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Sampling-based event-triggered consensus for multi-agent systems ☆

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ABSTRACT

This work considers the event-based consensus algorithms for multi-agent systems with periodic sampling schemes. An event-triggered consensus algorithm using sampled states are proposed. The measurement error in the event design is defined based on the state of local neighborhood center in order to reduce the number of events. Based on this algorithm, a self-triggered consensus protocol is proposed to further reduce the amount of communication and state sampling, as well as the number of events and thus the controller updates. It is noted that by introducing the periodic sampling scheme, Zeno behaviors can be naturally avoided in both of the algorithms. The effectiveness of the proposed algorithms is illustrated by simulation examples.

1. Introduction

Multi-agent control systems have been studied extensively in the past decade due to their significance in various industrial and military applications [1]. Typical research directions in this field include consensus [2,3], topology analysis and algorithms [4], connectivity control [5–7], leader–follower tracking [8], coverage control [9], and competition behaviors [10]. When implementing these control algorithms in practical systems, traditional periodic sampling technique is often adopted, e.g., in some industrial process control systems [11]. Digital platforms on these systems using periodical sampling usually require high frequency of communication and controller update. However, a practical agent, such as a personal robot, a quadrotor vehicle, or a mobile sensor, may only have limited onboard energy resources and low-level communication and actuating capabilities. Using periodical sampling will waste much energy and consequently shorten the lifespan of the system. To solve this problem, the event-triggered control strategy has been proposed and then studied for single-loop control systems in recent years [12–14]. In an event-triggered control system, state sampling and controller update is required to be conducted only when some “events” occur. These events are generated when some state-dependent equations or inequalities, called as the event conditions, are satisfied. In contrast with traditional periodic sampling control technique, the event-triggered controller can be considered as one of the nonuniform sampled controllers [15,16]. The event conditions are properly designed based on the control task. Then the amount of sampling and controller update can be significantly reduced by the event-triggered strategy.

Since the energy consumption may be significantly reduced by event-triggered controllers, this control approach is quite suitable for multi-agent systems, where a practical agent may suffer from the above-mentioned resource limitations. Using such a technique, each agent determines its sampling time instants, called as the event time instants, by some prescribed event conditions defined based on the measurement of the neighbors' states. Moreover, each agent execute only the inter-agent communication and control input updating at these event instants. By virtue of this, the frequency of communication and the number of controller updates can be significantly reduced. The control input will be held to be invariant in the uneven sampling time intervals to lighten the control effort. These improvements may save plenty of energy and lengthen the lifespan of the system. Currently, the event-triggered control approach has been utilized in multi-agent systems and some important results have been obtained, see [17–23].

Although the existing works in the event-triggered multi-agent control have obtained some important results, two problems have not been addressed and still warrant further investigation. One problem is that digital platforms in practical agents can only afford discreet signal transmission. Thus a sampling period has to be adopted and the events may occur only at the sampling time instants. Another problem is that an event-triggered control system may exhibit Zeno behavior since the overall system is hybrid. A Zeno execution implies that there exists infinitely many transitions in a finite time period, which is not allowed for a real physical system [24]. It is quite complex to analyze Zeno behavior in an event-triggered system, especially for distributed

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multi-agent systems. Motivated by these two problems, we present novel event-based consensus algorithms for multi-agent systems based on sampled data in this work. A sampling time period is adopted for the agent group and the triggering events can only occur at the sampling time instants. We firstly propose the event-triggered consensus algorithm by using measurement error of the local center state in the event design. Since the communication can only happen at the sampling time instants and the control input of each agent is unchanged during the event time interval, continuous monitoring of the neighbor states is no longer needed and moreover the number of control input updates is reduced. Since the periodic sampling scheme is used, the inter-event time interval is at least bounded from below by the sampling period and thus the Zeno behavior can be excluded.

The proposed consensus algorithms have several important advantages when compared with the existing works. In this work, one of the major objectives is to reduce the number of events and thus reduce the amount of communication among agents. This will save much energy for the agent group and thus lengthen the lifespan of the system in practice. The reduction of events is realized by two design approaches. Firstly, the measurement error in the event design is different from existing works such as [17,20]. In this work, the augmented state generated from the neighbor states is employed to define the measurement error for event design. It can be noted that such a design can significantly reduce the number of events and thus the amount of communication. Secondly, based on the proposed event-triggered algorithm, a self-triggered strategy has also been developed to relax the requirement that each agent should communicate with its neighbors at each sampling time instant. As a result, only at the event time instants, an agent transmits its state information to its neighbors. Thus the communication amount will be further significantly reduced when compared with that in [20]. Another improvement when compared with the similar sample-data-based event-triggered control in [20] is that the constraint on the sampling period h has been relaxed by using the event-triggered state as the threshold in the event design. Then h can be chosen as a larger positive real number than that in [20], which can be more practical in real multi-agent systems.

