Design of non-fragile state estimators for discrete time-delayed neural networks with parameter uncertainties ☆

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ABSTRACT

This paper is concerned with the problem of designing a non-fragile state estimator for a class of uncertain discrete-time neural networks with time-delays. The norm-bounded parameter uncertainties enter into all the system matrices, and the network output is of a general type that contains both linear and nonlinear parts. The additive variation of the estimator gain is taken into account that reflects the possible implementation error of the neuron state estimator. The aim of the addressed problem is to design a state estimator such that the estimation performance is non-fragile against the gain variations and also robust against the parameter uncertainties. Sufficient conditions are presented to guarantee the existence of the desired non-fragile state estimators by using the Lyapunov stability theory and the explicit expression of the desired estimators is given in terms of the solution to a linear matrix inequality. Finally, a numerical example is given to demonstrate the effectiveness of the proposed design approach.

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1. Introduction

The past two decades have witnessed a surge of interest on both theoretical investigations and algorithm developments of the recurrent neural networks (RNNs) due mainly to their remarkable ability to exhibit dynamic temporal behavior in order to extract/detect/ approximate functional information from complicated or imprecise data. So far, the RNNs have come to play a more and more important role in a variety of application areas including pattern recognition, image processing, optimization calculation and so on. On the other hand, since early 90s, the time-delay has been recognized as a ubiquitous phenomenon that could cause undesired oscillations or even instability in both biological and man-made neural networks [2]. As an active research branch, the study on the dynamical behaviors of RNN with time-delays has recently attracted an ever-increasing interest from many communities including neural science, signal processing and control engineering. Accordingly, in the past years, a large number of results have been available in the literature on the

dynamics analysis issues (e.g. stability, synchronization and estimation) for RNNs with various kinds of time-delays such as constant, time-varying, discrete, distributed or mixed delays, see [1,6,11,13,18,22,28] for some representative works. Very recently, in [21], the passivity analysis problem has been investigated for a class of switched neural networks subject to stochastic disturbances and time-varying delays by using the average dwell-time approach. In [33], the adaptive synchronization problem has been addressed for memristor-based neural networks with time-varying delays by virtue of the differential equation theory with discontinuous right-hand sides.

In many practical applications, the actual values of the neural states of a RNN are vitally important. For example, in optimization problems, the RNNs can be implemented physically in designated hardware such as application-specific integrated circuits where the optimization is carried out in a truly parallel and distributed manner. In this case, the neuron states are closely related to the equilibria as well as the decision-making solutions. However, because of the large scale of the RNN as well as the implementation cost, the neuron states are often not fully observable and only the network outputs are available that contain partial information about the network states. As such, accurate estimation of the neuron states through measured outputs becomes an essential prerequisite for successful accomplishment of certain tasks such as approximation and optimization by using RNNs. In [34], the problem of state estimation has been first proposed for neural networks with time-varying delays. Since then, such a problem has

received considerable research attention for both continuous- and discrete-time neural networks, see e.g. [15,17,23,25,30,34,35].

As is well known, the parameter uncertainties are often unavoidable in real systems due to modeling inaccuracies and/or changes in the environment. In recent years, a great deal of effort has been devoted to the robustness analysis for uncertain systems [7,12,16,29]. Despite the rich body of literature on the state estimation issues for RNN with parameter uncertainties, most results obtained so far have been based on the assumption that the desired state estimator can be realized precisely. Such an assumption is, however, not necessarily true in certain engineering practice. When implementing a state estimator digitally, the implementation errors are often inevitable due probably to analogue-to-digital conversion, rounding errors, finite precision or internal noise. As discussed in [19], a small or even tiny drift/fluctuation/error with the parameter implementation of the designed controller/estimator could lead to unexpected fragility of the closed-loop system as a whole. In other words, the parameters of the actually implemented controller/estimator might have slight deviations from their expected values, and therefore designed controller/estimator should have certain degree of tolerance or non-fragility against the possible parameter deviations. In the past decade, the problem of non-fragile control has gained much attention with respect to the implementation errors in controllers/estimators [24,27,32,36,37]. However, when it comes to the discrete-time RNNs with time-delays, the non-fragile state estimation problem has not been fully studied yet, not to mention the case where the uncertainties also enter into other network parameters. It is, therefore, the main purpose of this paper to shorten such a gap.

Motivated by the above discussion, we aim to design a non-fragile state estimator for a class of discrete-time neural networks with parameter uncertainties. A sufficient condition for the asymptotic stability of the error dynamics of the state estimation is obtained and the gain matrix of the state estimator is derived by solving a linear matrix inequality (LMI). A numerical example is presented to demonstrate the effectiveness of the theoretical results obtained. The main contribution of this paper lies mainly on the problem addressed and the model proposed, which is twofold as follows: (1) the non-fragile state estimation problem is put forward for discrete time-delay neural networks in the presence of parameter uncertainties in all network parameters; and (2) the network output is quite general that is subject to nonlinear disturbances.

