Neurocomputing 191 (2016) 296–303

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Stabilization for linear uncertain systems with switched time-varying delays

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ARTICLE INFO

Article history:
Received 6 September 2015
Received in revised form
16 November 2015
Accepted 14 January 2016
Communicated by Xudong Zhao
Available online 8 February 2016

Keywords:
Switched systems
Time-varying delays
Stabilization
Controller design

ABSTRACT

This paper is concerned with the robust stabilization problem for a class of uncertain linear time-delay systems with large delay periods. The aim of this paper is to design a state feedback controller such that the closed-loop system is exponentially stable and preserves a desired performance in the presence of large delays. In order to deal with the large delay periods, the original system is converted into a switched delay system, in which the delays are time-varying. The delay-dependent condition is established in terms of matrix inequalities. The desired controllers can be solved by the cone complementary linearization method. An example is presented to illustrate the effectiveness of the proposed technique.

1. Introduction

Time-delays are often encountered in many practical systems, such as tandem mills, long transmission lines in pneumatic systems, networked control systems, biological systems and multi-agent systems [1–4]. Time-delay is one of the main resources of instability and poor performance of the system. In the last decades, much work has been done to the stability analysis and control synthesis of time-delay systems (see, for example [5–7], and references therein). The existing criteria can be classified into two categories, namely, delay-independent criteria [5,8], and delay-dependent criteria [9–11].

Switched systems are hybrid dynamical systems composed of a group of subsystems and a rule that governs the switching among them. In the last two decades, there have been increasing interests in the switched systems, see, for example, [12–16]. The authors in [12,13] studied the linear switched systems, while the authors investigated the nonlinear systems in [13–15]. A switched time-delay system is a switched system whose subsystems are time-delay systems. In the research of switched time-delay system, an assumption that the delay is within a certain upper bound is usually adopted in existing literatures. Such an upper bound usually can ensure the stability of the switched time-delay systems. The stability analysis and control synthesis are all investigated under such an assumption. However, if the delay exceeds the bound, the existing methods will fail to analyze such time-delay systems. Therefore, it is of importance to study this new case, because it usually occurs in the research of practical systems. As far as we have known, only a few papers have studied such problems, such as [17–19]. As pointed out in [19], when the delays exceed such an upper bound, the delays are called large delays.

On the other hand, the problem of robust stabilization of systems with uncertainties has also received much attention in recent years. Guaranteed cost control aims to design a feedback controller such that the asymptotic stability and an adequate level of performance of the closed-loop system are ensured for all admissible uncertainties. Since it was first put forward by [20], many significant results have been proposed based on it. The guaranteed cost control problem for a class of linear time-delay systems was solved in [21], where the desired memoryless state feedback controller was constructed by the linear matrix inequality (LMI) approach. The authors in [22] studied the guaranteed cost control problem for linear system with delays in both state and control input. The guaranteed cost control problem for uncertain discrete time system was investigated in [23]. The linear quadratic guaranteed cost control problem for a class of impulsive switched systems was studied in [24]. However, up to date, the guaranteed cost control problem for time-delay systems with large delay periods is still unconsidered yet, which motivates the investigation of this paper.

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http://dx.doi.org/10.1016/j.neucom.2016.01.050
0925-2312/

In this paper, the attention is focused on the guaranteed cost control problem for a class of linear systems with large delay periods and parameter uncertainties. The original system containing large delay periods is converted into a switched time-delay system consisting of two subsystems. One subsystem, which is stable, is used to denote the small delay periods; the other one, which may not be stable, is used to denote the large delay case. The issue we address here is to design a feedback controller such that the above switched time-delay system is robustly stable and a satisfactory control performance is guaranteed for all admissible uncertainties. Sufficient conditions for the solvability are provided in terms of matrix inequalities. The desired feedback controllers can be obtained by solving the matrix inequalities, which is implemented by the cone complementary linearisation method proposed in [20].

The reminder of this paper is organized as follows: In Section 2, some preliminaries and necessary definitions are provided and the investigated problems are formulated later. Two useful lemmas and the main results are presented in Section 3. In Section 4, a numerical example is given to show the effectiveness of the proposed method. The conclusions are drawn in the last section.

Notation: Throughout the paper, we denote by $\overline{\mathfrak{N}}$ the nonnegative integer set $\{0,1,2,\ldots\}$. $\mathfrak{R}^n$ denotes the n-dimensional Euclidean space. For real matrices X and Y, the notation $X \ge Y$ (resp. $X > Y$) means that the matrix $X-Y$ is positive semi-definite (resp. positive definite). The superscript 'T' stands for vector or matrix transpose. tr (M) denotes the trace of matrix M. $\delta_{\min}(P)$ and $\delta_{\max}(P)$ are used to represent the minimum and maximum eigenvalue of matrix P, respectively.

