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# General decay synchronization stability for a class of delayed chaotic neural networks with discontinuous activations

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ABSTRACT

This paper is concerned with the synchronization problem for a class of delayed chaotic neural networks with discontinuous activations. First a lemma which concerns stability in general decay rate is constructed. Based on this lemma, the general decay synchronization stability criteria of discontinuous neural networks are derived via a designed controller. The general decay synchronization is obtained by introducing a decay function and it contains exponential synchronization and polynomial synchronization as its two special cases. Finally, two examples are given to verify the effectiveness of the obtained results.



## 1. Introduction

Since Pecora and Carroll firstly introduced chaos synchronization in 1990 [1], chaos synchronization has been extensively studied due to its potential applications such as secure communication, information processing, and biological systems [2–7]. It is shown that delayed neural networks can exhibit chaotic behavior provided that the parameters and delays are appropriately chosen [8]. Therefore, synchronization and chaotic control of neural networks has been one of the hot research topics in the past decades. Moreover, lots of synchronization results have been obtained under different control approaches, such as feedback control [9–13], adaptive control [14–17], impulsive control [18], sampled-date control [19], intermittent control [20], and finite-time control [21].

It is worth noting that the activations of neural networks model in these papers are assumed to be continuous. A recent paper [22] has pointed out the interest for studying global convergence of general neural networks with discontinuous neuron activations. Discontinuous neuron activations are frequently encountered in the practical applications, and the system of neural networks with discontinuous activations has been proved really useful as an ideal model for the case where the gain of the neuron amplifiers is very high [23]. So recently, dynamical behaviors including the stability and synchronization of delayed neural networks with discontinuous activations have received a great deal of attention and have been extensively studied in the literature [24–32]. In [25,26], quasi-synchronization of discontinuous neural networks was investigated, i.e. the synchronization error can only be controlled within a small region around zero. It is also reported in [25] that complete synchronization cannot be achieved between the identical drive and response systems due to the discontinuity of activation functions.

In light of the above analysis, in this paper, we study the synchronization problem for a class of delayed neural networks with discontinuous activations. There are three advantages that make our approach attractive. Firstly, differential inclusion, nonsmooth analysis and control theory are employed to handle system with discontinuous right-hand sides. Secondly, a new crucial lemma which includes and extends the classical exponential stability theorem is constructed. The new lemma provides a new result on the stability in general decay rate by introducing a decay function. Then synchronization in general decay rate for discontinuous neural networks is obtained by using the lemma. Thirdly, the complete synchronization in general decay rate studied in our paper contains exponential synchronization and polynomial synchronization as its two special cases.

The rest of this paper is organized as follows. The system and some preliminaries are introduced in Section 2. In Section 3, by constructing a new lemma, the general decay synchronization criteria are established for discontinuous delayed neural networks via a nonlinear controller. Then, numerical simulations are given to

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demonstrate the effectiveness of the obtained results in Section 4. Finally, conclusions are drawn in Section 5.

*Notations*: Through this paper, $R_+$ denotes the set of all positive real numbers, $R^n$ denotes the $n$-dimensional Euclidean space and $R^{n\times n}$ denotes the set of all $n\times n$ real matrices. For any vector $x\in R^n$, its Euclidean norm is denoted as $\|\cdot\|$, i.e. $\|x\|=\sqrt{x^Tx}$. $A^T$ and $A^{-1}$ stand for the transpose and the inverse of the matrix $A$, respectively; $A>0 (A\ge 0)$ means that the matrix $A$ is symmetric and positive definite (semi-positive definite); $\lambda_{\max}(A)$ denotes the maximum eigenvalue of matrix. $\|A\|_\infty=\max_i\{\sum_{j=1}^n|a_{ij}|\}$, $\|A\|_1=\max_j\{\sum_{i=1}^n|a_{ij}|\}$. diag$(\cdot)$ denotes a block-diagonal matrix. $I$ is the identity matrix with appropriate dimension. sign$(\cdot)$ denotes the signum function.

