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Robust stability analysis for discrete-time neural networks with time-varying leakage delays and random parameter uncertainties

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ABSTRACT

This paper is concerned with the problem of robust stability analysis for discrete-time neural networks with time-varying coupling delays, random parameter uncertainties and time-varying leakage delays. The uncertainties enter into the system parameters in a random way and such randomly occurring uncertainties obey certain mutually uncorrelated Bernoulli-distributed white noise sequences. The important feature of the results reported here is that the probability of occurrence of the parameter uncertainties are known a priori. Constructing suitable Lyapunov–Krasovskii functional (LKF) terms, sufficient conditions ensuring the stability of the discrete-time neural networks are derived in terms of linear matrix inequalities (LMIs). Finally, numerical examples are rendered to exemplify the effectiveness of the proposed results.

1. Introduction

Over the past decades, neural networks have received considerable research attention and successfully applied in different areas such as image processing, signal processing, fault diagnosis, pattern recognition, and so on. It is well-known that, both in biological and artificial neural networks, the interactions between neurons are generally asynchronous which inevitably result in time-delays that are often a source of undesirable complex dynamical behaviours such as instability, oscillation, chaotic and poor performance [12,19,23]. In electronic implementations of neural networks, the delays are usually time-varying due to the finite switching speed of amplifiers. In a neural network model, data or axon signal transmission is always accompanied by a non-zero interval delay between the initial and delivery time of messages or signals. Interval time-varying delay is a time-delay that varies in an interval in which the lower bound is not restricted to be 0. A typical example of dynamical systems with interval time-varying delay is networked control systems [3]. Further, the time-delay in the leakage term has a great impact on the dynamical behaviour of neural networks. It is known from the literature on population dynamics [2] that time-delays in the stabilizing negative feedback terms have a tendency to destabilize a system. Since time-delays in the leakage terms are usually not easy to handle, such delays have been rarely considered in the neural network literature so far. To practice, the leakage time-delay is not a constant, so we ought to consider the model with the time-varying leakage delay. Thus, the stability analysis of neural networks with time-varying delay in leakage term has primary significance in the research of neural networks.

Further, it is well-known that in practical situations, uncertainties have great impact on the performance of the neural networks. In neural networks, the connection weights of the neurons depend on certain resistance and capacitance values that include modeling errors or uncertainties [9]. The deviations and perturbations in parameters are the main sources of uncertainty. The problem of exponential stability has been studied for a class of discrete-time stochastic neural networks with time-varying delay and norm-bounded uncertainties [11]. The network-induced phenomena would lead to abrupt structural and parametric changes in practical engineering applications. The parameter uncertainties may be subject to random changes in environmental circumstances, for instance, network-induced random failures and repairs of components, changing subsystem interconnections, sudden environmental disturbances, etc. These parameter uncertainties may occur in a probabilistic way with certain types and intensity. Hence, it is significant to consider the random parameter uncertainties when designing the networked systems, for example, see [6,8].

Recently, stability analysis problem for stochastic neural networks has been investigated and less conservative results have been derived using probability distribution of time-varying delay

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[5]. State estimation problem for neural networks with Markovian jumps has been studied in [18,22]. The stability of stochastic neural networks with constant time-delay in leakage term has been investigated in [14]. Although the stability analysis of neural networks has received much attention, so far very few results have been reported on the stability analysis of discrete-time neural networks with time-delays in leakage (or "forgetting") term. Stochastic disturbances are mostly inevitable owing to noise in electronic implementations and certain stochastic inputs could make a neural network unstable. The stability of stochastic discrete-time neural networks with discrete time-varying delays and leakage delay has been well investigated in [4, 15]. Authors in [14] have presented a robust analysis approach to stochastic stability of the uncertain Markovian jumping discrete-time neural networks with time-delay in the leakage term. A delay-dependent robust synchronization analysis has been studied for coupled stochastic discrete-time neural networks with interval time-varying delays in network coupling, a time-delay in leakage term and parameter uncertainties [17]. Robust stability problem for discrete-time uncertain neural networks with time-varying leakage delays has been considered in [7]. Unfortunately, to the best of authors' knowledge, stability analysis of discrete-time neural networks with time-varying network coupling delay, time-varying leakage delay and random parameter uncertainties has not been investigated yet. Thus, the proposed neural networks model and its applications are closed to the practical networks.

