Neurocomputing 191 (2016) 374–379

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief Paper

Finite-time control of switched stochastic delayed systems ☆

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ARTICLE INFO

Article history:
Received 7 August 2015
Received in revised form
28 October 2015
Accepted 21 January 2016
Communicated by Lixian Zhang
Available online 8 February 2016

Keywords:
Switched stochastic delayed systems
Mean-square finite-time stability
Average dwell time
Mode-dependent Lyapunov function

ABSTRACT

This paper is devoted to the finite-time stability analysis and control for switched stochastic delayed systems (SSDSs). The issue of mean-square finite-time stability for nonlinear switched stochastic delayed systems (NSSDSs) is considered. A stability criterion with average dwell time constraint is proposed to ensure the mean-square value of state is not larger than a prescribed threshold during a given time interval. This result can be extended to analysis and design for nonlinear/linear switched stochastic delayed systems. Then, based on partial-mode-dependent/mode-dependent Lyapunov function approaches, mean-square finite-time stability conditions for linear switched stochastic delayed systems (LSSDSs) are developed. Subsequently, both partial-mode-dependent and mode-dependent state feedback controllers for LSSDSs are designed, respectively. Finally, an illustrative example is provided to demonstrated the effectiveness of the method.

1. Introduction

Finite-time stability of dynamical systems is firstly introduced in 1960s [1]. A system is said to be finite-time stable if its state retains in a certain range [1] or converges to zero [2,3] within a prescribed time interval $t \in [0, T]$. Compared with Lyapunov stability which considers asymptotic feature of a system as $t \to \infty$, finite-time stability mainly focuses on qualitative transient property in given certain finite time. In many practical applications, as we know, the qualitative features in transient processes are especially important. An asymptotic system with large overshooting is unacceptable. Thus, the theoretical and applied studies on finite-time stability and control have received considerable attention in the past decades. The problem of finite-time attitude control for spacecraft has been considered in [4,5]. A finite-time gain-scheduled controller for bioreactor systems with partially known transition jump rates has been designed in [6]. Some sufficient conditions on finite-time stabilization for impulsive systems have been proposed by [7,8]. Finite-time dynamic output feedback control for discrete-time linear systems has been investigated in [9]. Finite-time and practical stability of linear systems with delay have been discussed, based on Lyapunov stability theory [10]. Jiang et al. have applied differential inclusions theory and Lyapunov function approach to finite-time synchronization control for memristor-based recurrent neural networks [11].

Recently, more and more attention has been paid to finite-time analysis and design of switched systems, since switched systems can be used to describe lots of real-world plants with switchings. Different from asymptotic/exponential stability analysis and design in [12–18], finite-time analysis, control and state observation problems for switched systems have been studied by [19,20]. Just as in [2,3], the states of all subsystems of [19,20] converge to zero in finite time. Meanwhile, if some important variables of the systems are required to be not larger than prescribed values, finite-time stability as in [1] should be considered for switched systems. On the basis of Lyapunov function method and linear matrix inequality (LMI) technique, the issues of finite-time analysis and control for switched systems with state-dependent switching have been addressed in [21,22]. More recently, applying Lyapunov-like functions, [23] has considered finite-time stability of nonlinear switched systems with finite-time unstable subsystems. Based on average dwell time (ADT) approach, uniform finite-time H_∞ control method for discrete-time nonlinear switched systems has been provided in [24]. Finite-time stability analysis and control for continuous-time fractional order switched systems have been investigated by employing the ADT technique [25]. For uncertain switched neutral systems with both stable and unstable subsystems, [26] has dealt with finite-time H_∞ control with admissible ADT in terms of a binary mode-dependent Lyapunov function. It can be seen from (13) and (14) of [26] that the matrices Q, Z, T are independent on the systems modes, thus the Lyapunov function in [26] is only partial mode-dependent.

