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Passivity of linearly coupled reaction–diffusion neural networks with switching topology and time-varying delay

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ABSTRACT

This paper studies the passivity of a general array model of coupled reaction–diffusion neural networks (CRDNNs) with switching topology and time-varying delay. By exploiting the Lyapunov functional method and some inequality techniques, several sufficient criteria are established to ensure the input strict passivity and output strict passivity of the proposed network model. Moreover, we reveal the relationship between passivity and stability of CRDNNs. Based on the obtained passivity results and relationship between passivity and stability, a synchronization criterion is presented for CRDNNs. Finally, two numerical examples are provided to demonstrate the correctness and effectiveness of the theoretical results.

1. Introduction

Various systems in nature and society, such as communication networks, social networks, collaborative networks, power grids, cellular networks, World Wide Web, metabolic systems, epidemic spreading networks, can be modeled as complex networks. Therefore, complex networks have long been regarded as a fundamental tool to understand dynamical behavior and the response of real system, and the analysis and control of dynamical behaviors in complex networks have received much attention in recent years [1–3].

As a special class of complex networks, coupled neural networks (CNNs) have found widely applications in many fields, such as pattern recognition, image processing, and optimization problems [4–9]. It is well known that these practical applications are heavily dependent on the dynamical behaviors of CNNs. In recent years, the analysis and control of dynamical behaviors in CNNs have become a hot topic. Liang et al. [10] investigated the robust synchronization of CNNs with stochastic discrete-time delay. In [11], the authors studied the synchronization problem of linearly coupled delayed neural network, in which the activation function is discontinuous and the coupling configuration matrix is not limited to symmetry or irreducibility. By constructing proper state feedback controller and adaptive controller, several criteria for synchronization were established. Yang and Cao [12] discussed the global chaotic synchronization of general coupled neural networks. Several sufficient conditions were developed to guarantee global synchronization by utilizing adaptive pinning feedback control schemes. Unfortunately, diffusion effects have not been taken into consideration in these existing works [10–12]. Strictly speaking, the diffusion phenomena could not be ignored in neural networks and electric circuits once electrons transport in a non-uniform electromagnetic field. However, very few researchers have investigated the CNNs with reaction–diffusion terms [13–17]. Wang et al. [13] studied the synchronization of CNNs with reaction–diffusion, and established several criteria for synchronization by utilizing adaptive feedback control technique. In [16], the authors investigated global exponential synchronization in an array of linearly diffusively coupled reaction–diffusion neural networks (CRDNNs) with time-varying delays by adding impulsive controller to a small fraction of nodes.

But in these existing works on CRDNNs [13–17], they always assume the topology structure is fixed. Practically, this assumption is very restrictive and only covers a few ideal situations. In many real-world networks, the connection topology may change very quickly by switches [18,19]. Therefore, it is very interesting to further investigate CRDNNs with switching topology.

Passivity is part of a broader and a general theory of dissipativeness [20]. The main point of passivity theory is that the passive properties of systems can keep the systems internally stable. The passivity theory was firstly proposed in the circuit analysis [21], and since then has found successful applications in diverse areas

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such as stability [22,23], complexity [24], signal processing [25], chaos control and synchronization [26,27], and fuzzy control [28]. In order to better analyze the dynamical behavior of complex networks, the passivity has also received a lot of attention in recent years and many important results on this topic have been reported [29–34]. In [29], the authors investigated input passivity and output passivity for a complex network with non-linear, time-varying, non-symmetric and delayed coupling, respectively. Song and Cao [30] studied the passivity of uncertain neural networks with leakage delay and time-varying delay as well as generalized activation functions by employing a combination of Lyapunov–Krasovskii functionals, Newton–Leibniz formulation and the free-weighting matrix method. In [31], the problem of passivity analysis was discussed for discrete-time stochastic Markovian jump neural networks with both discrete and distributed delays. Unfortunately, in these existing works [29–31], they assume that the node state, the input and output variables are only functions of time. But the node state, the input and output variables also intensively depend on space variable in many practical situations. Therefore, it is important to study the passivity of complex dynamical networks with spatially and temporally varying state, input and output variables [36,35]. Wang and Wu [35] investigated the robust passivity of a class of parabolic complex networks with multiple time-varying delays. In [36], the authors gave the passivity definition for the case where input and output variables are varied with the time and space variables, and established several sufficient conditions to guarantee the passivity of reaction–diffusion neural networks. To our knowledge, very few researchers have discussed the passivity of CRDNNs [37,38], in which the input and output variables are varied with the time and space variables. Especially, the passivity of the CRDNNs with switching topology has not yet been investigated.

Motivated by the above discussions, the objective of this paper is to investigate the input strict passivity and output strict passivity of CRDNNs with switching topology. By utilizing the Lyapunov functional method combined with some inequality techniques, several sufficient conditions are presented, ensuring the input strict passivity and output strict passivity of CRDNNs with switching topology. The passivity theory has long been a nice tool for analyzing the synchronization of the complex networks, but the relationship between passivity and synchronization of the CRDNNs with switching topology has not yet been investigated. Therefore, the relationship between passivity and synchronization of the CNNs with switching topology and reaction–diffusion terms is also considered in this paper.

The rest of this paper is organized as follows. In Section 2, the considered model is presented and some preliminaries are given. Section 3 is devoted to establishing some passivity criteria and revealing the relationship between passivity and synchronization. In Section 4, two simulation examples are provided to illustrate the effectiveness of the theoretical results. Finally, we conclude this paper and propose some further work in Section 5.

