Neurocomputing 189 (2016) 80–85

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief papers

Fuzzy-model-based admissibility analysis for nonlinear discrete-time descriptor system with time-delay ☆

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ARTICLE INFO

Article history:
Received 5 November 2015
Received in revised form
7 December 2015
Accepted 19 December 2015
Communicated by Hongyi Li
Available online 30 December 2015

Keywords:
T–S fuzzy model
Descriptor system
Time-delay
Discrete-time

ABSTRACT

This paper is concerned with the admissibility analysis problems of discrete-time Takagi–Sugeno fuzzy descriptor system with time-varying delay. The upper and lower bound of time-delay is assumed to be available online. By defining new Lyapunov functions and by making use of novel technique, delay dependence condition is given for the admissibility analysis of the systems without matrix decoupling and matrix transformation. The merit of the proposed conditions lies in their less conservativeness, which is achieved by combining some bounding inequalities for cross products between two vectors. Finally, these conditions are shown to be the effectiveness via two examples.

1. Introduction

Over the past two decades, there are growing interests in the fuzzy control of complex nonlinear systems and, in particular, Takagi–Sugeno (T–S) fuzzy-model-based control theory in various fields. It has been proved that T–S fuzzy models can approximate any smooth nonlinear system to any accuracy on a compact set, which is realized by piecewise smoothly connecting a family of local linear models with fuzzy membership functions. This "blending" makes T–S fuzzy models similar to linear systems, and the stability analysis and synthesis can be derived by making full use of the fruitful results on linear systems [1,5,6,17,11,21,22, 25,26,28,31,32,34,37]. So far, a great number of results have been reported for T–S fuzzy systems. To mention a few, robust control [30]; robust H_∞ filtering and state estimation [4,7,38]; stochastic fuzzy control [27], adaptive fuzzy control [12,13,24]; model reduction [14].

Descriptor systems are also referred to as implicit systems, generalized state-space systems, differential-algebraic systems or semi-state systems [2,29]. Descriptor systems have been extensively studied in the past years due to their applications in circuits, economics, and many other fields. The descriptor systems describe a larger class of realistic systems than the regular ones; they not only can formulate the dynamic behaviors of systems but also can additionally embrace extra algebraic constraints. A distinctive characteristic of descriptor systems is the possible impulse behavior, which is harmful to the physical system and is undesired in system control. Recently, a class of fuzzy descriptor systems has been presented in [23], whose stability and stabilization problems are addressed based on the linear matrix inequality (LMI) approach. By using of many locally linear descriptor system models, T–S fuzzy descriptor systems approximate or represent a wider class of systems including physical models and nondynamic constraints, which takes advantages of the redundancy of descriptor systems. It gives a novel approach for the research of the nonlinear descriptor system and the time-varying descriptor system. In [8], based on the delay-partitioning technique, the sliding mode control of continuous fuzzy descriptor systems with time-delay is discussed. In [9,33], the robust stability of uncertain discrete-time T–S fuzzy descriptor systems was studied by different ways, which do not take into accounting of stabilization problems. In [3,18,35,36], the stability analysis and synthesis for continuous fuzzy descriptor systems was addressed, respectively. To the best of our knowledge, very little attention has been paid to the problem of delay-dependent admissibility for discrete-time T–S fuzzy descriptor systems with time-varying delay, which motivates the work of this paper.

☆This work is supported in part by the National Natural Science Foundation of China under Grant 61463001, the Natural Science Foundation of Heilongjiang Province of China under Grants F201331 and F201440. Project funded by Heilongjiang Province Postdoctoral Science Foundation under Grant No. LBH-Z15043.
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http://dx.doi.org/10.1016/j.neucom.2015.12.053
0925-2312/

In this paper, the admissibility analysis for time-delay discrete T–S fuzzy descriptor systems is considered. A delay-dependent strict LMIs sufficient condition for discrete-time T–S fuzzy descriptor with time-delay to be regular, causal and stable is given. Furthermore, as a corollary, the stability problem of the normal state-space T–S fuzzy systems is solved. The proposed conditions are advantageous in terms of less conservativeness, which is achieved by circumventing the utilization of bounding inequalities for cross products between two vectors. Two numerical examples to illustrate the effectiveness of the method are given in the paper.

The rest of the paper is organized as follows. Section 2 formulates the system descriptions and the problem under consideration. Admissibility analysis for discrete T–S fuzzy descriptor systems is presented in Section 3. Illustrative examples are given in Section 4 to demonstrate the effectiveness of the theoretical results. Conclusions are given in Section 5.

Notation: The notations used throughout the paper are standard. The superscript "T" denotes matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; the notation $P>0$ means that P is real symmetric and positive definite; I and 0 represent the identity matrix and zero matrix with compatible dimensions respectively. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. We use an asterisk ($*$) is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. System description and preliminaries

Consider a class of nonlinear discrete descriptor system with time-delay, which can be represented by the following T–S fuzzy model:

Plant rule i:

IF $\theta_1(k)$ is μ_{i1} and $\theta_2(k)$ is μ_{i2} and $\cdots$ and $\theta_p(k)$ is μ_{ip}, THEN

$$Ex(k+1)=A_ix(k)+A_{di}x(k-d(k)),$$
$$x(k)=\phi(k), k=-d_2,\ \ldots,1,\ 0, \tag{1}$$

where $x(k)\in\mathbb{R}^n$ is the state vector, μ_{ij} is the fuzzy set, r is the number of IF–THEN rules, and $\theta(k)=[\theta_1(k),\theta_2(k),\ldots,\theta_p(k)]$ is the premise variables; the matrix E may be singular, we shall assume that rank $(E)=g\le n$; $d(k)$ is a time-varying delay and satisfies $d_1\le d(k)\le d_2$, where d_1 and d_2 are positive integers representing minimum and maximum delays, respectively. $\phi(k)$ is the compatible initial condition; A_i and A_{di} are known real constant matrixes with appropriate dimensions of the ith local model of the T–S fuzzy descriptor system.

