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Simultaneous balancing and trajectory tracking control for two-wheeled inverted pendulum vehicles: A composite control approach

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ABSTRACT

This paper is concerned with a novel composite controller for an underactuated two-wheeled inverted pendulum vehicle with an unstable suspension subject to nonholonomic constraint. The presented composite controller is consisted with an adaptive sliding mode technique to construct an additional disturbance-like signal, and particularly a direct fuzzy controller to approximate the optimal velocity tracking control effort by the adaptive mechanism. Unlike the traditional control strategies, this composite control approach offers a coordinate control objective for the vehicle with an unstable equilibrium and second-order nonholonomic constraints. In addition, with the aid of a posture controller aiming to the tracking error dynamics, the nonholonomic vehicle can track an arbitrary trajectory given by the earth-fixed frame. Numerical simulation results confirm that the proposed controllers can maintain the balance of the unstable suspension, drive the vehicle globally to track any reference trajectory, and guarantee all the signals of the closed-loop system convergent within a compact set.

1. Introduction

Two-wheeled inverted pendulum vehicles attract much attention in the last few decades due to their special advantages such as compact construction, convenient operation, high maneuverability and so on, e.g., see [1–5] and the reference herein. This kind of vehicle is similar to the cart-pendulum platform which is extensively investigated to verify the effectiveness of the nonlinear control methodologies, but compared with the cart-pendulum platform, the vehicle has more application value, especially in the unknown, dynamic and unstructured environment. However, high performance motion control of the wheeled inverted pendulum vehicle is still a challenging work for the control community up to now [6–8]. On the one hand, when the vehicle moves, it is always hypothesized that the ground can provide sufficient friction to prevent the slipping and sliding (i.e., pure rolling), and then the nonholonomic constraint is taken place. On the other hand, consider that the underactuated vehicle body cannot recover itself once the body falls down so that the mobile platform is required to realize trajectory tracking and hold the vehicle unstable suspension upright simultaneously. Therefore, the nonholonomic constraint and underactuated problem are two main challenges for the trajectory tracking control of this special mobile platform.

As far as the nonholonomic constraint is concerned, it is well known that the linear control cannot directly be used to realize global tracking to an arbitrary trajectory by Brockett theory in [9]. Then, many efforts, such as time-vary control and switching approach, have been widely studied to deal with the nonholonomic constraint up to now. By choosing appropriate Lyapunov candidates, the mobile platform suffered from nonhlonomic constraint can be resolved effectively, e.g., [10,11]. These works provide reference for the tracking control of two-wheeled inverted pendulum vehicles.

Another challenge arises from the underactuated behaviors of the vehicle body around its unstable equilibrium. Since all the vehicle position and posture variables must be controlled only by the two driving motors installed in the corresponding wheels, the two-wheeled inverted pendulum vehicle is a class of underactuated systems whose control inputs are less than the degree of freedoms (see [12–16] for example). Meanwhile, it is noted that, in practice, the tilt angle of the vehicle suspension should be held within an acceptable domain around the unstable equilibrium in the first place because the vehicle body cannot recover its upright equilibrium once it falls down. By exploring the vehicle's dynamic behaviors in detail, the dynamics of tilt angle related to the vehicle body can be separated from the

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Nomenclature

x, y	current position of the vehicle on *XOY* plane
τ_l, τ_r	output torques of left and right wheel motors
θ	tilt angle of the vehicle body
φ	rotational angle of the vehicle
υ	longitudinal velocity of the vehicle
ω	rotational velocity of the vehicle
x_v	longitudinal displacement along the movement direction
m_p	mass of the inverted pendulum
m_c	mass of the chassis
m_w	mass of the each wheel
J_v	moment of inertia of the chassis and pendulum about the *Z*-axis
J_c	moment of inertia of the chassis about the *Y*-axis
J_w	moment of inertia of the wheel with respect to the *Y*-axis
r	radius of the wheels
l	distance from the body center of gravity to the wheel axis
g	gravity acceleration
d	distance between the two wheels along the axle center

longitudinal and rotational movements and can be considered as an underactuated subsystem [17]. Further investigation implies that making the underactuated tilt angle subsystem stable relies on the coupled dynamics of the two control inputs, and the dynamics of the underactuated subsystem can be regarded as a second-order nonholonomic constraint [18–20]. To the best of our knowledge, it has not been fully investigated when the nonholonomic constraints are coupled with the underactuated issues, which motivates this study.

Therefore, a composite control approach is presented to deal with the aforementioned problems, by which a coordinate control objective can be achieved. By analyzing the dynamic behaviors of the underactuated subsystem, it states that the accelerated velocity can be taken as a virtual control input to stabilize the unstable suspension considering the limitation of the system control inputs. This acceleration can be introduced as a disturbance-like signal added to the velocity tracking controller, and then a composite control comes into being. In particular, velocity tracking controller is designed by the fuzzy logic control in the presented study since the fuzzy mechanism has preferable robustness to reject the disturbance-like signal [21–24]. Besides, the direct fuzzy control is utilized rather than the indirect fuzzy mechanism because the former needs no updated model information which will result in the complicated calculation and even lead to the instability of the closed-loop system. At the same time, the updated laws are applied so as to reduce the dependency on the vehicle mechanical parameters and enhance the adaptivity of the controllers in dynamic environment. In addition, a posture controller is employed to contribute to driving the vehicle to track an arbitrary trajectory given by the earth-fixed frame.

