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Generalized LMI observer design for discrete-time nonlinear descriptor models

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ABSTRACT

The present paper provides a systematic way to generalize Takagi–Sugeno observer design for discrete-time nonlinear descriptor models. The approach is based on Finsler's lemma, which decouples the observer gains from the Lyapunov function. The results are expressed as strict LMI constraints. To obtain more degrees of freedom without altering the number of LMI constraints and thus relax the conditions, delayed Lyapunov functions and delayed observer gains are considered. Even more relaxed results are developed by extending the approach to α-sample variation. The effectiveness of the proposed methods is illustrated via examples.

1. Introduction

A large family of nonlinear models can be represented as Takagi–Sugeno (TS) models [1]. Several methods to obtain a TS representation exist; the most common are via linearization in several operational points [2] and using the sector nonlinearity approach [3]. During the last years, the sector nonlinearity approach has been employed since the resulting TS model exactly represents the original nonlinear model in a compact set of the state space. A TS model is a collection of linear models interconnected by membership functions (MFs), which are nonlinear and hold the convex sum property [4]. The analysis of TS models is performed through the direct Lyapunov method and one of the main goals is to express the conditions in terms of linear matrix inequalities (LMIs) [5,6]. Using the sector nonlinearity, the number of linear models (vertices) exponentially increases with the number of nonlinearities in the original model. For example, mechanical systems can involve a high number of states and numerous nonlinearities, thus resulting in a standard TS representation with a large number of rules, increasing the computational cost in a way that it can be difficult to handle with the actual LMI solvers [4,7,8].

Since the pioneering results of the non-quadratic approach [9], the analysis and design conditions for discrete-time TS models have witnessed interesting improvements [9–14]. Recently, a non-quadratic Lyapunov function using past samples in its MFs has been proposed in [15] for the observer design and generalized for state feedback controller design in [16].

For systems represented via nonlinear descriptor models [17], an interesting way to handle them has been given in [18]: a TS descriptor representation. This extension of TS models arises when applying twice the sector nonlinearity methodology: once for the right-hand side of the equation and another for the left-hand side. Generally, a TS descriptor model reduces the number of linear models and also the number of LMI constraints with respect to standard ones [8,19–21]. Moreover, the so-called descriptor redundancy [22] has been used to obtain relaxed conditions for those models that do not appear in a natural descriptor form [23–25]. The motivation of the work is twofold. The first one is that numerous models, for example in the mechanical field [8,19,20], do belong naturally to this family of models, and their study appeals to specific tools. The second is, – next to proposing LMI constraint solutions – to derive, depending on some complexity parameter, conditions that are less and less conservative.

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When the state vector is not fully available an observer has to be implemented. The observer design for descriptor models has been discussed in [26–29]. For nonlinear systems with a constant rank-deficient descriptor matrix, few results exist that involve LMI conditions [30,31]. Moreover, existing results include restrictions such as linear output matrices, i.e., linear measurements.

Previous results on descriptor models only consider a constant rank-deficient descriptor matrix. This paper develops conditions for the observer design for nonlinear descriptors with a non-constant full-rank descriptor matrix. In a sense, it extends the standard TS observer results [7,32,33] to the TS descriptor framework. LMI conditions are obtained via non-quadratic Lyapunov functions and Finsler's lemma. Finsler's lemma is used to avoid the explicit substitution of the closed-loop dynamics of the estimation error [34] and to decouple the Lyapunov matrices from the observer gains [15,21]. Furthermore, the paper provides a general framework which encompasses previous results for observer design for discrete-time descriptor models, both linear and TS. At last, the discrete nature of the Lyapunov function via α-sample variation is exploited as in [11] to obtain results whose conservativeness decreases according to a complexity parameter, i.e. α the number of past samples considered in the Lyapunov function.

The paper is divided as follows: Section 2 introduces the discrete-time TS descriptor model, provides useful notations, and motivates this research via an example; Section 3 presents and discusses the main results on the observer design and illustrates them; Section 4 extends the results using α-sample variation; Section 5 concludes the paper and gives some perspectives.

