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Robust exponential stability of uncertain impulsive delays differential systems

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ABSTRACT

This paper deals with the robust stability of a class of uncertain impulsive control systems with infinite delays. By employing the formula for the variation of parameters and estimating the Cauchy matrix, several criteria on robust exponential stability of the systems are derived, these criteria are less restrictive than those in the earlier publications. Moreover, the criteria can be applied to stabilize the unstable continuous systems with infinite delays and uncertainties by utilizing impulsive control. Finally, two numerical examples are given to illustrate the effectiveness and advantages of the proposed method.

1. Introduction

Recently, impulsive control has attracted great interest of many researchers [1–6]. Such control systems arise naturally in a wide variety of applications, such as dosage supply in pharmacokinetics [1], orbital transfer of satellite [2,3], ecosystems management [4,5] and control of saving rates in a financial market [6]. Moreover, time delays and uncertainties [7–11] occur frequently in engineering, biological and economical systems, and sometimes they depend on the histories heavily and result in oscillation and instability of systems [8,12]. [14–22] are the cases of finite delays. Yang and Xu presented several interesting criteria on robust stability for uncertain impulsive control systems with time-varying delays [16]. Liu established several criteria on asymptotic stability for impulsive control systems with time delays [18]. [23–25] are the cases of infinite delays. However, the corresponding theory for impulsive control systems with infinite delays has been relatively less developed. In fact, an infinite delays deserve study intensively because they are not only an extension of finite delays but also describing the adequate mathematical models in many fields [17]. Therefore, it is necessary to further investigate the stability of uncertain impulsive control systems with infinite delays. Meanwhile, it is challenging to address the issue since we must utilize impulsive effects to handle the instability which may be caused by the infinite delays and uncertainties. Hence, techniques and methods for uncertain impulsive control systems with infinite delays should be further developed and explored.

This paper is inspired by [16]. In this paper, we present some criteria for the robust exponential stability of uncertain impulsive control systems with infinite delays by using the formula for the variation of parameters and estimating the Cauchy matrix. More importantly, the robust stability criteria do not require the stability of the corresponding continuous systems and so it can be more widely applied to stabilize the unstable continuous systems with infinite delays and uncertainties by using impulsive control. Finally, two examples are given to show the effectiveness and advantages of the obtained results.

2. Preliminaries

Let $N = 1, 2, \cdots, I$ be the identity matrix, $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ be the smallest and the largest eigenvalues of a symmetrical matrix, respectively. For $\phi: \mathbb{R} \to \mathbb{R}^n$, denote $\phi(t^+) = \lim_{s \to 0^+} \phi(t+s), \phi(t^-)$

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$=\lim_{s\to 0^-}\phi(t+s)$. For $x\in\mathbb{R}^n$ and $A\in\mathbb{R}^{n\times n}$, let $\|x\|$ be any vector norm, $\|\phi\|_\alpha=\sup_{s\le 0}\|\phi(s)\|$ and denote the induced matrix norm and the matrix measure, respectively, by

$$\|A\|=\sup_{x\neq 0}\frac{\|Ax\|}{\|x\|},\quad \mu(A)=\lim_{h\to 0^+}\frac{\|I+hA\|-1}{h}.$$

The usual norms and measures of vectors and matrices are:

$$\|x\|_1=\sum_{j=1}^n|x_j|,\quad \|A\|_1=\max_{1\le j\le n}\sum_{i=1}^n|a_{ij}|,$$

$$\mu_1(A)=\max_{1\le j\le n}\left\{a_{jj}+\sum_{i\neq j}^n|a_{ij}|\right\};$$

$$\|x\|_2=\sqrt{\sum_{j=1}^n x_j^2},\ \|A\|_2=\sqrt{\lambda_{\max(A^TA)}},$$

$$\mu_2(A)=\frac{1}{2}\lambda_{\max}(A+A^T);$$

$$\|x\|_\infty=\max_{1\le i\le n}|x_i|,\ \|A\|_\infty=\max_{1\le i\le n}\sum_{j=1}^n|a_{ij}|,$$

$$\mu_\infty(A)=\max_{1\le i\le n}\left\{a_{ii}+\sum_{j\neq i}^n|a_{ij}|\right\}.$$

