Neurocomputing 179 (2016) 298–306

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Observer-based consensus of second-order multi-agent systems without velocity measurements

Chenglin Wen [a], Fang Liu [b], Qiang Song [a,*], Xiaoliang Feng [a]

[a] College of Electrical Engineering, Henan University of Technology, Zhengzhou 450001, China
[b] School of Information Engineering, Huanghuai University, Zhumadian, Henan 463000, China

ARTICLE INFO

Article history:
Received 2 September 2015
Received in revised form
24 October 2015
Accepted 27 November 2015
Communicated by Guang Wu Zheng
Available online 17 December 2015

Keywords:
Second-order consensus
Multi-agent system
Distributed observer
Double integrator
Harmonic oscillator
Time delay

ABSTRACT

This paper investigates the consensus of second-order multi-agent systems without measuring the velocity states of the agents, where each agent can be either a double integrator or a harmonic oscillator. By utilizing the position information of the agents, a distributed observer-based protocol is proposed to solve the second-order consensus problem of multi-agent systems with or without time delay. The observer-based consensus problem is converted to the stability analysis for a set of second-order quasi-polynomials through coordinate transform and system decomposition. Then, some necessary and sufficient conditions are derived for reaching second-order consensus in multi-agent systems with or without time delay, respectively, and it is shown that the consensus can be achieved if and only if the communication delay is less than a critical value. Simulation examples are given to verify the theoretical analysis.

1. Introduction

In the past few decades, the consensus problem has been a focal point of many different disciplines due to its important role in the investigation of the distributed coordination of multi-agent systems. The consensus in a multi-agent system is said to be reached if all the agents can eventually agree on some common value under some control algorithms based on local information. Much progress has been made in the consensus of multi-agent systems with first-order dynamics [1–3], second-order dynamics [4–9], higher-order dynamics [10–13] and fractional-order dynamics [14,15]. In particular, much effort has been devoted to the second-order consensus problem due to many practical applications of second-order multi-agent systems, such as the flocking of unmanned air vehicles [16,17] and the formation control of multiple mobile robots [18,19]. It is worth mentioning that the first-order consensus problem is closely related to the synchronization problem of complex networks [20–23].

A second-order linear multi-agent system is composed of a set of interconnected systems, where each agent may be a double integrator [4–6], a spring-mass system [24], or a harmonic oscillator [25,26]. Moreover, it is worth mentioning that a harmonic oscillator and a double integrator can be described by a unified mathematical model. Some fundamental results have been obtained to address the second-order consensus of multi-agent systems with double-integrator dynamics [4–6]. In recent years, the consensus problem of coupled harmonic oscillators has been intensively investigated by using many different control strategies [25–29]. It is now well known that the existence of a directed spanning tree in network topology is a necessary condition for reaching second-order consensus [4], and both the real and imaginary parts of the nonzero eigenvalues of network Laplacian matrix are important to discuss the consensus problem [5].

Since the time delay is inevitable in many physical systems [30–32], some researchers have studied the consensus in a network of double integrators or harmonic oscillators with time delay. For coupled double-integrators, Yu et al. [5] showed that the communication delay should be less than some critical value and Meng et al. [6] further analyzed the effects of input and communication delays on second-order consensus. As for coupled harmonic oscillators, Zhang et al. [27] considered the consensus in the network with delayed velocity coupling, while Song et al. [29] investigated the consensus using delayed position information and proved that the time delay should be chosen from a set of bounded intervals.

Note that most consensus protocols for coupled double integrators [4–6] and coupled harmonic oscillators all require velocity states [25–28]. However, for a second-order multi-agent system without being equipped with velocity sensors, the velocity information will not be available. Moreover, in some practical cases, the

* Corresponding author.
E-mail address: qsongseu@gmail.com (Q. Song).

http://dx.doi.org/10.1016/j.neucom.2015.11.087
0925-2312/

velocity might be more difficult to be accurately measured compared with the position. In recent years, by using current position measurements, some distributed observer-based consensus protocols have been developed for coupled double integrators [33–36] and coupled harmonic oscillators [37], where only the algorithm in [36] assumes general directed network topology. As far as we know, very few reduced-order observer-based consensus protocols have been designed for second-order multi-agent systems with directed topology and time delay.

