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Brief Papers

Image reconstruction from random samples using multiscale regression framework

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ABSTRACT

Preserving edge details is an important issue in most of the image reconstruction problems. In this paper, we propose a multiscale regression framework for image reconstruction from sparse random samples. A multiscale framework is used here to combine the modeling strengths of parametric and non-parametric statistical techniques in a pyramidal fashion. This algorithm is designed to preserve edge structures using an adaptive filter, where the filter coefficients are derived using locally adapted kernels which take into account both the local density of the available samples, and the actual values of these samples. As such, they are automatically directed and adapted to both the given sampling geometry and the samples' radiometry. Both the upscaling and missing pixel recovery processes are made locally adaptive so that the image structures can be well preserved. Experimental results demonstrate that the proposed method achieves better improvement over the state-of-the-art algorithms in terms of both subjective and objective quality.

1. Introduction

The emergence of high definition displays in recent years, along with rapid increase of cheaper digital imaging devices has resulted in the need for fundamentally new image processing algorithms. In this work, we address the issue of missing information corrupted by the limitations of the imaging system as well as degradation processes such as compression [1], in a different way. This work concentrates on a data-adaptive multiscale regression framework for reconstruction and enhancement of randomly sampled images.

The projection onto convex set (POCS) based Papoulis–Gerchberg (PG) algorithm [2–3] and Delaunay triangulation based interpolation [4] are two classic image reconstruction algorithms. Michael and David proposed a sparse representation-based morphological component analysis (MCA) method [5], which separates the image into texture and piecewise smooth (cartoon) parts. It is exploiting both the variational and the sparsity mechanisms. The method combines the basis pursuit denoising (BPDN) algorithm and the total-variation (TV) regularization scheme.

The maximum likelihood estimation by random sample and the local optimization (MLESAC) method [6] is a robust estimator which adopts maximum likelihood theory with local optimization (LO). Guided-MLESAC [7] introduced by Tordoff and Murray completely utilizes matching prior probabilities, which makes sampling more efficient and finally achieves Bayesian maximum likelihood estimation, but it requires high cost in calculation. AMLESAC mentioned by Konouchin [8] adopts modified median estimator method to estimate initial value of parameter in the model and enhances likelihood of the output model. The algorithm of MLESAC generally uses some iterative optimized algorithm. It also introduces an accelerated algorithm MLESAC, which embeds LO into the iterative steps of MLESAC, then guides the iteration through the result of LO [6].

Classical Kernel Regression [10] is another well known, non-parametric point estimation procedure. KR approach was useful for handling image reconstruction from very sparse samples [10,13]. The non-parametric KR method for image processing which was a variant of the famous Nadaraya–Watson (NW) estimator [9] was introduced in a nonlocal means denoising algorithm [18,19]. By using the kernel functions driven by the distances between existing pixels within a large neighborhood, these non-local type of non-parametric image models made the algorithm more robust in recovering very sparse image samples [10]. But, the classic KR-based method does not explore the inter-scale similarity.

Takeda et al. generalized this technique to spatially adaptive steering kernel regression [10], which preserved and restored details with minimal assumptions on local signal and noise models. The hybrid image reconstruction (HIR) algorithm [11] proposed by Guangtao and Zang combined the linear

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autoregressive (AR) parametric model [12] and the kernel regressive (KR) non-parametric model [9] systematically to improve the modeling efficiency. HIR algorithm utilized the context vectors in vicinity without considering the local orientation along the image structures. By incorporating the steering KR technique into the multiscale framework, both in the restoration of missing pixels as well as the upscaling of lower scale image to higher resolutions, we could reconstruct different edge structures more efficiently.

From the above analysis, it is noted that, a unified scheme for reconstructing the edge, smooth and texture regions with affordable computational complexity is desirable. The essence of this reconstruction is to assign the missing pixels according to the rest of the image, i.e., being a conditional expectation estimation problem. The missing pixels are considered as a vector, and its expectation is given by its surrounding available pixels. Moreover, the multiscale approach [13,14] is utilized to combine the parametric and non-parametric techniques in a single framework. The missing pixels are successively recovered from significant pixel loss (from low to high frequencies), and the restored image at a particular level is in turn used for estimation of the next level. This approach is shown to be extremely effective for sparse random samples. Many large-scale structures can be well recovered based upon the progressively computed low-level results, and this is impossible for traditional single level reconstruction algorithms. For the signal recovery and upscaling, a data-adaptive filter [9] is used here, which directs the kernel along the edges, rather spread across it. The filter coefficients are derived depending on the dominant orientation calculated from the local covariance matrices in the selected window. The original image with only randomly selected samples is successively downsampled to form a multiscale pyramid by replacing the low resolution pixel by the average of the available high resolution pixels. The missing pixels in the lowest resolution image (highest level L_2) can be recovered using a data-adaptive non-parametric KR model. This recovery can be done iteratively to improve the estimation accuracy. The parametric AR model embedded into a data-adaptive KR framework is then used to upsample the recovered image to a higher resolution.