Consider the uncertain infinite delays differential system:

$$\begin{cases}\dot{x}(t)=[A+\Delta A]x(t)+[B+\Delta B]x(t-r(t))+[C+\Delta C]\int_0^{+\infty}h(s)x(t-s)\,ds, & t\neq t_k,\\ y(t)=Ex(t), & t\ge 0,\end{cases}\tag{1}$$

in which $x\in\mathbb{R}^n$ is the state variable, $y\in\mathbb{R}^m$ is the output variable, $r(t)$ is the time-varying delay function with $0\le r(t)\le\tau$, τ is a given positive constant, $A,B,C\in\mathbb{R}^{n\times n}$ and $E\in\mathbb{R}^{m\times n}$ are known constant matrices, $\Delta A,\Delta B$ and ΔC are the uncertain matrices, which vary within the range of

$$\|\Delta A\|\le a,\ \|\Delta B\|\le b,\ \|\Delta C\|\le c,\tag{2}$$

where a,b,c are known non-negative constant, $h(s)\in C(\mathbb{R}^+,\mathbb{R})$ satisfying $\int_0^{+\infty}|h(s)|e^{\eta s}\,ds<\infty$, where $\eta>0$ is a given constant.

An impulsive control law of (1) can be presented in form of the following control sequence $\{t_k,U(k,x(t_k^-))\}$ (see [6,18]):

$$0\le t_0<t_1<\cdots<t_k<\cdots,\ \lim_{k\to+\infty}t_k=+\infty,$$

$$\Delta x(t_k)=x(t_k^+)-x(t_k^-)=U(k,x(t_k^-)),k\in N.$$

Let $U(k,x)=B_ky,B_k\in\mathbb{R}^{n\times m}$ and $C_k=B_kE$. Then we obtain the uncertain impulsive control system with infinite delays as follows:

$$\begin{cases}\dot{x}(t)=[A+\Delta A]x(t)+[B+\Delta B]x(t-r(t))+[C+\Delta C]\int_0^{+\infty}h(s)x(t-s)\,ds, & t\neq t_k,\\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=C_kx(t_k^-), & k\in N,\\ x(t)=\phi(t), & t\le 0,\end{cases}\tag{3}$$

where $\phi:(-\infty,0]\to\mathbb{R}^n$ is continuous. In particular, if $h(s)=0$, then system (3) becomes the impulsive control system with finite delays given in [16]:

$$\begin{cases}\dot{x}(t)=[A+\Delta A]x(t)+[B+\Delta B]x(t-r(t)), & t\neq t_k,\\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=C_kx(t_k^-), & k\in N,\\ x(t)=\phi(t), & -\tau\le t\le 0,\end{cases}\tag{4}$$

where $\phi:[-\tau,0]\to\mathbb{R}^n$ is continuous. Also, if $\Delta A=\Delta B=\Delta C=0$, then system (3) becomes the deterministic impulsive control systems with infinite delays:

$$\begin{cases}\dot{x}(t)=Ax(t)+Bx(t-r(t))+C\int_0^{+\infty}h(s)x(t-s)\,ds, & t\neq t_k,\\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=C_kx(t_k^-), & k\in N,\\ x(t)=\phi(t), & \alpha\le t\le 0.\end{cases}\tag{5}$$

We always assume that the solution $x(t)$ of (3) are right continuous at $t=t_k$, i.e., $x(t_k)=x(t_k^+)$. That is, the solutions are the piecewise continuous functions with discontinuities of the first kind only at $t=t_k,k\in N$. For more details on the impulsive delay systems, one can refer to [1,13,17] and references therein.