Motivated by the above-mentioned discussions, this paper studies the observer-based consensus for a class of second-order multi-agent systems, where each agent can be a double integrator or a harmonic oscillator. The main contribution of this paper is two-fold. First, by utilizing the position information of the agents, we propose a distributed observer approach to solve the second-order consensus problems for coupled harmonic oscillators or coupled double integrators under directed network topology with or without communication delay. Then, through coordinate transformation and system decomposition, the consensus problem is converted to the stability analysis for a set of a second-order quasi-polynomials, based on which some necessary and sufficient conditions are derived for reaching second-order consensus in multi-agent systems with or without time delay, respectively. Second, we analyze the effects of communication delay on second-order consensus of multi-agent systems under the proposed observer-based protocol: on the one hand, it is shown that the consensus can be reached only when the communication delay is less than some critical value; on the other hand, it is found that the final consensus state is only determined by network topology, initial state and a model parameter of multi-agent system, indicating that the final consensus state is independent of the communication delay.

The rest of this paper is organized as follows. Section 2 designs an observer-based consensus protocol for second-order multi-agent systems based on position measurements. In Section 3, the consensus problem is transformed and analyzed via coordinate transformation. Sections 4 and 5 provide some necessary and sufficient conditions for reaching second-order consensus in multi-agent systems without and with time delay, respectively. Numerical examples are given in Section 6 to verify the theoretical analysis. Finally, some concluding remarks and future trends are stated in Section 7.

Notation: Let $\mathbb{Z}^+$, $\mathbb{R}$ and $\mathbb{C}$ represent the sets of nonnegative integers, real numbers and complex numbers, respectively. For $z \in \mathbb{C}$, let $\bar{z}$, $\mathrm{Arg}(z)$, $|z|$, $\mathrm{Re}(z)$ and $\mathrm{Im}(z)$ be its conjugate, principal argument, modulus, real part and imaginary part, respectively, where $-\pi < \mathrm{Arg}(z) \le \pi$. Denote the imaginary unit by $\mathbf{i} = \sqrt{-1}$. Let I_n be the n-dimensional identity matrix, $1_n \in \mathbb{R}^n$ ($0_n \in \mathbb{R}^n$) be the vector of all ones (zeros), and O_n be the square matrix with all entries being zeros. A vector X is called nonnegative and denoted as $X \ge 0$ if all its elements are nonnegative. The determinant of the square matrix A is denoted by $\det(A)$. The traditional sign function is denoted by $\mathrm{sign}(\cdot)$.

2. Observer-based consensus protocol for second-order multi-agent system utilizing position information

In this section, a distributed reduced-order observer-based approach using position information is proposed to achieve second-order consensus in coupled harmonic oscillators or coupled double integrators.

Consider the following multi-agent system composed of N identical agents with second-order dynamics:

$$\dot{x}_i(t) = v_i(t)$$
$$\dot{v}_i(t) = -\alpha x_i(t) + u_i(t), \quad i = 1, \dots, N, \tag{1}$$

where $\alpha \ge 0$ is a constant, $x_i \in \mathbb{R}^n$ and $v_i \in \mathbb{R}^n$ are the position and velocity states of the ith agent, respectively, and $u_i \in \mathbb{R}^n$ is the control input to be designed to achieve consensus, that is, $\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0$ and $\lim_{t\to\infty} \|v_i(t) - v_j(t)\| = 0$, $\forall i,j = 1, \dots, N (i \ne j)$. For notational brevity, it is assumed $n = 1$ in this paper. Nevertheless, by using the Kronecker product [38], the theoretical results can be easily extended to the higher-dimensional case with $n \ge 2$.

