Neurocomputing 179 (2016) 290–297

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Second-order consensus in directed networks of identical nonlinear dynamics via impulsive control

Shasha Yang [a,*], Xiaofeng Liao [a], Yanbing Liu [b]

[a] College of Computer Science, Chongqing University, Chongqing 400044, PR China
[b] College of Computer Science, Chongqing University of Posts and Telecommunications, Chongqing 400044, PR China

ARTICLE INFO

Article history:
Received 18 June 2015
Received in revised form
29 September 2015
Accepted 30 November 2015
Communicated by W. Lu
Available online 15 December 2015

Keywords:
Second-order consensus
Multi-agent systems
Nonlinear oscillators
Impulsive effects
Switching topology

ABSTRACT

This paper investigates the problem of second-order impulsive consensus for multi-agent systems where each agent can be modeled as an identical nonlinear oscillator. Several fundamental consensus criteria are delivered based on algebraic graph theory and stability theory of impulsive differential equations by designing the suitable impulsive control protocols. Sufficient conditions are given to guarantee the consensus of the networked nonlinear oscillators. Finally, simulation results are presented to validate the effectiveness of theoretical analysis.

1. Introduction

A multi-agent system is always composed of many interconnected agents, in which agents represent individual elements with their own dynamics and edges represent the relationships between their dynamics. Multi-agent systems are ubiquitous in the real world, such as electrical power grids, global economic markets, social networks and so on [1–4]. During the past few years, there have been increasing research activities in the field of consensus analysis for the networked multi-agent systems (MAS) due to its wide applications. The research on such problem not only helps better understand the mechanisms of natural collective phenomena, such as avoiding predators and increasing the chance of foraging food, but also provides useful ideas to develop formation control for coordination of multiple mobile autonomous robots [6–9].

In the real world, many evolutionary processes may experience abrupt changes of states at certain time instants. These changes may be due to changes in the external environment disturbances or the interconnections of subsystems. Moreover, these abrupt changes may occur at prescribed moments and triggered by specified events along a particular trajectory. Then, to describe systematically on evolutionary of a real process with a short-time disturbance, it is natural to omit the duration of the disturbance and just assume these perturbations to be instantaneous, that is, in the pattern of impulses. The authors in 5,9–14,20] applied impulsive control to enhance network synchronization and consensus. In [19], the consensus problems for multi-agent networks under directed communication graphs are discussed. The motions of agents are described by impulsive differential equations, and thus, consensus algorithms can be designed. In paper [21], the problem of impulsive consensus of multi-agent systems is investigated.

Owing to the engineering applications, second-order systems where agents are governed by both position and velocity states have received considerable interest [15–18,22–25,33,34]. Unlike the first-order system described in [15] which demonstrated underlying network containing directed spanning tree, and the authors did not guarantee second-order consensus. Some significant conditions were further derived. In [16] proposed constant velocity model. Subsequently, time-varying velocity and nonlinear dynamics were considered in [17]. The authors in [18] found both real and imaginary parts of eigenvalues of the corresponding Laplacian matrix were closely relative to necessary and sufficient conditions of second-order consensus in MADS. In this paper, the problem of second-order impulsive consensus of multi-agent systems is investigated where each agent can be modeled as an identical nonlinear oscillator. Several fundamental consensus criteria are obtained based on algebraic graph theory and stability theory of impulsive differential equations by designing the suitable impulsive control protocols. The structure of this paper is outlined as follows. Some basic preliminaries are introduced in

* Corresponding author.
E-mail address: ysscqu@163.com (S. Yang).

http://dx.doi.org/10.1016/j.neucom.2015.11.088
0925-2312/

Section 2, which are topology description and mathematical preliminaries respectively. In Section 3, we formulate the second-order impulsive consensus problem for networked nonlinear oscillators and introduce an impulsive control protocol. Section 4 further investigates the second-order impulsive consensus of multi-agent systems with fixed or switching topologies. Numerical examples are given to demonstrate the effectiveness and the correctness of theoretical results in Section 5. Finally, the concluding remarks are also given in Section 6.