In order to prove our main results, we need the following definitions and lemmas:

Definition 2.1 (*Lakshmikantham et al. [1]*). Assume that $x(t)=x(t,t_0,\phi)$ to be the solution of (3) through (t_0,ϕ). Then the zero solution of (3) is said to be globally exponentially stable, if for any initial data $x_{t_0}=\phi$, there exist two positive numbers $\lambda>0,M\ge 1$ such that $\|x(t)\|\le M\|\phi\|_\alpha e^{-\lambda(t-t_0)},t\ge t_0$.

Definition 2.2 (*Yang and Xu [16]*). The uncertain impulsive dynamical system (3) is called robustly exponentially stable, if the zero solution $x=0$ of the system is globally exponentially stable for any $\Delta A,\Delta B,\Delta C$ satisfying (2).

The following lemma introduces a norm of vector, a matrix norm and a matrix measure (we call them P-norm and P-measure).

Lemma 2.1 (*Yang and Xu [16]*). *Let $P\in\mathbb{R}^{n\times n}$ be symmetrical and positive definite. Then $\|x\|_P=\sqrt{x^TPx}$ is a norm of a vector $x\in\mathbb{R}^n$, the induced norm and measure of matrix $A\in\mathbb{R}^{n\times n}$ are, respectively,*

$$\|A\|_P=\|DAD^{-1}\|_2,\mu_P(A)=\mu_2(DAD^{-1}),$$

where $D^TD=P$.

Lemma 2.2 (*Yang and Xu [16]*). *Let $W(t,t_0)$ be the Cauchy matrix of the linear systems:*

$$\begin{cases}\dot{x}(t)=Ax(t), & t\neq t_k,\\ \Delta x(t_k)=x(t_k^+)-x(t_k^-)=C_kx(t_k^-), & k\in N.\end{cases}\tag{6}$$

Given a constant $\|I+C_k\|\le\gamma$ for all $k\in N$, we have the following:

Case (1) if $0<\gamma<1$ and $\rho=\sup_{k\in N}\{t_k-t_{k-1}\}<\infty$, then

$$\|W(t,t_0)\|\le\frac{1}{\gamma}e^{\left(\mu(A)+\frac{\ln\gamma}{\rho}\right)(t-t_0)},\quad t\ge t_0;$$

Case (2) if $\gamma\ge 1$ and $\theta=\inf_{k\in N}\{t_k-t_{k-1}\}>0$, then

$$\|W(t,t_0)\|\le\gamma e^{\left(\mu(A)+\frac{\ln\gamma}{c}\right)(t-t_0)},\quad t\ge t_0.$$

3. Main results

Theorem 3.1. *Let $\rho=\sup_{k\in N}\{t_k-t_{k-1}\}<\infty$. Suppose that there exists a constant $0<\gamma<1$ satisfy $\|I+C_k\|\le\gamma$ and*

$$\frac{a+b+\|B\|+(c+\|C\|)\mathbb{M}}{\gamma}+\mu(A)+\frac{\ln\gamma}{\rho}<0,$$

where $\mathbb{M}=\int_0^{+\infty}|h(s)|e^{\eta s}\,ds$. Then the zero solution of the system (3) is robustly exponentially stable.

Proof. Since $\frac{a+b+\|B\|+(c+\|C\|)\mathbb{M}}{\gamma}+\mu(A)+\frac{\ln\gamma}{\rho}<0$, then we choose small enough $\lambda\in(0,\eta)$ such that

$$\frac{a+(b+\|B\|)e^{\lambda\tau}+(c+\|C\|)\mathbb{M}}{\gamma}+\mu(A)+\frac{\ln\gamma}{\rho}+\lambda<0.$$

Furthermore, for any $\varepsilon\in(0,\lambda)$, we have

$$0\le\frac{a+(b+\|B\|)e^{(\lambda-\varepsilon)\tau}+(c+\|C\|)\mathbb{M}}{\gamma}\le-\mu(A)-\frac{\ln\gamma}{\rho}-(\lambda-\varepsilon).\tag{7}$$