Neural Networks 79 (2016) 108–116

Contents lists available at ScienceDirect

Neural Networks

journal homepage: www.elsevier.com/locate/neunet

Global exponential stability of complex-valued neural networks with both time-varying delays and impulsive effects[☆]

Qiankun Song [a,*], Huan Yan [b], Zhenjiang Zhao [c], Yurong Liu [d,e]

[a] Department of Mathematics, Chongqing Jiaotong University, Chongqing 400074, China
[b] School of Information Science and Engineering, Chongqing Jiaotong University, Chongqing 400074, China
[c] Department of Mathematics, Huzhou Teachers College, Huzhou 313000, China
[d] Department of Mathematics, Yangzhou University, Yangzhou 225002, China
[e] Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 15 November 2015
Received in revised form 30 January 2016
Accepted 17 March 2016
Available online 6 April 2016

Keywords:
Complex-valued neural networks
Time-varying delay
Impulsive effects
Exponential stability
Complex-valued linear matrix inequality

ABSTRACT

In this paper, the global exponential stability of complex-valued neural networks with both time-varying delays and impulsive effects is discussed. By employing Lyapunov functional method and using matrix inequality technique, several sufficient conditions in complex-valued linear matrix inequality form are obtained to ensure the existence, uniqueness and global exponential stability of equilibrium point for the considered neural networks. Moreover, the exponential convergence rate index is estimated, which depends on the system parameters. The proposed stability results are less conservative than some recently known ones in the literatures, which is demonstrated via two examples with simulations.

1. Introduction

In the past decades, stability analysis of various classes of neural network models such as Hopfield neural networks, Cohen–Grossberg neural networks, cellular neural networks, and bidirectional associative memory neural networks has been extensively investigated as stable neural networks have been successfully applied to some practical engineering problems such as signal processing, pattern classification, associative memory design and control and optimization (Arik, 2004). However, in electronic implementation of neural networks, due to the communication between the neurons, some time delay parameters must be introduced into the equations that describe the neural network model (Faydasicok & Arik, 2013). The analysis of the time delays on the stability of neural networks is important as a stable neural network without time delays can exhibit an unstable behavior in the presence of time delays (Jian & Zhao, 2015). The mathematical modeling of delayed neural networks depends on how the delay parameters are introduced into system equations of neural networks (Marcus & Westervelt, 1989). Therefore, the problem of stability analysis of neural networks with delays has become interesting and received increasing attention, see Arik (2004), Cao and Song (2006), Cao and Wan (2014), Chen, Lu, and Chen (2005), Dharani, Rakkiyappan, and Cao (2015), Faydasicok and Arik (2013), Jian and Zhao (2015), Kwon, Park, Lee, Park, and Cha (2013), Marcus and Westervelt (1989), Park, Kwon, and Lee (2008), Wen, Zeng, and Huang (2012), Zeng and Wang (2006), Zhu and Cao (2011) and the references herein.

On the other hand, an impulsive phenomenon exists universally in a wide variety of evolutionary processes where the state is changed abruptly at certain moments of time, involving such fields as chemical technology, population dynamics, physics and economics (Xu & Yang, 2005). It has also been shown that an impulsive phenomenon exists likewise in neural networks (Balasubramaniam & Vembarasan, 2011). For instance, during the implementation of electronic networks, when a stimulus from the body or the external environment is received by receptors, the electrical impulses will be conveyed to the neural networks and an impulsive phenomenon which is called impulsive perturbations arises naturally (Rakkiyappan, Chandrasekar, Lakshmanan, Park, & Jung, 2013). The impulsive perturbation of neural networks can affect the dynamical behaviors of the neural networks, same as time delays effect (Stamova, Stamov, & Li, 2014). Therefore, it is

☆ This work was supported by the National Natural Science Foundation of China under Grants 61273021, 61473332 and 61403051.

