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Delay-partitioning approach to stability analysis of generalized neural networks with time-varying delay via new integral inequality ☆

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ABSTRACT

On the basis of establishing a new integral inequality composed of a set of adjustable slack matrix variables, this paper mainly focuses on further improved stability criteria for a class of generalized neural networks (GNNs) with time-varying delay by delay-partitioning approach. A newly augmented Lyapunov–Krasovskii functional (LKF) containing triple-integral terms is constructed by decomposing integral interval. The new integral inequality together with Peng–Park's integral inequality and Free-Matrix-based integral inequality (FMII) is adopted to effectively reduce the enlargement in bounding the derivative of LKF. Therefore, less conservative results can be expected in terms of e_s and LMIs. Finally, two numerical examples are included to show that the proposed method is less conservative than existing ones.

1. Introduction

In recent years, much effort has been made in the stability analysis of generalized neural networks (GNNs) model, due to the fact that the GNNs include static neural networks (SNNs) and local field neural networks (LFNNs) as their special cases [1–4]. As pointed out by [1], stability analysis of GNNs has provided a unified frame suitable for both SNNs and LFNNs. On the other hand, during the implementation of artificial NNs, the finite switching speed of amplifiers and the inherent communication time between the neurons inevitably introduce time delay, which might cause oscillation, divergence, and even instability. Therefore, the stability analysis and synthesis of the delayed neural networks (DNNs) have attracted a large number of researchers [2–23]. The criteria developed can be classified into two major categories: the delay-independent case [5–7] and the delay-dependent case [2–4,14,15,20]. Since the time delay encountered in neural networks is usually not very big [2], delay-dependent criteria, which include the information of time delay, are less conservative.

The main goal of delay-dependent stability analysis is to reduce the conservatism of the derived condition and obtain the maximum admissible upper bounds (MAUBs) of time delay that guarantees the stability of NNs. It is well-known that the reduction of conservatism in delay-dependent stability criteria can be achieved mainly from two aspects: construction of appropriate LKFs and utilization of tighter bounding techniques to bound the derivatives of LKFs. As far as construction of LKFs are concerned, delay-slope-dependent LKF [21], discretized LKF [24], triple integral form LKF [25,26], delay-partitioning-dependent augmented LKF [9,15,27–29] have been introduced to reduce the conservativeness of the derived results. On the other hand, Jensen inequality [24], free-weighting matrices techniques [30], convex combination technique [31], reciprocally convex combination (RCC) technique [32], Peng–Park's integral inequality [37], Wirtinger-based integral inequality [33] and Free-Matrix-based integral inequality (FMII) [34] are more or less tighter bounding techniques for estimating the derivatives of LKFs.

During the last decade, the stability analysis of GNNs with time-varying delay with LMI based method has become a hot research topic, see, e.g., [1–4] and references therein. In [1], delay-independent/-dependent stability criteria have been established by employing LKF approach for GNNs with interval time-varying delays, and these stability criteria have provided a unified frame suitable for both SNNs and LFNNs. Based on constructing LKF including more information on activation functions and delay

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upper bounds, [2] has derived less conservative stability criteria for GNNs with two time-varying-delay components, and most commonly used techniques for treating the derivative of the LKF have been reviewed. With a suitably augmented LKF and modified Wirtinger-based integral inequality, sufficient conditions for guaranteeing the asymptotic stability of the GNNs with time-varying delay are derived in terms of LMIs in [3]. Most recently, by constructing an augmented LKF and utilizing the FMII, which encompasses the Wirtinger-based inequality and is more tighter than exiting ones [34], to bound the derivative of the augmented LKF, less conservative stability criteria for GNNs with time-varying delay have been derived in [4]. However, when revisiting the aforementioned works for stability analysis of delayed GNNs, it is found that these works still leave plenty of room for improvement because (i) the LKFs constructed in these papers are just the LKFs without delay-partitioning augmented terms and (ii) over-bounding techniques have been employed to bound the derivatives of the LKFs, which are the origin of conservatism. Whereas, as pointed out by [27], via delay-partitioning approach, less and less conservative results can be expected as the fractioning becomes thinner. Therefore, these criteria can be further improved by taking advantage of delay-partitioning-dependent augmented LKF and establishing more tighter bounding technique to bound the derivative of the augmented LKF.

Motivated by the above discussion, the aim of this paper is to develop further less conservative stability criteria for GNNs with time-varying delay via delay-partitioning approach. The main contribution of this paper lies in the following aspects: firstly, a new integral inequality composed of a set of adjustable slack matrix variables is established to bound the crucial double integral terms $-\int_{\alpha}^{\beta}\int_{t+\theta}^{t+\beta}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\,ds\,d\theta$; secondly, a newly augmented LKF containing triple-integral terms $\sum_{i=1}^{m}\int_{-i\delta}^{-(i-1)\delta}\int_{\theta}^{-(i-1)\delta}\int_{t+\alpha}^{t}\dot{x}^{\mathrm{T}}(s)V_i\dot{x}(s)\,ds\,d\alpha\,d\theta$ is constructed by decomposing integral interval, and the $[P_{ij}]_{(m+1)\times(m+1)}/[\mathcal{X}_{ij}]_{m\times m}/[\mathcal{X}'_{ij}]_{(m-1)\times(m-1)}$-dependent sub-LKFs are also incorporated in the LKF, which enable the relationships between the augmented state vectors $[x^{\mathrm{T}}(t), x^{\mathrm{T}}(t-\delta), \ldots, x^{\mathrm{T}}(t-m\delta)]^{\mathrm{T}}$ and $[\frac{1}{\delta}\int_{t-\delta}^{t}x^{\mathrm{T}}(s)\,ds, \frac{1}{\delta}\int_{t-2\delta}^{t-\delta}x^{\mathrm{T}}(s)\,ds, \ldots, \frac{1}{\delta}\int_{t-m\delta}^{t-(m-1)\delta}x^{\mathrm{T}}(s)\,ds]^{\mathrm{T}}$ to be taken into full consideration; thirdly, the novel integral inequality together with FMII and Peng–Park's integral inequality is adopted to effectively reduce the enlargement in bounding the derivative of the augmented LKF as much as possible. Therefore, less conservative results can be achieved in terms of e_s and LMIs; finally, two numerical examples are included to show the effectiveness and the benefits of the proposed method.