Motivated by the above, in this paper we aim to establish the robust stability conditions for a class of discrete-time neural network systems with time-varying leakage delay and randomly occurring uncertainties. The Lyapunov–Krasovskii functional (LKF) is chosen to be augmented type which utilizes more information about the system. It is worth pointing out that in this work LKF is developed with more decision variables to exploit the information of both the lower and upper bounds of the time-varying transmission and leakage delays. A new set of zero equations are added to the derivative of the LKF and employing reciprocal convex lemma to the augmented terms, sufficient stability conditions are established in terms of LMIs. The derived stability conditions are depended on the lower and upper bounds of the transmission delay as well as the leakage delay. The feasibility of derived criteria can be easily checked by resorting to Matlab LMI Toolbox. Finally, numerical examples are included to illustrate the effectiveness of the proposed results.

This paper is organized as follows. Problem formulation and preliminaries are given in Section 2. Section 3 gives the sufficient stability conditions for the discrete-time neural networks system. Numerical examples are demonstrated in Section 4 to illustrate the effectiveness of the proposed method. Finally, conclusions are drawn in Section 5.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript T and (-1) denote the matrix transposition and matrix inverse, respectively. Matrices, if they are not explicitly stated, are assumed to have compatible dimensions. $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. I is an identity matrix with appropriate dimension. $\mathbb{E}$ denotes the mathematical expectation.

2. Problem description and preliminaries

Consider the following discrete-time delayed neural networks system with time-varying leakage delay

$$y(k+1)=Ay(k-\sigma(k))+B\hat{f}(y(k))+C\hat{g}(y(k-\tau(k)))+J \tag{1}$$

where $y(\cdot)=[y_1(\cdot),\dots,y_n(\cdot)]\in\mathbb{R}^n$ is the state vector, $\hat{f}(\cdot)=[\hat{f}_1(\cdot),\dots,\hat{f}_n(\cdot)]^T\in\mathbb{R}^n$, $\hat{g}(\cdot)=[\hat{g}_1(\cdot),\dots,\hat{g}_n(\cdot)]^T\in\mathbb{R}^n$ denote the activation functions, $J=[J_1,\dots,J_n]^T$ is the external input vector. $\sigma(k)$ represents the leakage delay satisfying $0<\sigma_m\le\sigma(k)\le\sigma_M$, where σ_m and σ_M denote the lower and upper bounds of $\sigma(k)$. $\tau(k)$ describes the transmission delay satisfying $0<\tau_m\le\tau(k)\le\tau_M$, τ_m and τ_M are known positive integers representing the lower and upper bounds of $\tau(k)$.

Assumption 1 (*Liu et al. [13], Wang et al. [20]*). For any $s_1, s_2\in\mathbb{R}$, $s_1\ne s_2$, the continuous and bounded activation functions $\hat{f}_i(\cdot)$ and $\hat{g}_i(\cdot)$ satisfy

$$F_i^-\le\frac{\hat{f}_i(s_1)-\hat{f}_i(s_2)}{s_1-s_2}\le F_i^+,$$
$$G_i^-\le\frac{\hat{g}_i(s_1)-\hat{g}_i(s_2)}{s_1-s_2}\le G_i^+,\quad i=1,2,\dots,n,$$

where F_i^-, F_i^+, G_i^-, and G_i^+ are known constants.