2. Preliminaries

In this section, some basic definitions in graph theory and preliminary mathematical results are firstly introduced for subsequent use.

The mathematical notations which will be employed in the rest of the paper are presented as follows. Let R^n denote the n-dimensional real vector space. The Euclidean norms of a vector $x=(x_1,\dots,x_N)^T$ and a matrix $A\in R^{n\times n}$ are denoted by $\|x\| \triangleq \sqrt{\sum_{i=1}^{n} x_i^2}$ and $\|A\| \triangleq \sqrt{\lambda_{max}(A^TA)}$, respectively, where $\lambda_{max}(A^TA)$ is the maximum eigenvalue of the matrix A.

2.1. Topology description

A directed graph $\Re$ of order N consists of a vertex set $V=\{1,2,\dots,N\}$ and an ordered edge set $\zeta=\{(i,j): i,j\in V\}$. The set of neighbors of vertex i is denoted by $\Im_i=\{j\in V:(i,j)\in\zeta, j\neq i\}$. A directed path is a sequence of ordered edges of the form (S_{i_1},S_{i_2}), $(S_{i_2},S_{i_3}),\dots$, where $S_{i_j}\in V$ in a directed graph. A directed graph is said to be strongly connected, if there is a directed path from every node to every other node.

A weighted adjacency matrix $A=[a_{ij}]$, where $a_{ij}=0$ and $a_{ij}\le 0$, $i\neq j$. $a_{ij}>0$ if and only if there is an ordered edge (i,j) in $\Im$. The in-degree of vertex i is defined as follows $deg_{in}(i)=\sum_{j=1}^{n} a_{ji}$.

The out-degree of vertex is $deg_{out}(i)=\sum_{j=1}^{N} a_{ij}$.The vertex i is said to be balanced if and only if its in-degree and out-degree are equal, i.e. $deg_{in}(i)=deg_{out}(i)$. Let $\wp$ be the diagonal matrix with the out-degree of each vertex along the diagonal and call it the degree matrix of $\Im$. The Laplacian matrix of the weighted graph is defined as $L_{\Im}=\wp-A$. An important property of L is that the row sums of L are zero and thus $\mathbf{1}^T=(1,1,\dots,1)^T\in R^N$ is an eigenvector of L associated with zero eigenvalue. The graph is said to be balance if and only if every vertexes in-degree and out-degree are equal. i.e. $\sum_{j=1}^{n} a_{ji}=\sum_{j=1}^{n} a_{ij},\ i=1,2,\dots,N$. If the graph is balance, then $\mathbf{1}^TL=0$.

2.2. Mathematical preliminaries

Given $C=[c_{ij}]\in R^{N\times r}$, it is said that $C\ge 0$ (C is nonnegative) if all its elements c_{ij} are nonnegative, and it is said that $C>0$ (C is positive) if its entire element c_{ij} are positive. Further, $C\ge D$ if $C-D\ge 0$, and $C>D$ if $C-D\ge 0$. If a nonnegative matrix $C\in R^{n\times n}$ satisfies $C\mathbf{1}=\mathbf{1}$, then it is said to be stochastic. A square matrix $C\in R^{n\times n}$ is said to be doubly stochastic if both C and C^T are stochastic.

Let L be the graph Laplacian of the network. We refer to $P=I-\varpi L$ with parameter.