☆This work was supported partially by the National Basic Research Program of China (973 Program) under Grant 2012CB821200, the National Natural Science Foundation of China under Grants U1509205, 61333009, 61425009, 61473107 and 61104120, and the Zhejiang Provincial Natural Science Foundation of China under Grants R1100716, LR16F030003.

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http://dx.doi.org/10.1016/j.neucom.2016.01.042
0925-2312/© 2016 Published by Elsevier B.V.

In fact, many practical systems are perturbed by multiplicative stochastic noises [27–29], and the effects of stochastic perturbations should be taken into account when investigating finite-time analysis and design [29,30]. Via stochastic Lyapunov approach, finite-time stability conditions in probability, and in mean and mean square have been established for Itô-type stochastic linear systems [29,30]. Based on stochastic Lyapunov method and convex optimization approach, mean-square finite-time stability and synthesis for various Itô stochastic systems results have been provided recently [31–33]. And, Ref. [34] has concerned with H_∞ control for stochastic linear switched systems under admissible switching signal satisfying average dwell time restriction. However, to the best of our knowledge, the problems of finite-time stability analysis and feedback control for switched stochastic delayed systems (SSDSs) have not been fully considered, which motivates this paper.

This paper is concerned with finite-time stability analysis and feedback stabilization for switched stochastic delayed systems (SSDSs) of Itô-type. First, a mean-square finite-time stability criterion for nonlinear switched stochastic delayed systems (NSSDSs) is obtained, with switching signal satisfying average dwell time (ADT) condition. It is a general result and can be used to stability analysis and design for nonlinear/linear SSDSs. By means of the obtained finite-time stability condition for NSSDSs and partial-mode-dependent Lyapunov method as in [26], finite-time mean-square stability for linear switched stochastic delayed systems (LSSDSs) is investigated. Then, a feedback stabilization controller is designed to guarantee the finite-time stability of the resulting closed-loop systems. Furthermore, to overcome the conservatism, mode-dependent Lyapunov function is also used for finite-time stability analysis and control of LSSDSs. Finally, an illustrative example shows the usefulness of the method.

Notations: Throughout this paper, the notations are standard. $\mathbf{R}^n$ is an n-dimensional Euclidean space; $\mathbf{R}^{n\times m}$ represents the set of all $n\times m$ real matrices; $|\cdot|$ denotes the Euclidean norm or the induced 2-norm as appropriate; $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ are the minimal and maximal eigenvalues of a symmetric matrix, respectively; $\mathrm{tr}\{\cdot\}$ stands for the trace of a matrix; $\mathrm{diag}\{A_1, A_2, \ldots, A_n\}$ represents a block diagonal matrix with diagonal matrix blocks $A_1, A_2, \ldots, A_n$; $\mathcal{E}\{\cdot\}$ is the expectation operator. $Z^{\frac{1}{2}}$ is the square root matrix of the positive definite and symmetric matrix $Z>0$. $(\Omega, \mathcal{F}, \mathcal{P})$ is a complete probability space, where Ω is the sample space, and $\mathcal{F}$ is a σ-algebra of subsets of Ω called events, and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $*$ denotes the symmetric term in a symmetric matrix.

2. Problem formulation and preliminaries

We first consider the following nonlinear switched stochastic delay system (NSSDS):

$$
\begin{aligned}
dx(t) &= f_{\sigma(t)}(t, x(t), x(t-h))\,dt \\
&\quad + g_{\sigma(t)}(t, x(t), x(t-h))\,dw(t) \\
x(\vartheta) &= \phi(\vartheta), \quad \forall \vartheta \in [-h, 0]
\end{aligned} \tag{1}
$$

where $x(t)\in\mathbf{R}^n$ is the state vector, $h>0$ is the delay, $\phi(\cdot)$ is the known initial condition that is assumed to be continuously differentiable on $[-h,0]$; $w(t)$ is a r-dimensional Wiener process; $\sigma(t)$ is a switching signal, a piecewise constant and right-continuous function on t, defined on a finite set $\mathbf{S}=\{1,2,\ldots,N\}$, where $N>1$ is the number of subsets. For a switching series $0<t_1<t_2<\cdots$, the $\sigma(t_k)$-th subsystem is activated when $t\in[t_k, t_{k+1})$. In this paper, for brevity, the switching instant is denoted as $\sigma(t)=i$. Subsequently, the nonlinear functions $f_{\sigma(t)}(t, x(t), x(t-h))$, $g_{\sigma(t)}(t, x(t), x(t-h))$ are abbreviated as $f_i(t)$, $g_i(t)$ in the following.