The rest of this paper is organized as follows. Section 2 presents a brief review of graph theory and the definitions of Zeno behaviors. Then the new event-triggered consensus algorithm is developed in Section 3. Section 4 presents the improved self-triggered consensus algorithm based on the event-triggered algorithm. In Section 5, a numerical example is provided to illustrate the proposed algorithm. Finally the paper is concluded in Section 6.

2. Preliminaries

In the coordination control of multi-agent systems, it is conventionally assumed that each agent can obtain the state information of the other agents through agent-to-agent communication. The communication network can be represented by a graph, called the communication graph. An undirected graph G of order N is denoted by $G=(V,E)$, where $V=\{1,2,\ldots,N\}$ is the vertex set and each vertex can represent an agent. $E \subseteq V \times V$ is the edge set, representing the communication links. If agents i and j can communicate with each other, there is a communication link between them, denoted by $(i,j) \in E$. Then agents i and j are called communication neighbors of each other. The communication neighbor set of agent i is denoted by $N_i=\{j \in V \mid (i,j) \in E\}$. The adjacency matrix of graph G is an $N \times N$ matrix $A=[a_{ij}]_{N\times N}$, with entries defined by

$$a_{ij}=\begin{cases} 1, & \text{if } (i,j) \in E, \\ 0, & \text{otherwise}. \end{cases} \tag{1}$$

The (in-)degree matrix $D=diag(d_1,d_2,\ldots,d_N)$ is a diagonal matrix, where $d_i=card(N_i)$ is the number of neighbors of agent i. Then the Laplacian matrix of G is defined by $L=D-A$. For an undirected graph G, the Laplacian matrix L is symmetric and positive semi-definite. If G is connected, L has a single zero eigenvalue with $\mathbf{1}_N$ being the corresponding eigenvector [3], where $\mathbf{1}_N$ is an order N column vector of all ones. The eigenvalues of L for an undirected graph are all nonnegative. Let these eigenvalues be listed by an ascending order

$$\lambda_1=0 \le \lambda_2 \le \cdots \le \lambda_N. \tag{2}$$

λ_2 is called as the algebra connectivity of a graph. If G is connected, λ_2 is positive [3].

In a control system with event-triggered strategy, both continuous dynamics and discrete transitions are involved. For such a hybrid system, a well-known phenomenon is the so-called Zeno behavior. A Zeno execution for a hybrid system is that many infinite discrete transitions occur in a finite period of time interval. Let the transition time instants for a hybrid system be represented by $t_0,t_1,\ldots,t_k,\ldots,k=0,1,\ldots$. Then the system exhibits a Zeno behavior if [24]

$$\lim_{k\to\infty} t_k=\sum_{k=0}^{\infty}(t_{k+1}-t_k)<\infty. \tag{3}$$

In a control system with event-triggered strategy, to avoid the Zeno behavior is critical since a practical system cannot support Zeno executions. In many existing works, an effective way to avoid the Zeno behavior is to show that there is a strictly positive time period in between two consecutive transition time instants, i.e., $t_{k+1}-t_k>c$ for all k with c being strictly positive.

3. Event-triggered consensus with sampling

Consider a multi-agent system with N agents in an $\mathbb{R}^n$ space. Each agent has the ability of computing, communicating and moving. Let the agent state be represented by $x_i(t) \in \mathbb{R}^n$ and the dynamic be

$$\dot{x}_i(t)=u_i(t), \tag{4}$$

where $u_i(t)$ is the control input. For clarity, in the sequel of this work we assume that $n=1$ and the results can be easily extended to the cases of $n=2,3$ by simple manipulations. The communication network of the agent group can be captured by an undirected graph G. The communication neighbor set of agent i, denoted by N_i, consists of all agents which has a communication link to agent i. To achieve consensus, each agent measures the local center state of its neighbors, which has the following form

$$q_i(t)=\sum_{j\in N_i}(x_j(t)-x_i(t)). \tag{5}$$

To reduce the communication and control loads, the sampling strategy will be used. Let h be the sampling period for the multi-agent system. This sampling period is fixed and identical for all agents. Then each agent is required to execute communication, state measurement, and control input updating actions only at the sampling time instants of the form

$$t=mh, \quad m=0,1,\ldots. \tag{6}$$

The measured local center state for agent i, which will be transmitted among agents for coordination control, is given by

$$q_i(mh)=\sum_{j\in N_i}(x_j(mh)-x_i(mh)). \tag{7}$$

In the controller design, the event-triggered scheme is adopted to reduce the resource consumption. We further require that each agent only update its control input at some specified event time instants, denoted by

$$t_0^i=0,t_1^i,\ldots,t_k^i,\ldots, \tag{8}$$