



# Neural-network-based adaptive tracking control for Markovian jump nonlinear systems with unmodeled dynamics<sup>☆</sup>



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## ABSTRACT

An adaptive control scheme is investigated for a class of strict-feedback Markovian jump nonlinear systems with unknown control gains and unmodeled dynamics. To deal with the unmodeled dynamics, an available dynamic signal is employed to construct appropriate Lyapunov functions. RBF neural networks are used to approximate the unknown nonlinear functions with Markovian switching. The approximation capability of neural networks is combined with the backstepping technique to avoid the inherent problem of controller complexity in traditional backstepping design method. It is proved that all the signals in the closed-loop system are uniformly ultimately bounded in probability and that the tracking errors signal converges to a small neighborhood of origin by choosing suitable design parameters. Simulation results illustrate the effectiveness of the proposed scheme.

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## 1. Introduction

Markovian jump system (MJS) is a two-level hybrid system, in which the first level is governed by a set of modes and the second level coordinates the switching among the modes [1]. MJS is also a stochastic switching system, in which continuous mode states are intertwined with random switching signals and the random switching signals obey the Markov process [2]. These systems are widely applied, such as economic systems, transportation systems, manufacturing systems, electric power systems and communication systems [3–9]. In recent years, the adaptive control problem for MJS has been extensively studied [10–17]. The adaptive estimation method was utilized to estimate unknown upper bounds of uncertainties and an adaptive backstepping controller was designed for stochastic jump systems with matched uncertainties and disturbances [10]. By on-line estimating the loss of effectiveness of actuators [11], Chen investigated the problem of sliding

mode control for stochastic Markovian jump systems with actuator degradation. With a new nonnegative function and the M-matrix approach [12–14], Zhou investigated the adaptive synchronization problem of stochastic neural networks with Markovian jumping parameters. It should be noted that these design methods were applied in a linear case, indicating that much stricter conditions, such as a global Lipschitz condition or linear growth condition, were imposed on the practical controlled system. The nonlinear controllers have been designed for Markovian jump systems with nonlinearities which were not constrained by linear bound [15–17]. By utilizing traditional backstepping technique [15], Zhu discussed the adaptive tracking control problem for a class of Markovian jump nonlinear systems disturbed by Wiener noise and designed an adaptive backstepping controller. However, the controller contains “interconnected” items caused by Markovian switching. Based on the standard assumption that the finite homogeneous Markov process was irreducible, the “interconnected” items were suppressed [16]. Based on the stability criteria and the existence theorem of strong solutions [16], Wu investigated an improved adaptive tracking controller design for stochastic nonlinear systems with stationary Markovian switching [17] and avoided the “interconnected” items problem. However, the aforementioned methods are inappropriate for Markovian jump nonlinear systems with unknown control gains.

To deal with unknown control gains and unknown nonlinear functions, as one of the effective control strategies, adaptive neural network (NN) control has been developed for strict-feedback

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nonlinear systems [18–24]. Gao [18], Chen [19], and Sun [20] used the norm quadratic of weight vector as estimation parameter instead of the weight vector elements [21,22]. Especially, Wang [23] and Zhou [24] estimated the upper bound of the norm quadratic of weight vector and only one online learning parameter was contained in the proposed controller, thus greatly reducing the number of adjustable parameters and the complexity of computation. But, if there is no a priori information about signs of control gains [18–24], the adaptive control for such systems becomes much more difficult. To relax this assumption, Ma [25] constructed an affine variable in the controller design for pure-feedback stochastic nonlinear systems. However, the aforementioned results cannot be simply extended to the controlled nonlinear systems with Markovian jumping parameters and unmodeled dynamics.

In fact, unmodeled dynamic issues widely exist in many industrial plants, which make the control synthesis more difficult. Assuming that dynamic uncertainties satisfied the triangularity condition and affine condition, Jiang [26,27] exhibited unmodeled dynamics. Afterwards, some interesting results on nonlinear systems with unmodeled dynamics have been proposed for deterministic systems [28,29] and stochastic cases [30–34]. However, the adaptive control problem for Markovian jump nonlinear systems with unmodeled dynamics has not been well addressed.

