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State estimation for uncertain Markovian jump neural networks with mixed delays

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ABSTRACT

This paper investigates the problem of state estimation for uncertain Markovian jump neural networks (NNs) with additive time-varying discrete delay components and distributed delay. By constructing a novel Lyapunov–Krasovskii function with multiple integral terms and using an improved inequality, several sufficient conditions are derived. Some improved conditions are formulated in terms of a set of linear matrix inequalities (LMIs), under which the estimation error system is globally exponentially stable in the mean square sense. Some numerical examples are provided to demonstrate the effectiveness of the proposed results.

1. Introduction

In the past few decades, neural networks have been extensively studied and have found application in a variety of areas, such as signal processing, pattern recognition, and combinatorial optimization. However, such applications heavily depend on the study of dynamical analysis of neural networks in the presence of time delays and parametric uncertainties [1–6,34,37]. In particular, time delays may cause undesirable dynamic network behaviors such as oscillation and instability and the connection weights of neurons depend on certain resistance and capacitance values that include uncertainties, such as modeling errors. Therefore, the problem of stability of delayed neural networks is importance in both theory and practice, there have been many important and interesting results [7–16,35].

Hybrid systems driven by continuous-time Markov chain have been used to model many practical systems, where they may experience abrupt changes in their structure and parameters. When the neural network incorporates abrupt changes in its structure, the Markovian jump linear system is very appropriate to describe its dynamics [17–23,36]. It is well known that the state estimation is one of the foundational problems in dynamics analysis for complex systems including recurrent neural networks, complex networks, genetic regulatory networks as well as general engineering systems. Over the past few decades, a lot of effective approaches have been proposed in this research area [24–27]. In addition, due to the modeling errors, parameter drifting, uncertainties occur so frequently that may lead to instability and poor performance of the neural networks. Parameter uncertainties have been mainly categorized as norm bounded uncertainties and interval uncertainties, while the interval type can be usually transformed into the norm-bounded type.For these two types of uncertainties, the state estimation problem and stability analysis have been investigated in [28,29,38] for neural networks. Stability and state estimation for uncertain neural networks with time varying delays and Markovian jump parameters have drawn considerable research attention. It is worth mentioning that, although there are already many works to deal with the problem of dynamic analysis to those neural networks, they are still conservative to some extent, for example, the technique to deal with the cross products in most of those works was Jensen inequality [30], it will lead to some conservativeness of the achieved results, which leaves great room for further research.

In this paper, the problem of state estimation for uncertain Markovian jump neural networks (NNs) with mixed delays is investigated. By constructing a novel Lyapunov–Krasovskii functional with multiple integral terms based on the idea of delay partitioning, and using reciprocally convex approach and an improved inequality, which provides more accurate upper bound than Jensen inequality for dealing with the cross-term. The improved stability criteria of the estimation error systems is obtained in the form of linear matrix inequalities. In fact, the system discussed in [31] is a special case of ours. Some numerical

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examples are provided to demonstrate the effectiveness of the proposed results.

Notation: Throughout this paper, $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For symmetric matrices X and Y, the notation $X>Y$ ($X\ge Y$) means that the matrix $X-Y$ is positive definite (nonnegative). The superscripts $'T'$ and $'-1'$ respectively stand for the transpose and inverse of a matrix. The symmetric block in a symmetric matrix is denoted by $*$.

2. Preliminaries

Consider the following Markovian jump neural networks with mixed delays:

$$\dot{x}(t)=-A(t,r_t)x(t)+B(t,r_t)\sigma(x(t))+B_1(t,r_t)\sigma(x(t-h_1(t,r_t)-h_2(t,r_t)))+D(t,r_t)\int_{t-d}^{t}\sigma(x(s))ds+J(r_t), \quad (1)$$

$$y(t)=C(r_t)x(t)+\phi(t,x(t)), \quad (2)$$

Remark 2.1. In the past few decades, the time delay in singular form in a state was paid more attention. However, in practical situations especially networked controlled systems, signals sometimes transmitted from one point to another two segments of networks [33]. Therefore, a system with two additive time varying delay components should be considered.