* Corresponding author.
E-mail addresses: qiankunsong@163.com (Q. Song), huanyancquc@163.com (H. Yan), zhaozjcn@163.com (Z. Zhao), liuyurong@gmail.com (Y. Liu).

http://dx.doi.org/10.1016/j.neunet.2016.03.007
0893-6080/

necessary to consider both impulsive effect and delay in the study of stability of neural networks (Chen, Lu, & Zheng, 2015).

As an extension of real-valued neural networks, complex-valued neural networks with complex-valued state, output, connection weight, and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves (Hirose, 1992). In fact, complex-valued neural networks (CVNNs) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons (Jankowski, Lozowski, & Zurada, 1996). Besides, CVNNs have more different and more complicated properties than the real-valued ones (Lee, 2001). Therefore it is necessary to study the dynamic behaviors of CVNNs deeply (Nishikawa, Iritani, Sakakibara, & Kuroe, 2005).

It is known that the main challenge is the choice of activation function in study for stability of complex-valued neural networks (Zhou & Song, 2013). Any regular analytic function cannot be bounded unless it reduces to a constant. This is known as the Liouville's theorem. That is to say, activation functions in complex-valued neural networks cannot be both bounded and analytic.

On the one hand, when activation functions can be expressed by separating their real and imaginary parts, some stability criteria were given for various CVNNs, for example, see Gong, Liang, and Cao (2015a, 2015b), Hu and Wang (2012), Liu and Chen (2016), Rakkiyappan, Velmurugan, and Cao (2015), Rakkiyappan, Velmurugan, and Li (2015), Velmurugan, Rakkiyappan, and Cao (2015), Xu, Zhang, and Shi (2014) and Zhou and Song (2013). In Zhou and Song (2013), the boundedness and complete stability of CVNNs with constant delay were investigated when activation functions were defined as $f(z) = \max(0, \mathrm{Re}(z)) + i\max(0, \mathrm{Im}(z))$. In Hu and Wang (2012), a class of CVNNs with constant delays was considered, and some sufficient conditions were obtained for assuring the stability of the equilibrium point of CVNNs. In Xu et al. (2014), the exponential stability of CVNNs with time-varying delays and unbounded distributed delays was considered by using the vector Lyapunov method. In Gong et al. (2015a), Rakkiyappan, Velmurugan, and Cao (2015) and Velmurugan et al. (2015), authors investigated the problem of μ-stability and multiple μ-stability for CVNNs with unbounded time-varying delays. In Liu and Chen (2016), authors considered a class of CVNNs with asynchronous time delays and proved the exponential convergence directly, while the existence and uniqueness of the equilibrium point is just a direct consequence of the exponential convergence. In Gong et al. (2015b), based on the matrix measure method and the Halanay inequality, global exponential stability problem was investigated for CVNNs with time-varying delays. In Rakkiyappan, Velmurugan, and Li (2015), the complete stability of CVNNs with time delay and impulsive effects was investigated, and some analytical results to guarantee the complete stability of equilibrium points were presented with the help of Lyapunov functionals, stability theory and impulses.

On the other hand, when the activation functions cannot be separated into their real and imaginary parts, some stability criteria of CVNNs were also obtained under assumptions condition that activation functions satisfy the Lipschitz continuity condition in the complex domain, for example, see Fang and Sun (2014), Pan, Liu, and Xie (2015), Song, Zhao, and Liu (2015) and Zhang, Lin, and Chen (2014). In Fang and Sun (2014) and Zhang et al. (2014), by constructing appropriate Lyapunov functional, several sufficient conditions to ascertain the existence, uniqueness, and globally asymptotical stability of the equilibrium point of CVNNs with constant delay were provided in terms of linear matrix inequality. In Pan et al. (2015), the global exponential stability of a class of CVNNs with time-varying delays was investigated by applying conjugate system of CVNNs, fixed point theorem, contraction mapping principle and a delay differential inequality. In Song et al. (2015), a class of CVNNs with probabilistic time-varying delays is considered, several delay-distribution-dependent sufficient conditions to guarantee the global asymptotic and exponential stability were obtained by constructing proper Lyapunov–Krasovskii functional and employing inequality technique.