2. Preliminaries and model foundation

Consider a class of uncertain systems with time delays described by

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+A_d x(t-\tau(t))+Bu(t),\\ x(t) &= \varphi(t), \quad t\in[-h_1,0],\end{aligned} \tag{1}$$

where $x(t)\in\mathfrak{R}^n$ and $u(t)\in\mathfrak{R}^m$ denote the system state and control input, respectively; A, A_d, B are constant matrices with appropriate dimensions; $\varphi(t)$ is a continuous differentiable vector-valued initial function defined on $[-h_1,0]$; $0\le\tau(t)\le h_1$ denotes the time-varying delay. ΔA is an unknown real-valued function representing the time-varying uncertainties, which is assumed to have the following form

$$\Delta A = DF(t)E, \tag{2}$$

where D and E are known constant matrices, and $F(t)$ is an unknown matrix function satisfying

$$F^T(t)F(t)\le I. \tag{3}$$

Lemma 1 (*Petersen and Hollot [25]*). *Let D,H and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t)F(t)\le I$. Then, for any scalar $\epsilon>0$*

$$DF(t)H+H^TF^T(t)D^T\le\epsilon DD^T+\epsilon^{-1}H^TH.$$

Definition 1 (*Sun et al. [19]*). If for $t\in[T_1,T_2)$, it holds that $h_1<\tau_2(t)\le h_2$, then the time interval $[T_1,T_2)$ is called a large delay period; and if for $t\in[T_3,T_4)$, it holds that $0\le\tau_1(t)\le h_1$, then the time interval $[T_3,T_4)$ is called a small delay period.

It should be noted that the actuator/controller failure and data-dropouts may lead to large delay periods. Under such a case, the time-varying delay $\tau(t)$ may exceed the upper bound h_1 and belongs to another interval, such as $\tau(t)\in(h_1,h_2]$. In this paper, we let $0\le\tau_1(t)\le h_1$ and $h_1<\tau_2(t)\le h_2$. Then, when large delays occur, system (1) can be rewritten as

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+A_d x(t-\tau_2(t))+Bu(t),\\ x(t) &= \varphi(t), \quad t\in[-h_2,0].\end{aligned} \tag{4}$$

If the large delay phenomenon occurs occasionally, systems (1) and (4) can be described by

$$\begin{aligned}\dot{x}(t) &= (A+\Delta A)x(t)+A_d x(t-\tau_{\sigma(t)}(t))+Bu_{\sigma(t)}(t),\\ x(t) &= \varphi(t), \quad t\in[-h_2,0],\end{aligned} \tag{5}$$

where $\sigma(t):[0,+\infty)\to\{1,2\}$ is a piecewise constant switching signal to be designed. $\sigma(t)=1$ (resp. $\sigma(t)=2$) denotes that system (5) is running in a small delay period (resp. large delay period). In this paper, the following assumption is adopted.

Assumption 1. The derivative of the time-varying delay $\tau_{\sigma(t)}(t)$ satisfies

$$\dot{\tau}_{\sigma(t)}(t)\le\tau_m<1. \tag{6}$$

Remark 1. In this paper, we consider the slow-varying delays, i.e., $\dot{\tau}_{\sigma(t)}(t)<1$. The slow-varying delays adopted in most existing literatures, such as, [26,13] usually guarantee that the time-delay systems are stable. However, the slow-varying delays may make the system unstable, thus further attention should be paid for such a case. In fact, such an assumption can also be relaxed, one can refer to Remark 5 for the relaxation.

Definition 2. System (5) is said to be exponentially stable under switching signal $\sigma(t)$ if the solution of system (5) satisfies

$$\|x(t)\|\le\kappa\|x_{t_0}\|_{c1}e^{-\beta(t-t_0)}, \quad \forall t\ge0, \tag{7}$$

for $\kappa\ge1$ and $\beta>0$, where $\|x(t)\|_{c1}=\max\left\{\sup_{-h_2\le\theta\le0}|x(t+\theta)|, \sup_{-h_2\le\theta\le0}|\dot{x}(t+\theta)|\right\}$.

The linear state feedback control law is given as

$$u_{\sigma(t)}=K_{\sigma(t)}x(t). \tag{8}$$

Thus, the resulting closed-loop system from system (5) and (8) is obtained as

$$\begin{aligned}\dot{x}(t) &= A_{\sigma(t)}x(t)+A_d x(t-\tau_{\sigma(t)}(t)),\\ x(t) &= \varphi(t), \quad t\in[-h_2,0],\end{aligned} \tag{9}$$

where

$$A_{\sigma(t)}=A+\Delta A+BK_{\sigma(t)}. \tag{10}$$

The weighted cost function associated with system (5) is given by

$$J=\int_0^{+\infty}e^{-\lambda t}\left[x^T(t)Qx(t)+u^T_{\sigma(t)}Ru_{\sigma(t)}\right]dt, \tag{11}$$

where λ is a positive constant, Q and R are positive definite weighted matrices given in advance.

Now, the guaranteed cost control problem for system (5) with large delay periods can be stated as:

Definition 3. If there exist a control law $u^*_{\sigma(t)}$ for the two subsystems and a switching signal $\sigma(t)$, and a scalar J^* such that for all admissible uncertainties, the closed-loop system is asymptotically stable and the value of cost function (11) satisfies $J\le J^*$, then system (5) is said to satisfy weighted guaranteed cost control. In this case, (8) is said to be a guaranteed cost state feedback control law.

In this paper, the main attention is to solve the guaranteed cost control problem of system (9) in the presence of the large delay periods. Hereinafter, some knowledge related to large delay period will be presented. A time sequence $0=t_0<t_1<t_2<\cdots$, which denotes the switching instants of the switching signal $\sigma(t)$ is adopted. Then we introduce another time sequence $\{p_0,p_1,\ldots\}$, which belongs to one of the subsequences of $\{t_0,t_1,\ldots\}$, and