## 2. System description and preliminaries

In this paper, we consider a class of chaotic neural networks with time-varying delay as follows:

$$\dot{x}(t)=-Dx(t)+Af(x(t))+Bg(x(t-\tau(t)))+J, \tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in R^n$ is the state vector. $D=\text{diag}(d_1,d_2,\ldots,d_n)$ is an $n\times n$ diagonal matrix with $d_i>0, i=1,2,\ldots,n$. $A=(a_{ij})_{n\times n}, B=(b_{ij})_{n\times n}\in R^{n\times n}$ are the connection weight matrix and delayed connection weight matrix, respectively. $f(x(t))=(f_1(x_1(t)),\ldots,f_n(x_n(t)))^T\in R^n$ and $g(x(t-\tau(t)))=(g_1(x_1(t-\tau(t))),\ldots,g_n(x_n(t-\tau(t))))^T\in R^n$ are the neuron activation functions. $\tau(t)$ is the time-varying delay. $J=(J_1,J_2,\ldots,J_n)^T$ is the external input vector.

Throughout this paper, the following assumptions are given for system (1).

(A1) For every $j=1,2,\ldots,n, f_j, g_j: R\to R$ are continuous except on a countable set of isolate points $\{\rho_k^j\}$, where the finite right and left limits $f_j^+(\rho_k^j), g_j^+(\rho_k^j)$ and $f_j^-(\rho_k^j), g_j^-(\rho_k^j)$ exist, respectively.

(A2) For each $j=1,2,\ldots,n$, there exist constants $h_j,k_j,r_j>0, s_j>0$, such that

$$\begin{aligned}&\sup|\xi_j-\zeta_j|\le h_j|u-v|+r_j,\\ &\sup|\varrho_j-\nu_j|\le k_j|u-v|+s_j,\end{aligned} \tag{2}$$

for all $u,v\in R$, where $\xi_j\in K[f_j(u)], \zeta_j\in K[f_j(v)], \varrho_j\in K[g_j(u)], \nu_j\in K[g_j(v)]$, $K[f_j(x)]=[\min\{f_j^-(x),f_j^+(x)\},\max\{f_j^-(x),f_j^+(x)\}]$, $K[g_j(y)]=[\min\{g_j^-(y),g_j^+(y)\},\max\{g_j^-(y),g_j^+(y)\}]$ for $x,y\in R$.

(A3) The time-varying delay $\tau(t)$ is bounded and there exist $\tau>0,\mu>0$ such that

$$0\le\tau(t)\le\tau,\quad \dot\tau(t)\le\mu<1, \tag{3}$$

for all $t\ge 0$.

**Remark 1.** It is worth noting that $\xi_j,\varrho_j$ may not be equal to $\zeta_j,\nu_j$ even $u=v$ if $u$ is a discontinuous point. So the constants $r_j,s_j$ in assumption (A2) are necessary, which is the essential difference between this paper and the previous literature where the Lipschitz condition was used.

Since system (1) is a discontinuous system, its solution is different from the classic solution and cannot be defined in the conventional sense. So we introduce the Filippov solution [34].

**Definition 1** (*Filippov [34]*). For a system with discontinuous right-hand sides:

$$\frac{dx}{dt}=F(x),\quad x(0)=x_0,\ x\in R^n,\ t\ge 0. \tag{4}$$

A set-valued map is defined as

$$\Phi(x)=\bigcap_{\delta>0}\bigcap_{\mu(N)=0}K[F(B(x,\delta)\backslash N)],$$

where $K[E]$ is the closure of the convex hull of set $E$, $E\subset R^n$, $B(x,\delta)=\{y:\|y-x\|<\delta, x,y\in R^n,\delta\in R^+\}$, and $N\subset R^n$, $\mu(N)$ is the Lebesgue measure of set $N$.

