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Passivity analysis of memristive neural networks with probabilistic time-varying delays ☆

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ABSTRACT

The passivity problem has been further researched for a class of memristive neural networks with probabilistic time-varying delays in this paper. Based on an effective Lyapunov functional and the Wirtinger-type inequality, sufficient conditions are presented to guarantee the passive performance of the memristive models. By establishing a stochastic variable with Bernoulli distribution, the information of probabilistic time-varying delays are considered, which were transformed into one with deterministic time-varying delay and stochastic parameters. Moreover, the range of the delays as well as the probability distribution of its variation are all taken into consideration, thus, the results derived in this paper are more reasonable. Finally, the advantages of the proposed techniques are demonstrated by two numerical examples.

1. Introduction

There are four fundamental circuit variables in circuit theory, namely, current, voltage, charge and flux. Among them, there should be six possible combinations, while five of them are well defined except the relationship between charge and flux. Based on the symmetric and logical properties of the four fundamental circuital variables, memristor was emerged [1], which was named as the missing fourth basic circuit element along with the resistor, capacitor and inductor. In late 2008, a successful physical realization of a very compact and nonvolatile nanoscale memristor device was published by Williams and coworkers [2]. Since then, this new nanometer circuit element has generated unprecedented worldwide interest.

Memristance (resistance of memristor) can be controlled by the voltage or current signal that is applied to it. When a sinusoidal, or any bipolar periodic signal is applied to the memristor, no matter positive or negative value, it exhibits a hysteresis loop in the v–i plane which is pinched at the origin. This pinched hysteresis loop is considered as a fingerprint of the memristor [3]. Because of this feature, many applications of memristors have been identified [4–6].

Time delay, which is a common phenomenon to depict the future state of a system that depends on the present state as well as the past one. For the dynamical behavior analysis of delayed neural networks, different types of time delays, such as constant delays, time-varying delays, and distributed delays, have been taken into account by using a variety of techniques. However, for the probabilistic reasons, time delay often occurring in a random way. To better reflect a realistic situation, probabilistic measurement delays have been introduced and the most popular way to describe probabilistic measurement delay is regarding it as a Bernoulli distributed white sequence.

Very recently, the dynamic behaviors of neural networks have been extensively investigated because of their great significance for both practical and theoretical purposes [7–15]. Among which, passivity is an important one, because a great number of systems should be effectively passive for attenuating noises, the problem with respect to robustness usually reduces to a subsystem or a modified system passive is another reason. The passivity theory, intimately related to the circuit analysis, treated as a useful and significant tool to analyze the stability of nonlinear systems, signal processing, chaos control, and so on [16]. In fact, the essence of the passivity theory is that it can

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keep the system internal stability based on energy-related considerations, i.e., passive system only burns energy, without energy production. Recently, the problem of passivity analysis for memristive neural networks has been investigated [17–21], the passivity conditions derived in these papers only containing the variation range of the delays and its derivative. However, with the rapid network delay tomography development, as well as wireless networks and phone recognition systems, the probability distribution of time varying delays could be estimated, which make it possible to employ such information to reduce the conservatism of these conditions.

Based on the above discussion, the literature on the passivity of memristive neural networks with probabilistic time-varying delays is very scarce, to shorten sup gap, it is of vital importance to formulate this model. Moreover, as one of the fundamental problems, passivity analysis still remains unsolved and challenging, which motivates the present study. The main contributions of this paper can be summarized as: (i) the information of neuron activation function and the involved probability values of the time-varying delays are adequately considered; (ii) the passive law obtained in this paper based on new inequality instead of Jensen's inequality, this makes the results received in this paper are less restrictive; (iii) instead of the theory of differential inclusion and set-value mapping which are used in [17,18], the parameters in this paper are divided into 2^{2n^2} cases, which is more reasonable; (iv) the conditions are expressed in terms of linear matrix inequalities (LMIs) which can be easily checked via Matlab LMI Toolbox, which overcomes the shortcomings of the results based on algebraic.

The remaining of this paper is organized as follows. In Section 2, the problem to be studied is formulated, and some preliminaries are introduced. The main result was given in Section 3. In Section 4, two numerical examples are given to show the validity of the developed results, and we conclude the paper in Section 5.