It should be noted that each agent in multi-agent system (1) is a *double integrator* [4–6] and a *harmonic oscillator* [25,26] when $\alpha = 0$ and $\alpha > 0$, respectively. Hence, a class of second-order multi-agent systems can be generalized by the network model (1).

Let $\mathcal{G}$ be the digraph describing the topology of network (1). Let $\mathcal{A} = (a_{ij})_{N\times N}$ be the adjacency matrix associated with $\mathcal{G}$ whose off-diagonal elements are defined as follows: if there is a directed link from node j to node i ($i \ne j$), then $a_{ij} > 0$; otherwise $a_{ij} = 0$. In this paper, the graph $\mathcal{G}$ is assumed to be simple without self-loops, that is, $a_{ii} = 0$ for all $i = 1,\dots,N$. On the basis of the adjacency matrix $\mathcal{A}$, the Laplacian matrix $L = (l_{ij})_{N\times N}$ is given by: $l_{ij} = -a_{ij} \le 0 (i \ne j)$ and $l_{ii} = \sum_{j=1, j\ne i}^N a_{ij}$ [2,4,5].

In [11], Li et al. proposed a distributed reduced-order observer-based consensus protocol for *delay-free* multi-agent systems with general linear dynamics by using the outputs of the agents. Inspired by the work in [11] and the technique for Luenberger reduced-order observer [39], we design the following observer-based approach for reaching consensus in network (1) based on delayed position states:

$$\dot{w}_i(t) = -\alpha x_i(t) - x_i(t) - w_i(t) + u_i(t),$$
$$u_i(t) = (k_1 + k_2) \sum_{j=1, j\ne i}^N a_{ij}(x_j(t-\tau) - x_i(t-\tau))$$
$$+ k_2 \sum_{j=1, j\ne i}^N a_{ij}(w_j(t-\tau) - w_i(t-\tau)), \quad i = 1, \dots, N, \tag{2}$$

where $w_i \in \mathbb{R}$ is the state of the observer, $\tau \ge 0$ is the communication delay, a_{ij} is the (i,j)-th element of the adjacency matrix $\mathcal{A}$, k_1 and k_2 are the coupling strengths to be designed.

Remark 1. For each agent in network (1), letting $z_i = (x_i, v_i)^T$, one has $\dot{z}_i = Az_i + Bu_i$, $y_i = Cz_i$, $i = 1, \dots, N$, where $A = \begin{pmatrix} 0 & 1 \\ -\alpha & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $C = (1\ \ 0)$. It is easy to verify that the matrix pair (A, B, C) is controllable and observable. In Ref. [39], some techniques have been proposed to design reduced-order observers for linear time-invariant systems. Considering the formula (3.5) in Ref. [39] and the structure of matrices A, B and C, let $A_{11} = 0$, $A_{12} = 1$, $A_{21} = -\alpha$, $A_{22} = 0$, $B_1 = 0$ and $B_2 = 1$. Then one can develop observer-based protocol (2) on the basis of the delayed position information.

Remark 2. Note that algorithm (2) is a kind of reduced-order observer-based protocol for multi-agent system (1) based on position states of the agents, and provides a unified approach to solve the second-order consensus problem for multi-agent systems composed of either double integrators or harmonic oscillators. In particular, the traditional separation principle in control theory still holds for the observer-based protocol (2), which will be further discussed in Remark 5.

Let $x(t) = (x_1(t), \dots, x_N(t))^T$, $v(t) = (v_1(t), \dots, v_N(t))^T$ and $w(t) = (w_1(t), \dots, w_N(t))^T$. By the definition of Laplacian matrix, the closed-loop network with dynamics (1) and (2) can be written as

$$\dot{x}(t) = v(t)$$
$$\dot{v}(t) = -\alpha x(t) - (k_1 + k_2)Lx(t-\tau) - k_2 Lw(t-\tau)$$
$$\dot{w}(t) = -(\alpha+1)x(t) - w(t) - (k_1 + k_2)Lx(t-\tau) - k_2 Lw(t-\tau). \tag{3}$$