Although Rakkiyappan, Velmurugan, and Li (2015) considered the impulsive effects on stability for CVNN with constant delay when activation functions can be expressed by separating their real and imaginary parts, to the best of the author's knowledge, however, very few results on the stability problem for impulsive CVNN with time-varying delays under assumptions condition that activation functions satisfy the Lipschitz continuity condition in the complex domain.

Motivated by the aforementioned discussions, this paper shall investigate the problem of global exponential stability for CVNNs with both time-varying delays and impulsive effects. The main contributions of this paper are the following aspects: (1) The activation functions have not been separated into their real and imaginary parts. (2) The established sufficient conditions to ensure the existence, uniqueness and global exponential stability of equilibrium point are expressed in terms of complex-valued linear matrix inequalities, which can be checked numerically using the effective YALMIP toolbox in MATLAB. (3) Compared with the results in Fang and Sun (2014), Gong et al. (2015b), Hu and Wang (2012), Pan et al. (2015) and Zhang et al. (2014), our results are less conservative and more general.

Notations: The notations are quite standard. Throughout this paper, N^+ denotes the set of positive integers; I represents the unitary matrix with appropriate dimensions; $\mathbb{C}^n$ and $\mathbb{C}^{n\times m}$ denote, respectively, the set of all n-dimensional complex-valued vectors and the set of all $n \times m$ complex-valued matrices. A^* shows the complex conjugate transpose of complex-valued matrix A. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are defined as the largest and the smallest eigenvalue of Hermitian matrix P, respectively. The subscript T denotes the matrix transposition. The notation $X > Y$ means that X and Y are Hermitian matrices, and that $X - Y$ is positive definite. i shows the imaginary unit, i.e., $i = \sqrt{-1}$. $|a|$ denotes the module of complex number $a \in \mathbb{C}$, and $\|z\|$ denotes the norm of $z \in \mathbb{C}^n$, i.e., $\|z\| = \sqrt{z^*z}$. If $A \in \mathbb{C}^{n\times n}$, denote by $\|A\|$ its operator norm, i.e., $\|A\| = \sup\{\|Ax\| : \|x\| = 1\} = \sqrt{\lambda_{\max}(A^*A)}$. For $A = (a_{ij})_{n\times n} \in \mathbb{C}^{n\times n}$, $|A| = |a_{ij}|$ denotes the modulus matrix of A. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Model description and preliminaries

In this paper, we consider the following CVNNs with time-varying delays

$$\begin{cases} \dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-\tau(t))) + J, & t \neq t_k, \\ z(t) = D_k h_k(z(t^-)) + E_k s_k(z(t^- - \tau(t^-))) + \bar{J}_k, & t = t_k, \end{cases} \quad (1)$$

for $t \geq 0$, $k \in N^+$, where $z(t) = (z_1(t), z_2(t), \ldots, z_n(t))^T \in \mathbb{C}^n$, $z_i(t)$ is the state of the ith neuron at time t; $f(z(t)) = (f_1(z_1(t)), f_2(z_2(t)), \ldots, f_n(z_n(t)))^T \in \mathbb{C}^n$, and $f(z(t-\tau(t))) = (f_1(z_1(t-\tau(t))), f_2(z_2(t-\tau(t))), \ldots, f_n(z_n(t-\tau(t))))^T \in \mathbb{C}^n$, are the vector-valued activation functions without and with time delays whose elements consist of complex-valued nonlinear functions; $\tau(t)$ corresponds to the transmission delay and satisfies $0 \leq \tau(t) \leq \tau$; $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} \in \mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix, where $c_i > 0$; $A \in \mathbb{C}^{n\times n}$ is the connection weight matrix, $B \in \mathbb{C}^{n\times n}$ is the delayed connection