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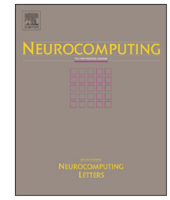

Brief Papers

Synchronization criteria of chaotic Lur'e systems with delayed feedback PD control ☆

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A B S T R A C T

This paper is concerned with the problem of global asymptotical synchronization of chaotic Lur'e systems by using a delayed feedback proportional-derivative (PD) control scheme. Based on Lyapunov functional approach, a stabilization condition for the synchronization error systems with delayed PD control is derived. The designed delayed PD feedback controller can ensure that the master and slave systems are asymptotically synchronous. The advantage of the constructed Lyapunov functional lies in the fact that it makes full use of the additional derivative state term. Furthermore, a less conservative synchronization criterion is proposed for the considered system only used proportional control. Compared with the existing ones, the merit of the proposed results in this paper lies in their reduced conservatism and application of the derivative control. The conditions represented in terms of linear matrix inequalities (LMIs) can be solved by the application of convex optimization algorithms. Finally, numerical examples are given to show the effectiveness of the proposed method and the improvement over some existing results.

1. Introduction

In the past few decades, chaotic synchronisation has been received much attention due to its potential applications in many fields, such as secure communications, chemical reactions and biological systems. The objective of master–slave synchronization is to control the slave system using the output of the master system such that the output of the slave system follows the output of the master system asymptotically. It has been known that many nonlinear systems, such as Chua's circuit [1], n-scroll attractors [2] and hyperchaotic attractors [3], can be represented in the Lur'e form which consist of a linear dynamical system and a feedback nonlinearity satisfying sector bound constraints. For this reason, the master–slave synchronization of chaotic Lur'e systems has become an important topic, and a lot of synchronization criteria have been proposed [4,5].

It is noted that time delay is an inherent feature of many physical processes, such as chemical processes, nuclear reactors, and biological systems, and may lead to instability or significantly deteriorate the performance of the corresponding closed-loop systems [6–22]. Hence, there are many researchers devote their effort on studying the delay effect on master–slave synchronization. In [23–27], synchronization conditions were derived by adopting the delayed feedback control approach, where the error system between master and slave systems was globally asymptotically stable with given the feedback gain matrix. However, the design problem of feedback controller was not considered. Thus, in [28], the controller design problem is solved by using a nonlinear programming, which is difficult to be handled owing to its nonlinearity. In order to overcome this difficulty, linear matrix inequality (LMI) technique has been used in [29–34]. Based on this approach, the authors in [29–32] derived some synchronization criterion by integral inequality and free-weighting matrix approach. In [33], a less conservative delay-dependent synchronization condition was proposed by using an augmented Lyapunov functional. Very recently, Ge et al. [34] employed delay decomposition method to obtain a larger delay bound. Although delay-partitioning approach can obtain tighter upper bounds than the results without delay-partitioning approach, time consuming and computational burden also become bigger. Hence, one motivation is to get the less conservative result than the ones proposed in [29–34] by constructing a modified Lyapunov functional and using some new technique to handle the terms of quadratic form without using delay-partitioning method.

On the other hand, the proportional-integral-derivative (PID) control has a long history in control engineering. Its simplicity in architecture makes it much easier to be understood and used by the control engineer than advanced controllers. PID control is a

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model-free linear control, and the control gains can be adjusted easily and separately. It should be pointed out that, compared with PID control, PD controls have also been constructed to give global asymptotic stabilization of many systems, mostly due to their conceptual simplicity and explicit tuning procedures [35–37]. And it is well known that the derivative control is to enhance the stability [38]. Hence, considering the popularity and simplicity of PD control in industrial applications, we focus on the delayed feedback PD controller design problem since there are few results on the synchronization problem by using delayed feedback PD control. Another goal of this paper is to find a simple and easy PD controller for deriving the less conservativeness synchronization criterion of chaotic Lur'e systems.

Based on the aforementioned discussions, in this paper, we will study the problem of master–slave synchronization of chaotic Lur'e systems by using delayed feedback PD control. A new augmented Lyapunov–Krasovskii functional, which has the relation with the derivative state terms, is constructed. Here, in order to obtain tighter upper bounds of integral terms of quadratic form and to trade off between time-consumption and improvement of the feasible region, Wirtinger-based integral inequality in [39] is used to derive the synchronization criterion of chaotic Lur'e systems. Furthermore, a less conservative synchronization condition can be obtained without derivative control. Finally, the effectiveness and reduced conservatism of the developed results are demonstrated by numerical examples.

The main contribution and novelty of this paper are summarized as follows:

(1) Compared with the literatures in [29–34], the derivative control is introduced to the synchronization of chaotic Lur'e systems. And an improved result is derived because of the existence of derivative control.
(2) Based on Lyapunov technique and the parameterized LMI technique, a less conservative synchronization criterion for chaotic Lur'e systems is obtained by only using delayed proportional control as many other literatures [29–34].

Notations: Throughout this paper, I denotes the identity matrix with appropriate dimensions, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, and $\mathcal{R}^{m\times n}$ is the set of all $m\times n$ real matrices. For symmetric matrices A and B, the notation $A>B$(respectively, $A\ge B$) means that the matrix $A-B$ is positive definite (respectively, nonnegative).