Inspired by the previous observation, a NN-based adaptive backstepping control scheme is investigated for a class of strict-feedback Markovian jump nonlinear systems with unmodeled dynamics and unknown nonlinear disturbances. The main contributions are summarized as follows. (1) Compared with existing results [18–24], the assumption about the sign of unknown control gains is abandoned by introducing intermediate control signals. (2) The neural-network-based adaptive control scheme is extended to Markovian jump nonlinear system. By designing NN-based backstepping control scheme, the inherent problem of controller complexity is avoided, which exists in the methods in [15–17] caused by repeated differentiations of virtual controllers. (3) Only one parameter is required to be estimated online and the computational burden is reduced greatly.

The rest of this paper is organized as follows. Section 2 begins with the problem statement and preliminaries. Section 3 gives the adaptive neural controller design and stability analysis. The effectiveness of the proposed scheme is illustrated through simulation analysis in Section 4. Conclusions are provided in Section 5.

## 2. Problem statement and preliminaries

Consider the following strict-feedback Markovian jump nonlinear system:

$$\begin{cases} \dot{\zeta} = q(y, \zeta, \eta_t) \\ \dot{x}_i = g_i(\bar{x}_i, \eta_t)x_{i+1} + f_i(\bar{x}_i, \eta_t) + \Delta_i(x, \zeta, \eta_t) & i = 1, 2, \dots, n-1 \\ \dot{x}_n = g_n(\bar{x}_n, \eta_t)u + f_n(\bar{x}_n, \eta_t) + \Delta_n(x, \zeta, \eta_t) \\ y = x_1 \end{cases} \quad (1)$$

where  $\bar{x}_i = [x_1, x_2, \dots, x_i]^T \in R^i$ ,  $x = \bar{x}_n$  are the state vectors;  $u \in R$  and  $y \in R$  denote input signal and output signal of the system, respectively;  $\zeta \in R^{n_\zeta}$  denotes the unmodeled dynamic;  $g_i(\cdot)$  are unknown control gains, and satisfy  $g_i(\cdot) \neq 0$ ;  $q(\cdot)$  and  $f_i(\cdot)$  are unknown nonlinear continuous functions;  $\Delta_i(\cdot)$  are unknown dynamic uncertainties;  $\eta_t, t \geq 0$  describes the mechanism of mode switching and  $\eta_t$  is a right-continuous Markov process taking values on a finite set  $M = \{1, 2, \dots, N\}$ . Let  $\Pi = \{\pi_{kp}\}$ ,  $k, p \in M$  be the transition rate matrix of the process, such that transition

probability of the system mode variable  $\eta_t$  satisfy

$$P(\eta_{t+\Delta t} = p | \eta_t = k) = \begin{cases} \pi_{kp}\Delta t + o(\Delta t) & \text{if } k \neq p \\ 1 + \pi_{kk}\Delta t + o(\Delta t) & \text{if } k = p \end{cases} \quad (2)$$

with  $\pi_{kp} \geq 0$  for  $k \neq p$  and  $\pi_{kk} = -\sum_{p=1, p \neq k}^N \pi_{kp}$ , where  $\Delta t > 0$  and  $\lim_{\Delta t \rightarrow 0} o(\Delta t)/\Delta t \rightarrow 0$ . Here, it is assumed that  $\eta_t$  is homogeneous and irreducible [16,17,35] and for transition rate matrix  $\Pi$ , there is a unique stationary distribution  $\xi = [\xi_1, \dots, \xi_N]$  satisfying

$$\xi_k > 0, \sum_{k=1}^N \xi_k = 1, \text{ and } \xi \Pi = 0, \quad \forall k \in M. \quad (3)$$

**Remark 1.** The nonlinear smooth functions  $f_i(\cdot)$  in system (1) are unknown and the strict conditions, such as a global Lipschitz condition or linear growth condition [4–6,12–14], are removed. The control gains are unknown functions rather than known constants [15–17]. In addition, unmodeled dynamic is considered. Therefore, the Markovian jump nonlinear system (1) is general.