Lemma 1 (Olfati-saber et al. [26]). *Let ς be a directed graph with n nodes and maximum degree $d=\max_i\left(\sum_{j\neq i} a_{ij}\right)$. Then the perron matrix P with parameter $\varpi\in(0,1/d]$ satisfies the following properties:*

1. *P is a row stochastic nonnegative matrix with a trivial eigenvalue of 1.*
2. *All eigenvalues of P are in a unit circle.*
3. *If ς is balanced graph, then P is a doubly stochastic matrix.*

Lemma 2 (Horn and Johnson [27]). *The Kronecker product of matrices A and B is defined as*

$$A\otimes B=\begin{pmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{n1}B & \cdots & a_{nn}B \end{pmatrix},$$

which satisfies the following properties:
1. $\|I\otimes A\|=\|A\otimes I\|=\|A\|$.
2. $(A+B)\otimes C=A\otimes C+B\otimes C$
3. $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$.

3. Problem formulation

The general second-order consensus protocol is described by

$$\begin{cases} \dot{x}_i(t)=\dot{v}_i(t) \\ \dot{v}_i(t)=u_i(t) \\ u_i(t)=\sum_{j\in N_i}^{N} a_{ij}\left(x_j(t)-x_i(t)\right)+\sum_{j\in N_i}^{N} a_{ij}\left(v_j(t)-v_i(t)\right) \end{cases} \tag{1}$$

where $x_i(t)\in R^n$ and $v_i(t)\in R^n$ are position and velocity of the ith agent respectively. $A=[a_{ij}]_{N\times N}$ is the adjacency matrix characterizing the topology structure of the network, N_i is the set of neighbors of agent i.

Definition 1. The multi-agent system (1) is said to achieve second order consensus, if for any initial conditions $\lim_{t\to\infty}\|x_j(t)-x_i(t)\|=0$, $\lim_{t\to\infty}\|v_j(t)-v_i(t)\|=0$, $\forall i,j=1,2,\dots,N$.

As we can see, the second-order consensus can be reached if the coupling strengths and spectra of Laplacian matrix satisfy some suitable conditions. However, for the real world multi-agent systems, the dynamics of velocity for each agent is often nonlinear. Moreover, it is much more difficult to obtain the continuous velocity information compared with the position information. To deal with these difficulties, an impulsive control technique is introduced, where each agent can update its position and velocity at impulsive instants. Therefore the multi-agent system with impulsive control signals is described by

$$\begin{cases} \dot{x}_i(t)=\dot{v}_i(t) \\ \dot{v}_i(t)=f(v_i(t),t)+u_i(t) \\ u_i(t)=\sum_{k=1}^{+\infty} h(t-t_k)b_k\sum_{j\in N_i}^{N} a_{ij}(x_j(t)-x_i(t))+\sum_{k=1}^{+\infty} h(t-t_k)c_k\sum_{j\in N_i}^{N} a_{ij}(v_j(t)-v_i(t)) \end{cases} \tag{2}$$

where the discrete instants t_k satisfy $0\le t_k<t_1<\cdots<t_{k-1}<t_k<\cdots$, and $\lim_{k\to+\infty}t_k=+\infty$ with $\tau_k=t_{k+1}-t_k$, $h(t)$ is the Dirac delta function, i.e., $h(t)=0$ for $t\neq 0$, and $\int_{-\infty}^{+\infty} h(t)dt=1$. The Dirac delta function has the fundamental property that $\int_{a-\varepsilon}^{a+\varepsilon} h(t)\sigma(t-a)dt=h(a)$ for $\varepsilon\neq 0$ and all continuous compactly supported functions $h(t)$. In many applications, the Dirac delta function is usually used to model a tall narrow spike function (an impulse). $b_k\in R$, $c_k\in R$, $k\in N^+$ are impulsive constants to be designed later. Without loss of generality, we assume that $x_i(t)$, $v_i(t)$ are left continuous at time t_k.That is $x_i(t_k)=x_i(t_k^-)$ and $v_i(t_k)=v_i(t_k^-)$.

Adopting a similar approach to that used in [28,29], form (1) and (2) we have $x_i(t_k+\varepsilon)-x_i(t_k-\varepsilon)=\int_{t_k-\varepsilon}^{t_k+\varepsilon}(f(x_i(s),s))ds$ where