2. Preliminaries

2.1. TS descriptor models

Consider the following discrete-time nonlinear descriptor model:

$$\begin{aligned} E(x_k)x_{k+1} &= A(x_k)x_k + B(x_k)u_k \\ y_k &= C(x_k)x_k, \end{aligned} \tag{1}$$

where $x_k \in \mathbb{R}^{n_x}$ is the state vector, $u_k \in \mathbb{R}^{n_u}$ is the control input vector, $y_k \in \mathbb{R}^{n_y}$ is the output vector, and k is the current sample. Matrices $A(x_k)$, $B(x_k)$, $C(x_k)$, and $E(x_k)$ are assumed to be smooth in a compact set Ω_x of the state space including the origin. Moreover, $E(x_k)$ is assumed to be a regular matrix, at least in the compact set Ω_x. In what follows, arguments will be omitted when they can be easily inferred. An asterisk $(*)$ will be used in matrix expressions to denote the transpose of the symmetric element; for in-line expressions it will denote the transpose of the terms on its left-hand side, i.e.

$$\begin{bmatrix} A & B^T \\ B & C \end{bmatrix} = \begin{bmatrix} A & (*) \\ B & C \end{bmatrix}, \qquad A+B+A^T+B^T+C = A+B+(*)+C.$$

Using the sector nonlinearity approach [4], the p_a nonlinear terms in the right-hand side of (1) are captured via the membership functions (MFs) $h_i(z)$, $i \in \{1,2,\ldots,2^{p_a}\}$. The p_e nonlinear terms in the left-hand side of (1) are grouped in MFs $v_j(z)$, $j \in \{1,2,\ldots,2^{p_e}\}$. These MFs hold the convex sum property in the compact set Ω_x, i.e., $\sum_{i=1}^{r_a} h_i(z)=1$, $h_i(z) \ge 0$, $\sum_{j=1}^{r_e} v_j(z)=1$, $v_j(z) \ge 0$ with $r_a = 2^{p_a}$ and $r_e = 2^{p_e}$. In this work, the MFs depend on the premise variables grouped in the vector $z \in \mathbb{R}^P$, $p = p_a + p_e$, which is assumed to be known [7].

Using the methodology stated above, from the nonlinear model (1) an exact TS descriptor model is obtained [19]:

$$\begin{aligned} \sum_{j=1}^{r_e} v_j(z)E_j x_{k+1} &= \sum_{i=1}^{r_a} h_i(z)(A_i x_k + B_i u_k) \\ y_k &= \sum_{i=1}^{r_a} h_i(z)C_i x_k, \end{aligned} \tag{2}$$

where matrices A_i, B_i, and C_i, $i \in \{1,2,\ldots,r_a\}$ represent the i-th linear right-hand side model (2) and E_j, $j \in \{1,2,\ldots,r_e\}$ represent the j-th left-hand side model of the TS descriptor model. The premise vector is assumed to be available in time; it does not have to be estimated.

2.2. Properties and lemmas

Generally in the TS-LMI framework, it is natural to obtain inequality conditions involving convex sums, for instance:

$$\sum_{i_1=1}^{r_a}\sum_{i_2=1}^{r_a} h_{i_1}(z(k))h_{i_2}(z(k))\Upsilon_{i_1 i_2} < 0, \tag{3}$$

where $\Upsilon_{i_1 i_2} = \Upsilon_{i_1 i_2}^T$ $i_i, i_2 \in \{1,2,\ldots,r_a\}$. In order to obtain LMI conditions, the MFs must be removed from (3). Throughout this paper, the following sum relaxation scheme will be employed.

Lemma 1. *[35] The double convex-sum (3) is negative if*

$$\begin{aligned} &\Upsilon_{i_i i_i} < 0, \quad \forall i_i \in \{1,2,\ldots,r_a\}, \\ &\frac{2}{r_a-1}\Upsilon_{i_i i_i} + \Upsilon_{i_i i_2} + \Upsilon_{i_2 i_i} < 0, \quad i_i, i_2 \in \{1,2,\ldots,r_a\}, \quad i_1 \ne i_2, \end{aligned} \tag{4}$$

hold.

Note that Lemma 1 is one the possible schemes to drop off the convex MFs from (3). Other schemes that include slack variables [36,37] exist in the literature and they apply directly on the results presented in this work.