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Mean-square stability analysis of discrete-time stochastic Markov jump recurrent neural networks with mixed delays ☆

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ABSTRACT

In this paper, the problem of mean-square stability analysis for discrete-time stochastic Markov jump recurrent neural networks with time-varying mixed delays is considered. The Markov jumping transition probabilities are assumed completely unknown but piecewise homogeneous, and the mixed time delays under consideration comprise both time-varying discrete delay and infinite distributed delay. In the framework of the delay partitioning approach, the informations of the delay distribution probability are fully considered. With a novel Lyapunov functional, a sufficient delay-dependent condition is established, which is characterized in terms of Linear Matrix Inequalities (LMIs). Finally, a numerical example is given to demonstrate the effectiveness of the obtained results.

1. Introduction

In the past few years, recurrent neural networks have been extensively investigated due to the fact that neural networks have successful applications in a variety of areas, such as associative memory, pattern recognition, signal processing and combinatorial optimization [1,2,7]. Dynamical properties (e.g. stability, instability, periodic oscillatory and chaos) of neural networks are the key to use. Furthermore, the stability of neural networks is a prerequisite for some optimization problems. It is worth pointing out that discrete-time neural networks (DNNs), specializing in implementing and applications, are more suitable to our digital life than continuous-time ones due to the fact that discrete-time systems can provide more convenient in simulating and computing than continuous-time systems. Therefore, DNNs have been extensively studied, see [3–5] and the references therein.

As a result of the finite switching speed of amplifiers and the inherent communication time of neurons, time delays are frequently encountered in neural networks in electronic implementations. And it has been shown that time delays may cause oscillation, divergence and instability. Therefore, time-delay neural networks have been an attractive and important subject of research in the past few years [6,8-17]. Based on the delay partitioning algorithm, [8] has represented $d(k)$ as $d_m+h(k)(d_m=\tau m)$ and introduced a new Lyapunov functional. [12] has modeled the recurrent neural network as a kind of stochastic system and assumed the stochastic variable to satisfy Bernoulli process and the similar method can be found in [35,36]. [14] has divided $[\tau_m, \tau_M]$ into n_1 parts and described the probability distribution of time-varying delay as Bernoulli stochastic variables, and such variables are considered in the Lyapunov functional to obtain less conservative results. On the other hand, another type of time-delay, namely, distributed delay, has attracted considerable interest, that's because neural network usually has a spatial nature and there exists an amount of parallel pathways with a variety of axon sizes and lengths [37]. Recently, several interesting research results for neural networks with mixed time delays have been obtained. For recurrent neural network with multiple discrete delays and infinity-distributed delays, sufficient conditions for checking the exponential stability of such systems were given via LMI in [11]. In [16], the exponential synchronization of neural networks with mixed delays (both discrete and finite-distributed time delays) under time-varying sampling was given.

In addition, as a special class of hybrid systems, Markov jump systems have the advantage of better describing practical systems with different structures due to random abrupt variations, and thus a large amount of attention has been devoted to the analysis and synthesis of Markov jump systems [18-34]. For example, the first attempt to investigate the stability of neural networks with

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Markov jump parameters and time delay has been made in [20], where some LMI-based stability criteria have been proposed for testing whether the network dynamics is stochastically exponentially stable in the mean square, i.e., independent of the time delay. In [24], the passivity analysis has been conducted for discrete-time stochastic neural networks with both Markovian jumping parameters and mixed time delays, and a delay-dependent passivity condition has been derived by introducing a Lyapunov functional that accounts for the mixed time delays. In [26], the sampled-data synchronization has been investigated for Markov jump neural networks with time-varying delay, and the desired mode-dependent sampled-data controller has been given. In addition, asynchronous l_2-l_∞ filtering for discrete-time stochastic Markov jump systems with randomly occurred sensor nonlinearities, Passivity-based non-fragile control for Markovian jump systems with aperiodic sampling, and the exponential synchronization of complex dynamical networks with Markovian jumping parameters using sampled-data and mode-dependent probabilistic time-varying delays are recently investigated in [28–30] respectively.