To simplify the notations, for $r_t=i$. $A(t,r_t),B(t,r_t),B_1(t,r_t),D(t,r_t),J(r_t),C(r_t),h_1(t,r_t)$ and $h_2(t,r_t)$ are denoted by $A_i(t),B_i(t),B_{1i}(t),D_i(t),J_i,C_i,h_{1i}(t)$ and $h_{2i}(t)$, respectively. We denote $h_i(t)=h_{1i}(t)+h_{2i}(t)$. For each $r_t\in\mathcal{F}$, (1), and (2) can be rewritten to the following form:

$$\dot{x}(t)=-A_i(t)x(t)+B_i(t)\sigma(x(t))+B_{1i}(t)\sigma(x(t-h_i(t)))+D_i(t)\int_{t-d}^{t}\sigma(x(s))ds+J_i, \quad (3)$$

$$y(t)=C_ix(t)+\phi(t,x(t)), \quad (4)$$

and

$$A_i(t)=A_i+\Delta A_i(t),\quad B_i(t)=B_i+\Delta B_i(t),$$
$$B_{1i}(t)=B_{1i}+\Delta B_{1i}(t),\quad D_i(t)=D_i+\Delta D_i(t), \quad (5)$$

where $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in\Re^n$ is the neuron state vector, $y(t)\in\Re^m$ is the network measurement. $\sigma(x(\cdot))=[\sigma_1(x_1(\cdot)),\sigma_2(x_2(\cdot)),\dots,\sigma_n(x_n(\cdot))]^T\in\Re^n$ denotes the neuron activation function, and $J=(J_1,J_2,\dots,J_n)^T\in\Re^n$ is a constant input vector, $\phi(t,x(t))$ is a nonlinear disturbance. $A_i\in\Re^{n\times n}$ is a positive diagonal matrix, B_i,B_{1i} and D_i are the connection weight matrices with appropriate dimension, C_i is a real known matrix, $h_{1i}(t)$ and $h_{2i}(t)$ are two mode-dependent time-varying delay. $\Delta A_i(t),\Delta B_i(t),\Delta B_{1i}(t)$ and $\Delta D_i(t)$ time-varying parameter uncertainties, which are assumed to be the form:

$$[\Delta A_i(t),\quad \Delta B_i(t),\quad \Delta B_{1i}(t),\quad \Delta D_i(t)]=H_iF_i(t)[E_{1i},E_{2i},E_{3i},E_{4i}], \quad (6)$$

where H_i,E_{1i},E_{2i},E_{3i} and E_{4i} are known real constant matrices, and $F_i(\cdot)$ are unknown time-varying matrix function satisfying

$$F_i^T(t)F_i(t)\le I, \quad (7)$$

The transition probability matrix of system (1) is given by

$$P\{r_{t+\Delta}=j|\,r_t=i\}=\begin{cases}\pi_{ij}\Delta+o(\Delta) & \text{if } (j\ne i)\\ 1+\pi_{ii}\Delta+o(\Delta) & \text{if } j=i\end{cases} \quad (8)$$

where $\Delta>0$, $\lim_{\Delta\to0^+}o(\Delta)/\Delta=0$, $\pi_{ij}\ge0$, for $j\ne i$ is the transition rate from i mode at time t to j mode at time $t+\Delta$, and for each $i\in\mathcal{F}$, $\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}$. Let $\{r_t\}_{t\ge0}$ be a right-continuous Markov chain defined on a complete probability space $(\Omega,\mathcal{F},P)$ and taking discrete values in a finite set $\mathcal{F}=1,2,\dots,N$ with generator $\prod=(\pi_{ij})_{N\times N}$.

Assumption 2.1. Suppose that $\sigma(\cdot)$ satisfies

$$[\sigma(x)-\sigma(y)-\Sigma_1(x-y)]^T[\sigma(x)-\sigma(y)-\Sigma_2(x-y)]\le0,\quad \forall x,\quad y\in\Re^n \quad (9)$$

where $\Sigma_1,\Sigma_2\in\Re^{n\times n}$ are known constant matrices.