The estimates on each level is refined by a non-parametric KR model which use the upsampled image from previous level as a prior estimate in the next level. Refined estimate is upscaled again by a parametric data-adaptive KR model, which can be in turn used as a prior for next level of reconstruction to get the final result.

The rest of this paper is organized as follows. Section 2 describes the underlying theory behind our work. The concept of parametric and non-parametric image modeling, multiscale approximation, kernel regression and soft-decision interpolation are discussed. The proposed multiscale regression framework with an algorithmic description is presented in Section 3. The implementation details and simulation results are given in Section 4. Also, the proposed method is compared with classical and the state-of- the-art image reconstruction algorithms in terms of both subjective and objective quality. Finally, the paper is concluded in Section 5.

2. Background theory

2.1. Image models

The estimation of the conditional expectation of Y given an observation of the context $X=\boldsymbol{x_i}$ is as follows

$$E(Y|X=\boldsymbol{x_i})=\sum_{i=1}^{n} y_i P(y_i|\boldsymbol{x_i}) \tag{1}$$

where $\{y_i\}$, $1 \le i \le n$ and $\{\boldsymbol{x_i}\}$, $1 \le i \le n$ are two sets of image pixel samples. Y is a dependent variable and $X \in R^m$ is an independent variable that represent a pixel and a set of pixels (context vector), respectively. In this paper, for image reconstruction, we use the existing pixel samples from the observation.

2.1.1. Parametric image model

Linear autoregressive (AR) model is widely used in signal processing. It is an effective structural constraint for solving various image processing problems such as predictive coding and image interpolation. A parametric image model can be derived by assuming a parametric relationship between y_i and $\mathbf{x_i}$. A linear regression model of y_i for $\mathbf{x_i}$ can be written as:

$$y_i=\sum_{j=1}^{m} \alpha_j \mathbf{x}_j + e_i \tag{2}$$

where α_j, $j=1,2,\ldots,m$ are the regression coefficients and $e_i \in R^m$ is an additive multivariate Gaussian zero mean noise term. This is known as the AR model. Writing (2) in vector form, we have

$$\mathbf{y}=\boldsymbol{A}\mathbf{x}+\mathbf{e} \tag{3}$$

where $A=\left[x_i^{(1)},x_i^{(2)},\ldots,x_i^{(n)}\right], A \in R^{m\times n}$ is the design matrix, and $\mathbf{x}=[\alpha_1,\alpha_2,\ldots,\alpha_n]^T, \mathbf{x}\in R^n$ is the parameter vector. The parameter vector can be estimated by the $\ell_2$minimization problem (minimization of error vector)

$$\hat{\mathbf{x}}=\underset{x\in R^n}{\operatorname{argmin}} \|\boldsymbol{y}-\boldsymbol{A}\boldsymbol{x}\|_2^2 \tag{4}$$

The AR models are solved by classical least square (LS) method. With the normality assumption of the noise term e_i, the least square estimation is also the maximum likelihood estimation of the model parameters. The LS problem in (4) is equivalent to solving an incompatible linear system

$$\mathbf{Ax}=\mathbf{Y} \tag{5}$$

where $\mathbf{Y}^T=[y_1,y_2,\ldots,y_n]$ and $\mathbf{X}^{\mathrm{T}}=[\mathbf{x_1},\mathbf{x_2},\ldots,\mathbf{x_n}]$, which has closed form solution as

$$\hat{\mathbf{x}}_{LS}=(\mathbf{A}\mathbf{A}^{\mathrm{T}})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Y} \tag{6}$$

for the over-determined system $(n>m)$.

Numerical stability is a major issue with the LS solution of the AR model. The problem is related to the rank condition of the design matrix. The probability of numerical rank deficiency is rather high due to discrete nature and structures of the digital images in case of natural images. Without proper care, numerical rank deficiency can adversely affect the parameter estimation of the AR model. In order to overcome this, rank revealing QR factorization is used to select an optimal subset from the design matrix. A truncated solution to the linear system can be calculated by removing the ill conditioned part of the right orthogonal matrix of the rank revealing QR decomposition [15].

2.1.2. Non-parametric image model

KR is a widely known non-parametric technique which is used for point estimation of probability functions, where the estimated distributions are smooth. The conditional expectation in (1) can be expressed as

$$y=E(Y|X=\mathbf{x}_i)\approx\frac{\sum_{j=1}^{n} K_h(\|\mathbf{x}_i-\mathbf{x}_j\|)y_j}{\sum_{j=1}^{n} K_h(\|\mathbf{x}_i-\mathbf{x}_j\|)} \tag{7}$$

where the kernels $K_o(x)=\frac{1}{o}K\left(\frac{x}{o}\right)$ can be chosen from functions that are non-negative, sum to 1 and symmetric around 0.

The non-parametric conditional expectation estimator in (7) is known as the Nataraya–Watson (NW) estimator, which is the weighted average of the observations $y_1,y_2,\ldots,y_n$with the weight in inverse proportion to the distances between $\mathbf{x}_j$, $1 \le j \le n$ and $\mathbf{x}_i$.