(P(\gamma),\rho(\gamma))$ are said contractively invariant if the time derivative of $V(x)$ along the trajectory of the system (2), $\dot{V}(x)<0$, $\forall x\in\varepsilon(P(\gamma),\rho(\gamma))\backslash\{0\}$ for some $u(t)$ [30].

Proposition 1 (*Zhou and Duan [30]*). *Assume that for a linear system (2), (A,B) controllable and $P(\gamma)>0$ satisfies (5). Then, under the linear feedback control law (4), the following statements hold*

(*a*) *For $\gamma>\max\{0,2\lambda_{max}(-A)\}$, matrix $(A-BB^TP(\gamma))$ is asymptotically stable and $P(\gamma)$ is monotonically increasing with respect to γ.*

(*b*) *The ellipsoids $\varepsilon(P(\gamma),\rho(\gamma))$ can be made contractively invariant if and only if $\rho(\gamma)<\rho^\star(\gamma)$. Here*

$$\rho^\star(\gamma)=\frac{4B^TP(\gamma)B}{(B^TP(\gamma)B-\gamma)^2} \tag{6}$$

where

$$B^TP(\gamma)B=n\gamma+2\sum_{i=1}^n\lambda_i(A) \tag{7}$$

and $n\ge 2$ is the order of the system.

3. Motivating example

Consider an anti-stable (all eigenvalues of adjacency matrix are positive) network of three nodes as shown in Fig. 1. The adjacency matrix for the network can be computed as

$$A=\begin{bmatrix}0.5 & 0.2 & 0.4\\ 0.1 & 0.3 & -0.7\\ -0.9 & 0.5 & 0.8\end{bmatrix}$$

Table 1 summarizes input matrix corresponding to different driver nodes. The given network is controllable by each of the nodes acting independently. Eigenvalues of the matrix A are