Assumption 2.2. Suppose that $\phi(\cdot)$ satisfies

$$[\phi(t,x)-\phi(t,y)-\Phi_1(x-y)]^T[\phi(t,x)-\phi(t,y)-\Phi_2(x-y)]\le0,\quad \forall x,\quad y\in\Re^n \quad (10)$$

where $\Phi_1,\Phi_2\in\Re^{m\times n}$ are known constant matrices.

Assumption 2.3. There exist scalars $\underline{h}_{1i}$, $\overline{h}_{1i}$, $\underline{h}_{2i}$, $\overline{h}_{2i}$, μ_{1i} and μ_{2i} such that for $r_t=i\in\mathcal{F}$

$$0\le\underline{h}_{1i}\le h_{1i}(t)\le\overline{h}_{1i},\quad 0\le\underline{h}_{2i}\le h_{2i}(t)\le\overline{h}_{2i},$$
$$\dot{h}_{1i}(t)\le\mu_{1i},\quad \dot{h}_{2i}(t)\le\mu_{2i}, \quad (11)$$

For $r_t=i\in\mathcal{F}$, a proper state estimator is constructed as

$$\dot{\hat{x}}(t)=-A_i(t)\hat{x}(t)+B_i(t)\sigma(\hat{x}(t))+B_{1i}(t)\sigma(\hat{x}(t-h_i(t)))+D_i(t)\int_{t-d}^{t}\sigma(\hat{x}(s))ds+J_i+K_i[y(t)-C_i(t)\hat{x}(t)-\phi(t,\hat{x}(t))], \quad (12)$$

where $\hat{x}(t)$ is an estimation of the state $x(t)$, and $K_i(i=1,2,\dots,N)$, to be determined, are the gain matrices. Define the error signal to be $e(t)=x(t)-\hat{x}(t)$. Then the estimation error system can be immediately obtained from (3), (4) and (12):

$$\dot{e}(t)=-(A_i(t)+K_iC_i)e(t)+B_i(t)f(e(t))+B_{1i}(t)f(e(t-h_i(t)))+D_i(t)\int_{t-d}^{t}f(e(s))ds-K_ig(e(t)), \quad (13)$$

Here, it is simply written $\sigma(x(t))-\sigma(\hat{x}(t))$ as $f(e(t))$ and $\phi(t,x(t))-\phi(t,\hat{x}(t))$ as $g(e(t))$ without any confusion. In addition, it follows from Assumptions 2.1 and 2.2 that

$$[f(e(t))-\Sigma_1e(t)]^T[f(e(t))-\Sigma_2e(t)]\le0,$$
$$[g(e(t))-\Phi_1e(t)]^T[g(e(t))-\Phi_2e(t)]\le0, \quad (14)$$

Lemma 2.1 (*Seuret and Gouaisbaut [32]*). *For a given matrix $R>0$, $h_m\le h(t)\le h_M$, and any appropriate dimension matrix X, which satisfies*

$$\begin{bmatrix}\overline{R} & X\\ * & \overline{R}\end{bmatrix}\ge0,$$

Then, the following inequality holds for all continuously differentiable function $e(t)$:

$$-\int_{t-h_M}^{t-h_m}\dot{e}^T(s)R\dot{e}(s)ds\le-\frac{1}{h_M-h_m}\alpha^T(t)\begin{bmatrix}\overline{R} & X\\ * & \overline{R}\end{bmatrix}\alpha(t),$$

$$\alpha(t)=[\alpha_1^T(t),\alpha_2^T(t),\alpha_3^T(t),\alpha_4^T(t)]^T,$$
$$\alpha_1(t)=e(t-h(t))-e(t-h_M),$$
$$\alpha_2(t)=e(t-h(t))+e(t-h_M)-\frac{2}{h_M-h(t)}\int_{t-h_M}^{t-h(t)}e(s)ds,$$
$$\alpha_3(t)=e(t-h_m)-e(t-h(t)),$$
$$\alpha_4(t)=e(t-h_m)+e(t-h(t))-\frac{2}{h(t)-h_m}\int_{t-h(t)}^{t-h_m}e(s)ds,$$