Shifting the equilibrium point of (1) to the origin, assume $y^*=[y_1^*,\ y_2^*,\dots,y_n^*]^T$ is an equilibrium point of (1) and let $x_i(k)=y_i(k)-y_i^*$, $f_i(x_i(k))=\hat{f}_i(y_i(k))-\hat{f}_i(y_i^*)$, $g_i(x_i(k-\tau(k)))=\hat{g}_i(y_i(k-\tau(k)))-\hat{g}_i(y_i^*)$. Then, the neural networks system (1) can be transformed as

$$x(k+1)=Ax(k-\sigma(k))+Bf(x(k))+Cg(x(k-\tau(k))), \tag{2}$$

where $x(k)=[x_1(k),\ x_2(k),\dots,x_n(k)]^T$, $x(k-\sigma(k))=[x_1(k-\sigma(k)), x_2(k-\sigma(k)),\dots,x_n(k-\sigma(k))]^T$, $f(x(k))=[f(x_1(k)), f(x_2(k)),\dots,f(x_n(k))]^T$, $g(x(k-\sigma(k)))=[g(x_1(k-\sigma(k))),\ g(x_2(k-\sigma(k))),\dots,g(x_n(k-\sigma(k)))]^T$. By Assumption 1, it can be verified readily that the functions $f_i(\cdot)$, $g_i(\cdot)$, $i=1,2,\dots,n$ satisfy $F_i^-\le(f_i(s_1-f_i(s_2))/(s_1-s_2)\le F_i^+$, $G_i^-\le(g_i(s_1)-g_i(s_2))/(s_1-s_2)\le G_i^+$ for any $s_1\ne s_2$ and $f_i(0)=g_i(0)=0$.

The initial condition associated with the model is

$$x(s)=\phi(s),\quad s=-\rho,\ -\rho+1,\dots,0. \tag{3}$$

where $\rho=\max\{\sigma_M,\ \tau_M\}$.

The following lemmas will be used in the sequel to establish the main results.

Lemma 2.1 (*Park et al. [16]*). *Let $f_1, f_2,\dots,f_N:\mathbb{R}^m\mapsto\mathbb{R}$ have positive values in an open subset D of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{\alpha_i|\alpha_i>0,\sum_i\alpha_i=1\}}\sum_i\frac{1}{\alpha_i}f_i(k)=\sum_i f_i(k)+\max_{g_{i,j}(k)}\sum_{i\ne j}g_{i,j}(k) \tag{4}$$

subject to

$$g_{i,j}:\mathbb{R}^m\mapsto\mathbb{R},\quad g_{j,i}(k)\triangleq g_{i,j}(k),\ \begin{bmatrix} f_i(k) & g_{i,j}(k)\\ g_{i,j}(k) & f_j(k)\end{bmatrix}\ge 0. \tag{5}$$

Proof.. The constraint in (5) implies that

$$\begin{bmatrix}\sqrt{\frac{\alpha_j}{\alpha_i}}\\ -\sqrt{\frac{\alpha_i}{\alpha_j}}\end{bmatrix}^T\begin{bmatrix} f_i(k) & g_{i,j}(k)\\ g_{i,j}(k) & f_j(k)\end{bmatrix}\begin{bmatrix}\sqrt{\frac{\alpha_j}{\alpha_i}}\\ -\sqrt{\frac{\alpha_i}{\alpha_j}}\end{bmatrix}\ge 0.$$

Then we have,

$$\sum_i\frac{1}{\alpha_i}f_i(k)=\sum_{i,j}\frac{\alpha_j}{\alpha_i}f_i(k)=\sum_i f_i(k)+\frac{1}{2}\sum_{i\ne j}\left(\frac{\alpha_j}{\alpha_i}f_i(k)+\frac{\alpha_i}{\alpha_j}f_j(k)\right)\ge\sum_i f_i(k)+\sum_{i\ne j}g_{i,j}(k).$$

Note that the equality holds for

$$\alpha_i=\frac{\sqrt{f_i(k)}}{\sum_j\sqrt{f_j(k)}},\ g_{i,j}(k)=\sqrt{f_i(k)f_j(k)}$$

which completes the proof.□

Lemma 2.2 (*Boyd et al. [1] Schur Complement*). *Given constant matrices Ω_1, Ω_2 and Ω_3 with appropriate dimensions, where $\Omega_1^T=\Omega_1$ and $\Omega_2^T=\Omega_2>0$, then*

$$\Omega_1+\Omega_3^T\Omega_2^{-1}\Omega_3<0,$$