The paper is organized as follows. In Section 2, the wheeled inverted pendulum is described and the reduced order dynamics with three subsystems is presented, in addition to nonholonomic constraint as well as physical properties of the vehicle; a disturbance-like signal is explored in Section 3; in the following, in Section 4, the specific control design algorithms for the associated subsystem are developed and meanwhile necessary stabilities are discussed; the feasibility and effectiveness of the proposed methods are confirmed by numerical simulation studies in Section 5, followed by the conclusions in Section 6.

2. System description and problem formulation

2.1. System description

A two-wheeled inverted pendulum vehicle (see in Fig. 1), which can also be considered as a wheeled mobile robot, is generally described as follows:

$$M(q)\ddot{q}+V_m(q,\dot{q})\dot{q}+G(q)=B(q)\tau-A^T\lambda \quad (1)$$

where $q=[q_1,q_2,q_3,q_4]^T=[x,y,\varphi,\theta]^T\in\mathbb{R}^4$ is the vector of generalized coordinates. Meanwhile, $M(q)\in\mathbb{R}^{4\times4}$ denotes the inertia matrix, $V_m(q,\dot{q})\dot{q}\in\mathbb{R}^4$ is the vector of coriolis and centrifugal forces, $G(q)\in\mathbb{R}^4$ is the vector of gravitational forces, $B(q)\in\mathbb{R}^{4\times3}$ denotes the matrix of control coefficients, $\tau\in\mathbb{R}^3$ is the vector of control inputs, $A^T\lambda\in\mathbb{R}^4$ is the vector of constraint forces, $A^T\in\mathbb{R}^4$ is a Jacobian matrix, and $\lambda\in\mathbb{R}$ is Lagrangian multipliers corresponding to the nonholonomic constraints, respectively. If q can be partitioned into $q_v=p[x,y,\varphi]^T$ and θ, then it follows that

$$M(q)=\begin{bmatrix}M_v & M_{v\theta}\\ M_{\theta v} & M_\theta\end{bmatrix};\ V_m(q,\dot{q})=\begin{bmatrix}V_v & V_{v\theta}\\ V_{\theta v} & V_\theta\end{bmatrix};$$

$$G(q)=\begin{bmatrix}G_v\\ G_\theta\end{bmatrix};\ B(q)=\begin{bmatrix}B_v & 0\\ 0 & B_\theta\end{bmatrix};\ \tau=\begin{bmatrix}\tau_u\\ 0\end{bmatrix}.$$

where M_v, M_θ are the inertia matrices for the mobile platform and the inverted pendulum; $M_{v\theta}$, $M_{\theta v}$ are the coupling inertia matrices of the mobile platform and the inverted pendulum; V_v, V_θ denote the centripetal and coriolis torques for the mobile platform and the inverted pendulum; $V_{v\theta}$, $V_{\theta v}$ are the coupling centripetal and coriolis torques of the mobile platform and the inverted pendulum; and G_v, G_θ are the gravitational torque vectors for the mobile platform and the inverted pendulum, respectively. τ_u denotes the control input vector for the mobile platform.

Assumption 1. There exists sufficient friction between the wheels of the mobile platform and the ground such that the assumption of nonholonomic constraint, i.e., no slipping and skidding, holds throughout.

2.2. Reduced dynamics

The vehicle subject to nonholonomic constraints can be described by $A^T\dot{q}_v=0$. Assume that the annihilator of the co-distribution spanned by the covector fields S_v is a one-dimensional smooth nonsingular distribution Δ on $\mathbb{R}^2$. This distribution Δ is spanned by a set of smooth and linearly independent vector fields $A_1(q)$ and $A_2(q)$, i.e., $\Delta=\text{span}\{A_1(q),A_2(q)\}$, which in the local coordinates satisfies the relation as $S^TA^T=0$ where $A\triangleq[A_1(q),A_2(q)]\in\mathbb{R}^{3\times2}$. Also, $S(q)=\text{diag}[S_v\ I]^T$ and S_v are the kinematic constraint matrix related to system nonholonomic constraint. Notice that A^TA is of full rank. Defining $\nu=[\dot{\varphi}\ \dot{x}_v]^T=[\omega\ \upsilon]^T\in\mathbb{R}^2$, the heading velocity of the platform can be governed by $\dot{q}_v=S_v(q)\nu$.

Letting $\dot{z}=[\omega\ \upsilon\ \dot{\theta}]^T$, and multiplying both sides of (1) by $\text{diag}[S^T\ I]$ to eliminate kinematic nonholonomic constraint $A^T\lambda$, the dynamics of wheeled inverted pendulum vehicle can be formulated as

$$\overline{M}(z)\dot{z}+\overline{V}_m(z,\dot{z})z+\overline{G}(z)=\overline{B}(z)\tau \quad (2)$$