Notation: Throughout this paper, the superscript "T" represents the matrix transposition. $\mathbb{R}$ denotes the set of real numbers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $\mathbb{R}^{n \times m}$ denotes the set of all $n \times m$ real matrices. $\mathbb{N}^+$ stands for the sets of positive integers, I and 0 denote the identity matrix and zero matrix of appropriate dimensions, respectively. We use $X > 0$ ($X < 0$) to denote a positive-definite matrix (negative-definite matrix) X. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$. If A is a matrix, $\lambda_{\min}$ stands for the smallest eigenvalue of A. The notation $*$ always denotes the symmetric block in a symmetric matrix, and diag $\{\cdots\}$ stands for a block-diagonal matrix.

2. Problem formulation

In this paper, we consider a discrete-time neural network described by the following dynamical equations:

$$\begin{cases} x(k+1) = (C+\Delta C)x(k) + (A+\Delta A)f(x(k)) + (B+\Delta B)f(x(k-d)) \\ x(k) = \phi(k), \quad k \in [-d, 0) \end{cases} \tag{1}$$

where $x(k) = [x_1(k), x_2(k), ..., x_n(k)]^T \in \mathbb{R}^n$ is the neural state vector;

$f(x(k)) = [f_1(x_1(k)), f_2(x_2(k)), ..., f_n(x_n(k))]^T$ represents the nonlinear activation function with the initial condition $f(0) = 0$; $C = \text{diag}\{c_1, c_2, ..., c_n\}$ is a positive diagonal matrix; $A = [a_{ij}]_{n \times n}$, $B = [b_{ij}]_{n \times n}$ are, respectively, the connection weight matrix and the delayed connection weight matrix; $d \geq 0$ denotes the discrete time-delay; $\phi(k)$ describes the initial condition. In addition, ΔA, ΔB and ΔC are time-varying parameter uncertainties that satisfy

$$[\Delta A \ \Delta B \ \Delta C] = M_1 F_1(k)[N_1 \ N_2 \ N_3] \tag{2}$$

where M_1, N_1, N_2, N_3 are known real-valued matrices with appropriate dimensions and $F_1(k)$ is an unknown matrix satisfying

$$F_1^T(k)F_1(k) \leq I, \quad \forall k \in \mathbb{N}^+ \tag{3}$$

Throughout this paper, we make the following assumption.

Assumption 1. For any $\alpha_1, \alpha_2 \in \mathbb{R}, \alpha_1 \neq \alpha_2$, the activation function $f(\cdot)$ satisfies

$$\gamma_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq \gamma_i^+, \quad (i = 1, 2, ..., n) \tag{4}$$

where γ_i^- and γ_i^+ are known constant scalars.

Remark 1. As shown in [22], the activation functions described in (1) are more general than the usual sigmoid functions and the commonly used Lipschitz conditions, where the constants γ_i^- and γ_i^+ are allowed to be positive, negative or zero. Therefore, such activation functions could be nonmonotonic and would induce less conservative results.

The outputs from the neural network (1) are of the following form:

$$y(k) = Dx(k) + Eg(k, x(k)) \tag{5}$$

where $y(k) = [y_1(k), y_2(k), ..., y_n(k)]^T$ represents the measurement output, D and E are known real-valued matrices with appropriate dimensions, and $g(k, x(k))$ is the neuron-dependent nonlinear disturbance that satisfies the following Lipschitz condition:

$$|g(k, \mu_1) - g(k, \mu_2)| \leq |G(\mu_1 - \mu_2)| \tag{6}$$

where G is a known constant matrix with appropriate dimension.

In order to estimate the state of the neural network (1) from available measurement output (5), we construct the following non-fragile state estimator:

$$\begin{cases} \hat{x}(k+1) = C\hat{x}(k) + Af(\hat{x}(k)) + Bf(\hat{x}(k-d)) + (K+\Delta K)[y(k) - D\hat{x}(k) - Eg(k, \hat{x}(k))] \\ \hat{x}(k) = \hat{\phi}(k), \quad k \in [-d, 0) \end{cases} \tag{7}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the state estimation, $\hat{\phi}(k)$ is the initial function of $\hat{x}(k)$, and K is the estimator gain parameter to be determined. ΔK quantifies the estimator gain variation in the following additive norm-bounded form:

$$\Delta K = M_2 F_2(k)N_4 \tag{8}$$

where M_2, N_4 are known real-valued matrices with appropriate dimensions and $F_2(k)$ is an unknown matrix satisfying

$$F_2^T(k)F_2(k) \leq I, \quad \forall k \in \mathbb{N}^+ \tag{9}$$

Letting the estimation error be $e(k) = x(k) - \hat{x}(k)$, the dynamics of the estimation error can be obtained from (1), (5) and (7) as follows:

$$\begin{aligned} e(k+1) = & (C - KD - \Delta KD)e(k) + \Delta Cx(k) + A\tilde{f}(e(k)) + \Delta Af(x(k)) \\ & + B\tilde{f}(e(k-d)) + \Delta Bf(x(k-d)) - (K+\Delta K)E\tilde{g}(e(k)) \end{aligned} \tag{10}$$

where

$$\tilde{f}(e(k)) := f(x(k)) - f(\hat{x}(k)),$$
$$\tilde{f}(e(k-d)) := f(x(k-d)) - f(\hat{x}(k-d),$$