The rest of this paper is organized as follows. The main problem is formulated in Section 2 and improved stability criteria for the GNNs with time-varying delay are derived in Section 3. In Section 4, two numerical examples are provided; and a concluding remark is given in Section 5.

Notations: Through this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices; the notation $A>(\geq)B$ means that $A-B$ is positive (semi-positive) definite; I (0) is the identity (zero) matrix with appropriate dimension; A^{T} denotes the transpose; $\|\bullet\|$ denotes the Euclidean norm in $\mathbb{R}^n$; "$*$" denotes the elements below the main diagonal of a symmetric block matrix; $C([-\tau,0],\mathbb{R}^n)$ is the family of continuous functions ϕ from interval $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $\|\phi\|_{\tau}=\sup_{-\tau\leq\theta\leq 0}\|\phi(\theta)\|$; let $x_t(\theta)=x(t+\theta)$, $\theta\in[-\tau,0]$. In addition, because many abbreviations are used in this paper, they are given in Table 1 for the convenience of the reader.

Table 1
Abbreviations and their definitions.

Abbreviation	Definition
He(A)	The sum of matrices A and A^{T}
GNNs	Generalized neural networks
FMII	Free-Matrix-based integral inequality
MAUBs	Maximum admissible upper bounds
e_s	Defined in (14)
$[P_{ij}]_{(m+1)\times(m+1)}$	Defined in Theorem 1
$[\mathcal{X}_{ij}]_{m\times m}$	Defined in Theorem 1
$[\mathcal{X}'_{ij}]_{(m-1)\times(m-1)}$	Defined in Theorem 1

2. Problem statement and preliminaries

Consider the following generalized NNs (GNNs) with time-varying delay and its equilibrium point being shifted to origin [1]:

$$\dot{x}(t)=-Ax(t)+W_1f(W_0x(t))+W_2f(W_0x(t-\tau(t))), \tag{1}$$

where $x(\cdot)=[x_1(\cdot),x_2(\cdot),\ldots,x_n(\cdot)]^{\mathrm{T}}\in\mathbb{R}^n$ is the neuron state vector; $f(x(\cdot))=[f_1(x_1(\cdot)),f_2(x_2(\cdot)),\ldots,f_n(x_n(\cdot))]^{\mathrm{T}}\in\mathbb{R}^n$ denotes the neuron activation function; $A=\mathrm{diag}\{a_1,a_2,\ldots,a_n\}$ is a diagonal matrix with $a_i>0, i=1,2,\ldots,n$, and $W_0,W_1,W_2\in\mathbb{R}^{n\times n}$ are the connection weight matrices between neurons; $\tau(t)$ is a time-varying delay satisfying

$$0\leq\tau(t)\leq\tau, \tag{2}$$

$$\dot{\tau}(t)\leq\mu, \tag{3}$$

where τ and μ are constants.

Remark 1. It is worth noticing that the model of GNNs (1) includes SNNs and LFNNs as its special cases [1]. In fact, (i) let $W_1=I, W_2=0$ and $W_0=W$, the GNNs model (1) reduces to the SNNs model; (ii) let $W_1=W, W_2=0$ and $W_0=I$, the GNNs model (1) reduces to the LFNNs model. On the other hand, it is generally known that the well-known Hopfield NNs and the cellular NNs can be modeled as an LFNNs model. Therefore, stability analysis for both SNNs and LFNNs (including Hopfield NNs and cellular NNs) models can be made in a unified frame based on the GNNs model.

Assumption 1 (*Liu et al.* [35]). The neuron activation functions $f_i(\cdot)$ $(i=1,2,\ldots,n)$ are continuous and bounded, and satisfy

$$k_i^-\leq\frac{f_i(x)-f_i(y)}{x-y}\leq k_i^+,\quad\forall x,y\in R, x\neq y, \tag{4}$$

where $f_i(0)=0$, and k_i^-,k_i^+ $(i=1,2,\ldots,n)$ are known real constants.

Remark 2. Assumption 1 on the activation function has originally proposed in [35], and such activation functions could be non-monotonic and more general than the usual sigmoid functions, since k_i^- and k_i^+ may be positive, zero or negative. Under Assumption 1, one has [21],

(i) for GNNs (1) and any positive diagonal matrix T,

$$x^{\mathrm{T}}(t)W_0^{\mathrm{T}}KTKW_0x(t)-f^{\mathrm{T}}(W_0x(t))\,Tf(W_0x(t))\geq 0, \tag{5}$$

where $K=\mathrm{diag}\{k_1,k_2,\ldots,k_n\}$, $k_i=\max\{|k_i^-|,|k_i^+|\}$;

(ii) for any $x,y\in R$,

$$[(f_i(x)-f_i(y))-k_i^-(x-y)][k_i^+(x-y)-(f_i(x)-f_i(y))]\geq 0; \tag{6}$$

and letting $y=0$ in (6), it gives that

$$[f_i(x)-k_i^-x][k_i^+x-f_i(x)]\geq 0,\quad\forall x\in R. \tag{7}$$

Before proceeding, recall the following lemmas which will be used throughout the proofs.