A solution (in Filippov's sense) of system (4) with initial condition $x(0)=x_0\in R^n$ is an absolutely continuous function $x(t), t\in[0,T], T>0$, which satisfy $x(0)=x_0$ and differential inclusion:

$$\frac{dx}{dt}\in\Phi(x),\quad \text{for a.a. } t\in[0,T].$$

Now we extend the concept of the Filippov solution to the discontinuous system (1) as follows:

**Definition 2** (*Forti et al. [23]*). A function $x:[-\tau,T)\to R^n, T\in(0,+\infty]$, is a solution (in Filippov's sense) of the discontinuous system (1) on $[-\tau,T)$, if:

(i) $x$ is continuous on $[-\tau,T)$ and absolutely continuous on $[0,T)$;

(ii) $x(t)$ satisfies

$$\dot{x}(t)\in-Dx(t)+AK[f(x(t))]+BK[g(y(t-\tau(t)))]+J,\quad \text{for a.a. } t\in[0,T). \tag{5}$$

Or equivalently,

(ii)′ there exist two measurable functions $\alpha=(\alpha_1,\alpha_2,\ldots,\alpha_n)^T$, $\beta=(\beta_1,\beta_2,\ldots,\beta_n)^T:[-\tau,T)\to R^n$, such that $\alpha(t)\in K[f(x(t))], \beta(t)\in K[g(x(t))]$ for a.a. $t\in[-\tau,T)$ and

$$\dot{x}(t)=-Dx(t)+A\alpha(t)+B\beta(t-\tau(t))+J,\quad \text{for a.a. } t\in[0,T). \tag{6}$$

**Definition 3** (*(IVP), Forti et al. [23]*). For any continuous function $\upsilon:[-\tau,0]\to R^n$ and any measurable selections $\chi(s)\in K[f(\upsilon(s))], \omega(s)\in K[g(\upsilon(s))]$ for a.a. $s\in[-\tau,0]$ by an initial value problem associated to (1) with initial condition $(\upsilon,\chi,\omega)$, we mean the following problem: find a couple of functions $[x,\alpha,\beta]:[-\tau,T)\to R^n\times R^n\times R^n$, such that $x$ is a solution of (1) on $[-\tau,T)$ for some $T>0$, $\alpha,\beta$ are the outputs associated to $x$, and

$$\begin{cases}\dot{x}(t) &= -Dx(t)+A\alpha(t)+B\beta(t-\tau(t))+J, \quad \text{for a.a. } t\in[0,T)\\ \alpha(t) &\in K[f(x(t))], \beta(t)\in K[g(x(t))], \quad \text{for a.a. } t\in[0,T)\\ x(s) &= \upsilon(s), \quad \forall s\in[-\tau,0],\\ \alpha(s) &= \chi(s), \quad \beta(s)=\omega(s), \text{ for a.a. } s\in[0,T).\end{cases} \tag{7}$$

**Lemma 1** (*Liu et al. [25]*). *Suppose that the assumptions (A1) and (A2) are satisfied, then there exists at least one solution of system (1) defined on* $[0,+\infty)$ *in the sense of Eqs. (7).*

*Consider system (1) as the drive system, then the controlled response system is*

$$\dot{y}(t)=-Dy(t)+Af(y(t))+Bg(y(t-\tau(t)))+J+u(t), \tag{8}$$

*where* $y(t)=(y_1(t),y_2(t),\ldots,y_n(t))^T\in R^n$ *is the state variable of system (8),* $u(t)$ *is the controller to be designed, the other parameters are the same as in system (1).*

In view of Definition 3 and Lemma 1, the IVP of system (8) is

$$\begin{cases}\dot{y}(t) &= -Dy(t)+A\eta(t)+B\theta(t-\tau(t))+J+u(t), \quad \text{for a.a. } t\in[0,T)\\ \eta(t) &\in K[f(y(t))], \quad \theta(t)\in K[g(y(t))], \text{ for a.a. } t\in[0,T)\\ y(s) &= \phi(s), \quad \forall s\in[-\tau,0],\\ \eta(s) &= \vartheta(s), \quad \theta(s)=\varsigma(s), \text{ for a.a. } s\in[0,T).\end{cases} \tag{9}$$

Define the synchronization error as $e(t)=(e_1(t),e_2(t),\ldots,e_n(t))^T=y(t)-x(t)$, then from (7) and (9), we can obtain the following synchronization error system:

$$\dot{e}(t)=-De(t)+A\pi(t)+B\varpi(t-\tau(t))+u(t), \tag{10}$$

where $\pi(t)=\eta(t)-\alpha(t), \varpi(t-\tau(t))=\theta(t-\tau(t))-\beta(t-\tau(t))$, $u(t)=(u_1(t),u_2(t),\ldots,u_n(t))^T$ is the control input that will be designed latter.