Best to the author's knowledge, however, most of the above-mentioned references assume that the transition rates in the Markov process are completely known or partly known. In fact, the assumption cannot always be satisfied in real application, and the ideal assumption inevitably limits the applications of the established results to some extent. Up to now, the problem for mixed delay Markov jump neural networks with unknown transition probabilities has not fully investigated, despite its practical importance.

In this paper, we deal with the mean-square stability analysis problem of discrete-time stochastic Markov jump recurrent neural networks with mixed delays and we assume the transition probabilities in the Markov process is completely unknown but piecewise homogeneous. By exploiting all possible probability distribution information in discrete delay, a sufficient condition is given by a new class of Lyapunov functional. Finally, we give a numerical example to demonstrate the feasibility and the effectiveness of the developed methods.

Notations. : The notations are fairly standard. R^n denotes the n-dimensional Euclidean space. The notation $X>Y(X\geq Y)$, where X and Y are symmetric matrices, means that $X-Y$ is a positive definite (positive semidefinite). $\boldsymbol{I}$ and $\boldsymbol{0}$ represent the identity matrix and a zero matrix, respectively; and $diag\{\dots\}$ stands for a block-diagonal matrix. The superscript "T" represents the transpose and the asterisk "*" in a matrix is used to represent the term which is induced by symmetry. $\|\bullet\|$ is the Euclidean norm in R^n. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

2. Problem formulation and preliminaries

Fix a probability space $(\Omega,\mathcal{F},\mathcal{P})$ and consider the following Markov jump neural network with mixed time-varying delays :

$$x(k+1)=C(r(k))x(k)+A(r(k))F(x(k))+B(r(k))G(x(k-\tau(k)))$$
$$+D(r(k))\sum_{m=1}^{+\infty}\mu_m\alpha(x(k-m))+J \tag{1}$$

where $x(k)$ satisfy $x(k)=[x_1(k),x_2(k),...,x_n(k)]^T\in R^n$, and $x_i(k)$ is the state of the ith neuron at time k; $C(r(k))=diag\{c_1,c_2,\dots,c_n\}$ describes the rate with which the each neuron will reset its potential to the resting state in isolation when disconnected from the networks and external inputs ($\|c_i\|<1$); $A(r(k))=\left(a_{ij}(r(k))\right)_{n\times n}$,$B(r(k))=\left(b_{ij}(r(k))\right)_{n\times n}$ and $D(r(k))=\left(d_{ij}(r(k))\right)_{n\times n}$ are respectively the connection weight matrix, the discretely delayed connection weight matrix and the distributional delayed connection weight matrix; $J=\left[J_1,J_2,\dots,J_n\right]^T\in R^n$ is the exogenous input; $G_i(x_i(k))$ and $F_i(x_i(k))$ denote the neuron activation functions of the ith neuron at time k; $\alpha_i(x_i(k))$ is nonlinear function of the ith neuron at time k, and satisfy

$$F(x(k))=\left[F_1(x_1(k)),F_2(x_2(k)),\dots,F_n(x_n(k))\right]^T\in R^n,$$
$$G(x(k))=\left[G_1(x_1(k)),G_2(x_2(k)),\dots,G_n(x_n(k))\right]^T\in R^n,$$
$$\alpha(x(k))=\left[\alpha_1(x_1(k)),\alpha_2(x_2(k)),\dots,\alpha_n(x_n(k))\right]^T\in R^n.$$

And, $\tau(k)$ denotes the discrete time-varying delay satisfying $\tau_m\leq\tau(k)\leq\tau_M$, where τ_m and τ_M are the minimum and the maximum of the allowable time delay bound; μ_m is a nonnegative constant and satisfies the convergent conditions, as $\sum_{m=1}^{+\infty}\mu_m<+\infty$, $\sum_{m=1}^{+\infty}mu_m<+\infty$. The parameter $r(k)$ is assumed to be a discrete-time homogeneous Markov chain taking values in a finite set $\mathcal{S}=\{1,2,\dots,s\}$ with transition probability matrix $\Pi\triangleq\{\pi_{pq}\}$ given by