The objective of the paper is to design an adaptive controller  $u$  for system (1) such that all signals in the resulting closed-loop Markovian jump system are bounded in probability. Moreover, the system output  $y$  can track the given desired signal  $y_d$  as close as possible.

To develop the control design, the following assumptions and lemmas are presented.

**Assumption 1.** There exist unknown smooth functions  $\rho_{i1}(\cdot) \geq 0$  and  $\rho_{i2}(\cdot) \geq 0$  such that

$$|\Delta_i(x, \zeta, \eta_t)| \leq \rho_{i1}(\|\bar{x}_i\|, \eta_t) + \rho_{i2}(\|\zeta\|, \eta_t), \quad i = 1, 2, \dots, n \quad (4)$$

where  $\rho_{i2}(\cdot)$  is a monotone-increasing function.

**Assumption 2.** ([26–34]). The  $\zeta$ -subsystem with input  $x_1$  is exponentially input-to-state practically stable (exp-ISpS), namely, there exist Lyapunov function  $V_0(\zeta, \eta_t)$ , class  $\kappa_\infty$  functions  $\underline{\kappa}(\cdot)$ ,  $\bar{\kappa}(\cdot)$ ,  $\chi(\cdot)$  and constants  $(\bar{c}_0 > 0$  and  $d_0 > 0)$  such that

$$\begin{aligned} \underline{\kappa}(\|\zeta\|) &\leq V_0(\zeta, \eta_t) \leq \bar{\kappa}(\|\zeta\|), \\ E[V_0(\zeta, \eta_t)] &\leq -\bar{c}_0 V_0(\zeta, \eta_t) + x_1^2 \chi(x_1^2) + d_0. \end{aligned}$$

**Remark 2.** Assumption 1 is a reasonable and general assumption on adaptive controller design and many engineering systems satisfy this condition. Compare with the parameterized conditions [26–34], this assumption is relaxed.

**Lemma 1.** (Young's inequality [25]). For  $\forall (x, y) \in R^2$ , the following inequality holds:

$$xy \leq \frac{\varepsilon^p}{p}|x|^p + \frac{1}{q\varepsilon^q}|y|^q, \quad (5)$$

where  $\varepsilon > 0$ ,  $p > 1$ ,  $q > 1$ , and  $(p-1)(q-1) = 1$ .

**Lemma 2.** (Gao et al. [18]). Based on Assumption 2, if  $V_0(\zeta, \eta_t)$  is an exp-ISpS Lyapunov function for the subsystem  $\dot{\zeta} = q(y, \zeta, \eta_t)$ , there is a finite time  $T_0 = T_0(c_0, v_0, \zeta_0, \eta_0) \geq 0$ , a non-negative bounded function  $D(t_0, t)$  and a dynamical signal described by  $\dot{v} = -c_0 v + x_1^2 \chi(x_1^2) + d_0$  such that

$$V_0(\zeta, \eta_t) \leq v(t) + D(t_0, t) \quad \forall t \geq t_0 \text{ and } D(t_0, T) = 0 \quad \forall T \geq t_0 + T_0, \quad (6)$$

where  $c_0 \in (0, \bar{c}_0)$  and  $v_0 = v(t_0) > 0$  are any constants,  $\eta_0$  denotes the initial mode,  $\zeta_0 = \zeta(t_0, \eta_0)$  and  $t_0$  are the initial state and initial time respectively.

**Lemma 3.** (Gao et al. [18]). For any non-decreasing function  $\gamma(\cdot)$ , and any  $x, y \geq 0$ , we have

$$\gamma(x+y) \leq \gamma(2x) + \gamma(2y). \quad (7)$$

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