2. Problem formulation and preliminaries

Consider the following master–slave type of Lur'e systems by using delayed feedback PD control:

$$\mathcal{M}:\begin{cases}\dot{x}(t)=Ax(t)+Hf(Dx(t)),\\ p(t)=Cx(t),\end{cases} \tag{1}$$

$$\mathcal{S}:\begin{cases}\dot{y}(t)=Ay(t)+Hf(Dy(t))+u(t),\\ q(t)=Cy(t),\end{cases} \tag{2}$$

$$\mathcal{U}:u(t)=K_1(p(t-\tau)-q(t-\tau))+K_2(\dot{p}(t-\tau)-\dot{q}(t-\tau)), \tag{3}$$

which consists of master system $\mathcal{M}$, slave system $\mathcal{S}$, and controller $\mathcal{U}$; the master and slave systems are delayed Lur'e systems with state vectors $x(t),y(t)\in\mathcal{R}^n$, respectively, $A\in\mathcal{R}^{n\times n},H\in\mathcal{R}^{n\times n_h},C\in\mathcal{R}^{l\times n},D\in\mathcal{R}^{n_h\times n}$ are known constant matrices, $u(t)\in\mathcal{R}^n$ is the slave system control input. $K_1\in\mathcal{R}^{n\times l}$ and $K_2\in\mathcal{R}^{n\times l}$ are the controller gain matrix to be designed. τ is time delay. It is assumed that $f(\cdot):\mathcal{R}^{n_h}\to\mathcal{R}^{n_h}$ is a diagonal nonlinearity with $f(\cdot)$ belonging to sector $[0,k]$ for $i=1,2,\dots,m$.

Now, we define the error signal $e(t)=y(t)-x(t)$.

$$\dot{e}(t)=Ae(t)+Hg(De(t),x(t))-K_1Ce(t-\tau)-K_2C\dot{e}(t-\tau), \tag{4}$$

where

$$g(De(t),x(t))=f(De(t)+Dx(t))-f(Dx(t)).$$

The description of error system in (4) combined with the nonlinear function which satisfied the sector $[0,k],i=1,2\dots,m$ is the characteristic of Lur'e systems. Then, the following inequality of the nonlinear function $g(De(t),x(t))$ is satisfied

$$g_i(d_ie(t),x(t))[g_i(d_ie(t),x(t))-kd_ie(t)]\le 0,\ \forall e(t),x(t)\in\mathcal{R}^n. \tag{5}$$

The main purpose of this paper is to design a controller (20) to achieve synchronization of the master system (18) and the slave systems (19). In other words, we are interested in finding the controller gain matrix K_1 and K_2 such that the error system (4) is asymptotically stable, which means that the master system and the slave system synchronizes.

The following lemma will be used for deriving synchronization criteria.

Lemma 1 (*Seuret and Gouaisbaut [39]*). *For any positive matrix M, and vector function $\dot{x}:[a,b]\to R^n$, the following inequality holds:*

$$\int_a^b \dot{x}(s)R\dot{x}(s)\,ds\ge\frac{1}{b-a}\begin{bmatrix}x(b)\\ x(a)\\ \frac{1}{b-a}\int_a^b x(s)\,ds\end{bmatrix}^T\Omega\begin{bmatrix}x(b)\\ x(a)\\ \frac{1}{b-a}\int_a^b x(s)\,ds\end{bmatrix} \tag{6}$$

where

$$\Omega=\begin{bmatrix}4M & 2M & -6M\\ * & 4M & -6M\\ * & * & 12M\end{bmatrix} \tag{7}$$

3. Main results

In this section, by use of augmented Lyapulov–Krasovskii functional, new delay-dependent synchronization criteria for systems (4) will be proposed.

Theorem 3.1. *For given scalars $\tau>0,\alpha$ and given matrix $K=\mathrm{diag}\{k_1,k_2,\dots,k_{n_h}\}$, the error system (4) is globally asymptotically stable if there exist positive matrix $\mathcal{P},\mathcal{Q},R$ and positive diagonal matrices $\Lambda=\mathrm{diag}\{\lambda_1,\lambda_2,\dots,\lambda_{n_h}\},T=\mathrm{diag}\{t_1,t_2,\dots,t_{n_h}\}$, and any matrix G,F_1,F_2 with appropriate dimensions such that*

$$\Sigma<0, \tag{8}$$

where

$$\begin{aligned}\Sigma=&[e_1\tau e_3e_2]\mathcal{P}[e_5e_1-e_2e_6]^T+[e_5e_1-e_2e_6]\mathcal{P}[e_1\tau e_3e_2]^T\\ &(e_1D^TK-e_4)\Lambda De_5^T+((e_1D^TK-e_4)\Lambda De_5^T)^T\\ &+[e_1e_5]\mathcal{Q}[e_1e_5]^T-[e_2e_6]\mathcal{Q}[e_2e_6]^T\\ &+\tau^2e_5Re_5^T-[e_1e_2e_3]\begin{bmatrix}4R & 2R & -6R\\ * & 4R & -6R\\ * & * & 12R\end{bmatrix}[e_1e_2e_3]^T\\ &+(e_1+\alpha e_5)\Phi+((e_1+\alpha e_5)\Phi)^T\\ &+e_1D^TKTe_4^T+(e_1D^TKTe_4^T)^T-2e_4Te_4^T,\end{aligned}$$

$$\Phi=[GA-F_1C0GH-G-F_2C].$$

$e_i\in R^{(4n+n_h+2n)\times n}(i=1,2,\dots,6)$ *are defined as block entry matrices,*