Remark 1. The interval-like type time delay $d(k)$ describes the real situation in many practical applications. A typical case can be found in networked control systems. The delays are the induced delays by the communication transmission (either from sensor to controller or from controller to actuator), which are actually time-varying, and its lower and upper delay bounds are assumed to be known without loss of generality.

Thus, the overall model of the discrete T–S fuzzy descriptor system with time-delay is inferred as follows:

$$Ex(k+1)=\sum_{i=1}^{r}h_i(\theta(k))[A_ix(k)+A_{di}x(k-d(k))],$$
$$x(k)=\phi(k),\quad k=-d_2,\ldots,1,0, \tag{2}$$

where

$$h_i(\theta(k))=\frac{\omega_i(\theta(k))}{\sum_{i=1}^{r}\omega_i(\theta(k))},\quad \omega_i(\theta(k))=\prod_{j=1}^{p}\mu_{ij}(\theta_j(k)),$$

with $\mu_{ij}(\theta_j(k))$ representing the grade of membership of $\theta_j(k)$ in μ_{ij}. Then, it is easy to see that for all k,

$$h_i(\theta(k))\ge 0,\quad \sum_{i=1}^{r}h_i(\theta(k))=1.$$

To facilitate the following discussion, a more compact presentation of system (2) can be rewritten as follows:

$$Ex(k+1)=A(\theta)x(k)+A_d(\theta)x(k-d(k)),$$
$$x(k)=\phi(k),\quad k=-d_2,\ldots,1,0. \tag{3}$$

where

$$A(\theta)=\sum_{i=1}^{r}h_i(\theta(k))A_i, A_d(\theta)=\sum_{i=1}^{r}h_i(\theta(k))A_{di}.$$

Before moving on, we introduce some definitions to be used in the proof of main results throughout this paper. Consider the discrete descriptor time-delay system:

$$Ex(k+1)=A_hx(k)+A_{dh}x(k-d). \tag{4}$$

Definition 1 (*Xu and Lam [29]*).

(I) The pair (E,A_h) is said to be regular if $\det(zE-A_h)$ is not identically zero.

(II) The pair (E,A_h) is said to be causal if $\deg(\det(zE-A_h))=\text{rank}(E)$.

Definition 2 (*Xu and Lam [29]*).

(I) For a given integer d, the discrete descriptor time-delay system (4) is said to be regular and casual, if the pair (E,A_h) is regular and casual.

(II) The discrete descriptor time-delay system (4) is said to be stable if for any scalar $\epsilon>0$, there exists a scalar $\delta(\epsilon)>0$ such that, for any compatible conditions $\phi(k)$ satisfying $\sup_{-d\le k\le 0}\|\phi(k)\|\le\delta(\epsilon)$, the solution $x(k)$ to the system (4) satisfies $\|x(k)\|\le\epsilon$ for any $k\ge 0$; moreover $\lim_{k\to\infty}x(k)=0$.

(III) The system (4) is said to be admissible if it is regular, causal and stable.

3. Main results

In this section, first of all, we consider the regularity, causality and stability for the systems (3), which is concluded in the following theorem.

Theorem 1. *The discrete T–S fuzzy descriptor system (3) is admissible if there exist symmetric positive-definite matrices $P>0$, $Q>0$, $T>0$, $Z_1>0$, $Z_2>0$, matrices L_1, L_2, L_3, M_1, M_2, M_3, N_1, N_2, N_3, and symmetric matrices S, such that the following LMI holds for $i=1,\ldots,r$*

$$\begin{bmatrix}\Omega_{1i}+\Omega_2+\Omega_2^T+\Omega_{3i} & \Omega_4\\ * & \Omega_5\end{bmatrix}<0 \tag{5}$$

where $R\in\mathbb{R}^{n\times(n-g)}$ is any matrix with full column rank and satisfying $E^TR=0$, and

$$\Omega_{1i}=\begin{bmatrix}A_i^TPA_i-E^TPE+A_i^TRSR^TA_i+\bar{d}Q+T & A_i^TPA_{di}+A_i^TRSR^TA_{di} & 0\\ * & A_{di}^TPA_{di}+A_{di}^TRSR^TA_{di}-Q & 0\\ * & * & -T\end{bmatrix},$$
$$\Omega_2=\begin{bmatrix}LE+ME & NE-LE & -NE-ME\end{bmatrix},$$
$$\Omega_{3i}=d_2\begin{bmatrix}A_i-E & A_{di} & 0\end{bmatrix}^T(Z_1+Z_2)\begin{bmatrix}A_i-E & A_{di} & 0\end{bmatrix},$$
$$\Omega_4=\begin{bmatrix}\sqrt{d_2}L & \sqrt{d_2-d_1}M & \sqrt{d_2}N\end{bmatrix},$$
$$\Omega_5=\text{diag}\{-Z_1,\ -Z_1,\ -Z_2\},$$