$Pr\{r(k+1)=q|r(k)=p\}=\pi_{pq}$ $\forall$ $p,q\in\mathcal{S}$, where $0\leq\pi_{pq}\leq1$ and $\sum_{q=0}^{s}\pi_{pq}=1$

Throughout the article, the following assumptions, definition and lemmas are needed:

Assumption 1. (Wang et al. [3], Liu et al. [5]) For any $x,y,\in \mathrm{R}(x\neq y), i\in\{1,2,\dots,n\}$, the activation functions $F(x(k))$, $G(x(k))$ and nonlinear function $\alpha(x(k))$ satisfy

$$f_i^-\leq\frac{F_i(x)-F_i(y)}{x-y}\leq f_i^+,\ g_i^-\leq\frac{G_i(x)-G_i(y)}{x-y}\leq g_i^+,\ \alpha_i^-\leq\frac{\alpha_i(x)-\alpha_i(y)}{x-y}\leq\alpha_i^+ \tag{2}$$

where $f_i^-,f_i^+,g_i^-,g_i^+,\alpha_i^-$ andα_i^+ are constants.

Under Assumption 1, [5] had got the results that the system has equilibrium points, let x^* be the equilibrium point of system (1) and shift it to the origin by letting $y(k)=x(k)-x^*$, then system (1) with stochastic disturbances can be rewritten as

$$y(k+1)=C(r(k))y(k)+A(r(k))f(y(k))$$
$$+B(r(k))g(y(k-\tau(k)))+D(r(k))\sum_{m=1}^{+\infty}\mu_m h(y(k-m))$$
$$+\sigma(k,y(k),y(k-\tau(k)),r(k))\omega(k) \tag{3}$$

where

$$y(k)=\left[y_1(k),y_2(k),\dots,y_n(k)\right]^T,\quad f(y(k))=F(x(k))-F(x^*)$$
$$g(y(k))=G(x(k))-G(x^*),\quad h(y(k))=\alpha(x(k))-\alpha(x^*)$$

$\omega(k)$ is a scalar Wiener process on a probability space $(\Omega,\mathscr{F},\mathcal{P})$ with $E\{\omega(k)\}=0,E\{\omega^2(k)\}=1,\ E\{\omega(i)\omega(j)\}=0(i\neq j)$;

From Assumption 1 , it is not difficult to conclude that

$$f_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq f_i^+,\ g_i^-\leq\frac{g_i(x)-g_i(y)}{x-y}\leq g_i^+,\ h_i^-\leq\frac{h_i(x)-h_i(y)}{x-y}\leq h_i^+$$

Assumption 2. The noise intensity function vector $\sigma(k,x,y)\mathbb{N}\times\mathbb{N}^n\times\mathbb{N}^n\to\mathbb{N}$ satisfies the Lipschitz condition and there exist constants $\varepsilon_1,\varepsilon_2$, such that the following inequality:

$$\sigma^T(k,x,y)\sigma(k,x,y)\leq\varepsilon_1x^Tx+\varepsilon_1y^Ty$$

Assumption 3. (Hou et al. [3]) The time-varying delay $\tau(k)$ is bounded, $0<\tau_m\leq\tau(k)\leq\tau_M$, and its probability distribution can be observed. Assume that integer $\tau(k)$ satisfies $\tau_i\leq\tau(k)\leq\tau_{i+1},d_i=\tau_i-\tau_{i-1}\neq0$ $(i=1,2,\dots,n_1)$, which means we divide $[\tau_m,\tau_M]$ into n_1 parts, and $Pr\{\tau(k)\epsilon[\tau_{i-1},\tau_i]\}=\rho_i=1-\tilde{\rho}_i$, where $0\leq\rho_i\leq1,\sum\rho_i=1,i=1,2,\dots,n_1$ and $\tau_0=\tau_m$, $\tau_{n_1}=\tau_M$.

To describe the probability distribution of time-varying delay, we define the following set $A_i=(\tau_{i-1},\tau_i],i=1,2,\dots,n_1$. Define