Notations: Throughout this paper, solutions of all the systems are intended in Filippov's sense. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. For any given matrix P, P^T represent the transpose of matrix P. The notation $X\geq Y$ (respectively, $X>Y$), where X and Y are real symmetric matrices, means that $X-Y$ is positive semi-definite (respectively, positive definite). The shorthand $\text{diag}(A_1,A_2,\ldots,A_n)$ denotes a block diagonal matrix with diagonal blocks being the matrices $A_1,A_2,\ldots,A_n$. Moreover, the asterisk $*$ in a matrix is used to denote term that is induced by symmetry matrices. $\mathbb{E}(\cdot)$ stands for the mathematical expectation operator.

2. Model description and preliminaries

2.1. Model description

In this paper, we consider the following general memristive model with probabilistic time-varying delays

$$\begin{cases}\dot{x}(t)=-Cx(t)+A(x(t))f(x(t))+B(x(t))f(x(t-\tau(t)))+u(t)\\ y(t)=f(x(t))\end{cases}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in\mathbb{R}^n$ is the state of the ith neuron at time t, $C=\text{diag}(c_1,c_2,\ldots,c_n)>0$, $A(x(t))=(a_{ij}(x_i(t))_{n\times n}$ and $B(x(t))=(b_{ij}(x_i(t))_{n\times n}$ are the self-feedback connection weight matrix, the connection weight matrix and the delayed connection weight matrix functions, respectively. $y(t)$ represents the output of the neural network, $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))^T$ is a external input vector, the neuron activation function $f(x(t))$ is a vector-valued differential activation function, which satisfies the following assumption.

(A_1): For all $x_1,x_2\in\mathbb{R}$, $x_1\neq x_2$, the neural activation function $f_j(\cdot)$ satisfies:

$$l_j^-\leq\frac{f_j(x_1)-f_j(x_2)}{x_1-x_2}\leq l_j^+,$$

where l_j^- and l_j^+ are known constant scalars and $f_j(0)=0$.

According to the current–voltage characteristic of a memristor, to better describe the memory function and the pinched hysteretic feature of a memristor, suppose that the state-dependent parameters in (1) satisfy the following conditions:

$$a_{ij}(x_i(t))=\begin{cases}a'_{ij}, & \text{sign}_{ij}\dfrac{df_j(x_j(t))}{dt}-\dfrac{dx_i(t)}{dt}<0\\ \text{unchanged}, & \text{sign}_{ij}\dfrac{df_j(x_j(t))}{dt}-\dfrac{dx_i(t)}{dt}=0\\ a''_{ij}, & \text{sign}_{ij}\dfrac{df_j(x_j(t))}{dt}-\dfrac{dx_i(t)}{dt}>0\end{cases}$$
$$b_{ij}(x_i(t))=\begin{cases}b'_{ij}, & \text{sign}_{ij}\dfrac{df_j(x_j(t-\tau(t)))}{dt}-\dfrac{dx_i(t)}{dt}<0\\ \text{unchanged}, & \text{sign}_{ij}\dfrac{df_j(x_j(t-\tau(t)))}{dt}-\dfrac{dx_i(t)}{dt}=0\\ b''_{ij}, & \text{sign}_{ij}\dfrac{df_j(x_j(t-\tau(t)))}{dt}-\dfrac{dx_i(t)}{dt}>0\end{cases}\tag{2}$$

where a'_{ij}, a''_{ij}, b'_{ij}, and b''_{ij} are known constants with respect to memristances, and, "unchanged" means that the memristance keeps the current value.

It is easy to see that each weight switches between two different constant values, i.e., $a_{ij}(x_i(t))$ may be a'_{ij} or a''_{ij}, and $b_{ij}(x_i(t))$ also have b'_{ij} or b''_{ij} this two choices. As a result, the combination number of the possible form of $A(x(t))$ and $B(x(t))$ is 2^{2n^2}, then, order these 2^{2n^2} cases in the following way:

$$(A_1,B_1),\quad (A_2,B_2),\ldots,(A_{2^{2n^2}},B_{2^{2n^2}}).$$

Then, at any fixed time $t\geq 0$, the form of $A(x(t))$ and $B(x(t))$ must be one of the 2^{2n^2} cases, which imply that, there exists some