We now present the finite-time stability definition of system (1) as follows.

Definition 1. For given scalars c_1, c_2, T with $c_1<c_2$ and a switching signal $\sigma(t)$, if

$$\sup_{-h\le s\le 0} \mathbf{E}\{|x(s)|^2\} < c_1 \Rightarrow \mathbf{E}\{|x(t)|^2\} < c_2, \quad t\in[0,T]$$

then system (1) is said to be mean-square finite-time stable with respect to (c_1, c_2, T, σ).

Definition 2. Let $N_{\sigma(t)}(t_0,t)$ denote the number of switches for a given switching signal on the time interval $[t_0,t]$, if the following is satisfied for a scalar $N_0\ge 0$:

$$N_{\sigma(t)}(t_0,t) \le N_0 + \frac{t-t_0}{\tau_a}$$

then τ_a is called the average dwell time and N_0 the chatter bound.

In the next section, we shall investigate mean-square finite-time stability of system (1), then extend it to the following linear switched stochastic delay system

$$dx(t) = [A_i x(t) + C_i x(t-h) + B_i u(t)]\,dt + [F_i x(t) + G_i x(t-h)]\,dw(t) \tag{2}$$

where $u(t)\in\mathbf{R}^m$ is the control input, $w(t)$ is a scalar Wiener process. The initial condition is given by $\phi(\cdot)$ as in system (1). A_i, C_i, B_i, F_i, G_i are known real matrices.

Meanwhile, in this paper, a state feedback controller

$$u(t) = K_i x(t) \tag{3}$$

is adopted to stabilize system (2), where $K_i\in\mathbf{R}^{n\times m}$ are gain matrices to be designed.

To fulfill the above-mentioned analysis and design objectives, the following lemmas are proposed.

Lemma 1 (*Itô formula, Mao [27]*). *If $V_i(x)$ is a positive definite, radially unbounded, twice continuously differentiable function, then its stochastic differential along system (1) is*

$$dV_i(x) = \mathcal{L}V_i(x)\,dt + \frac{\partial V_i}{\partial x} g_i\,dw(t)$$

where the infinitesimal operator reads as

$$\mathcal{L}V_i(x) = \frac{\partial V_i}{\partial t} + \frac{\partial V_i}{\partial x} f_i + \frac{1}{2}\mathrm{tr}\left\{ g_i^T \frac{\partial^2 V_i}{\partial x^2} g_i \right\}.$$

Lemma 2 (*Gronwall inequality, Chen et al. [33]*). *For given a non-negative function $V(t)$ and some constants C, A, if*

$$V(t) \le C + A\int_0^t V(s)\,ds, \quad 0\le t\le T$$

then there is

$$V(t) \le Ce^{AT}, \quad 0\le t\le T.$$

3. Main results

This section will focus on mean-square finite-time stability for nonlinear switched stochastic delay system (1). Then, based on the obtained result, mean-square finite-time stability and control for system (2) will be considered.

3.1. Finite-time stability of NSSDS

First, construct a radially unbounded and twice continuously differentiable function $V_i(x)$ $(i\in\mathbf{S})$ such that

$$\lambda_1|x|^2 \le V_i(x) \le \lambda_2|x|^2 + h\lambda_3|x_t|^2 \tag{4}$$

where $\lambda_k>0, k=1,2,3$, and $|x_t| \triangleq \sup_{\vartheta\in[-h,0]}|x(t+\vartheta)|$.