2. Network model and preliminaries

Let $\mathbb{R}=(-\infty,+\infty), \mathbb{R}^+=[0,+\infty), \mathbb{R}^n$ be the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ be the space of $n\times m$ real matrices. $0\le P\in\mathbb{R}^{n\times n}(0\ge P\in\mathbb{R}^{n\times n})$ means that matrix P is symmetric and semi-positive (semi-negative) definite. $0<P\in\mathbb{R}^{n\times n}(0>P\in\mathbb{R}^{n\times n})$ means that matrix P is symmetric and positive (negative) definite. I_n denotes the $n\times n$ real identity matrix. B^T denotes the transpose of matrix B. $\otimes$denotes the Kronecker product of two matrices. $\lambda_m(\cdot), \lambda_M(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. $\Omega=\{x=(x_1,x_2,\ldots,x_q)^T \,||x_s|<l_s, s=1,2,\ldots,q\}$ is an open bounded domain in $\mathbb{R}^q$ with smooth boundary $\partial\Omega, \overline{\Omega}=\Omega\cup\partial\Omega$ and mesΩ denotes the measure of Ω. For any $e(x,t)=(e_1(x,t),e_2(x,t),\ldots,e_n(x,t))^T\in\mathbb{R}^n, (x,t)\in\Omega\times\mathbb{R}$, $\|e(\cdot,t)\|_2$ denotes

$$\|e(\cdot,t)\|_2=\left(\int_\Omega\sum_{i=1}^n e_i^2(x,t)dx\right)^{1/2}.$$

In addition, we define $\|e(\cdot,t)\|_\tau=\sup_{-\tau\le\theta\le0}\|e(\cdot,t+\theta)\|_2$.

To facilitate the readers, the mathematical model of the CRDNNs with switching topology and time-varying delay is presented in a step-by-step format.

A single reaction–diffusion neural network with Dirichlet boundary conditions is described by the following partial differential equations (PDEs):

$$\frac{\partial w_i(x,t)}{\partial t}=d_i\Delta w_i(x,t)-a_iw_i(x,t)+J_i+\sum_{j=1}^n b_{ij}f_j(w_j(x,t)), \tag{1}$$

where $i=1,2,\ldots,n,n$ is the number of neurons in the network; $w_i(x,t)\in\mathbb{R}$ is the state of the ith neuron at time t and in space x; $x=(x_1,x_2,\ldots,x_q)^T\in\Omega\subset\mathbb{R}^q$; $\Delta=\sum_{s=1}^q\frac{\partial^2}{\partial x_s^2}$ is the Laplace diffusion operator on Ω; $d_i>0$ represents the transmission diffusion coefficient along the ith neuron; $f_j(\cdot)$ denotes the activation function of the jth neuron; $a_i>0$ represents the rate with which the ith neuron will reset its potential to the resting state when disconnected from the network and external input; b_{ij} denotes the strength of the jth neuron on the ith neuron; J_i is a constant external input.

Throughout this paper, the function $f_j(\cdot)(j=1,2,\ldots,n)$ satisfies the Lipschitz condition, that is, there exists positive constant ρ_j such that

$$|f_j(\xi_1)-f_j(\xi_2)|\le\rho_j|\xi_1-\xi_2|$$

for any $\xi_1,\xi_2\in\mathbb{R}$, where $|\cdot|$ is the Euclidean norm.

The initial value and boundary value conditions associated with system (1) are given in the form

$$w_i(x,0)=\phi_i(x),\quad x\in\Omega, \tag{2}$$

$$w_i(x,t)=0,\quad (x,t)\in\partial\Omega\times[0,+\infty), \tag{3}$$

where $\phi_i(x)(i=1,2,\ldots,n)$ is bounded and continuous on Ω.

We can rewrite system (1) in a compact form as follows:

$$\frac{\partial w(x,t)}{\partial t}=D\Delta w(x,t)-Aw(x,t)+J+Bf(w(x,t)), \tag{4}$$

where $w(x,t)=(w_1(x,t),w_2(x,t),\ldots,w_n(x,t))^T, D=\mathrm{diag}(d_1,d_2,\ldots,d_n)$, $B=(b_{ij})_{n\times n}$, $J=(J_1,\ J_2,\ldots,J_n)^T$, $A=\mathrm{diag}(a_1,\ a_2,\ldots,a_n)$, $f(w(x,t))=(f_1(w_1\ (x,t)),f_2(w_2(x,t)),\ldots,f_n(w_n(x,t)))^T$.

In this paper, we consider a complex dynamical network with switching topology and time-varying delay consisting of N such identical reaction–diffusion neural networks (4). The mathematical model of the network can be described as follows:

$$\begin{aligned}\frac{\partial z_i(x,t)}{\partial t}=&D\Delta z_i(x,t)-Az_i(x,t)+J+Bf(z_i(x,t))\\&+c\sum_{j=1}^N G_{ij}^{\sigma(t)}\Gamma z_j(x,t-\tau(t))+u_i(x,t),\end{aligned} \tag{5}$$

where $i=1,2,\ldots,N,N$ is the number of nodes in the network; $\tau(t)$ is the time-varying delay with $0\le\tau(t)\le\tau; z_i(x,t)=(z_{i1}(x,t),\ z_{i2}(x,t),\ldots,z_{in}(x,t))^T\in\mathbb{R}^n$ is the state vector of node i; $u_i(x,t)\in\mathbb{R}^n$ is the control input; c is a positive real number, which represents the overall coupling strength; $\Gamma=\mathrm{diag}(\gamma_1,\gamma_2,\ldots,\gamma_n)\in\mathbb{R}^{n\times n}$ is the positive definite inner coupling matrix, which describes the individual coupling between two nodes; $\sigma:[0,+\infty)\to M=\{1,2,\ldots,m\}$ is a switching signal. For each $k\in M$, $G^k=(G_{ij}^k)_{N\times N}$ represents the topological structure of network and coupling strength between nodes, where ${G_{ij}}^k$ is defined as follows: if there exists a connection