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Brief Papers

Adaptive pinning control of cluster synchronization in complex networks with Lurie-type nonlinear dynamics ☆

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ABSTRACT

In this paper, cluster synchronization problem is investigated for a kind of complex dynamical networks with Lurie-type non-linear dynamics. With the aid of a local update law, a pinning adaptive control strategy is proposed to solve the cluster synchronization problem of the networks. Some simple criteria for cluster synchronization are presented for the networks with external disturbances and time delays. Firstly, sufficient conditions are established to realize cluster synchronization of the networks without disturbances and delays. Then, the cluster synchronization problem of the networks with external disturbances is considered. Finally, the cluster synchronization in the networks with time-varying delays is further studied. Numerical examples are given to verify and illustrate the theoretical results.

1. Introduction

The collection behaviors of complex dynamics networks have been received much attention from various areas due to its important applications in engineering control, social and ecological science, etc. As one of the fundamental and significant topics, synchronization and control have been extensively investigated during the past few decades. There are various types of synchronization problems that have been developed in the literature such as complete synchronization, phase synchronization, impulsive synchronization [1], projective synchronization, generalized synchronization and cluster synchronization, each of which plays an important role in the study of complex dynamics networks.

Complete synchronization of complex dynamical networks [2–6] used to study all nodes approaching to a common behavior. In particular, consensus of multi-agent systems with first-order dynamics can be regarded as a special case of it [7–11]. Cluster synchronization [12–21] focuses on when the set of nodes in the network is divided into several clusters, all individuals in the same cluster realize synchronization, but there is no synchronization among different clusters. Cluster synchronization is a common phenomenon and has its broad potential applications in theoretical and engineering aspects. Due to the complicated goals in practice, the interconnected individuals may evolve into different subgroups with their own specific goals. Cluster synchronization can be regarded as an extension of the synchronization problem and reduces to complete synchronization if all individuals have only one goal. It can be observed in flocks of birds [22], opinion formation of social networks [23], and circuits [24]. For instances, in flocks of bark-foraging birds, the birds will be naturally divided into communities. When a team of autonomous vehicles is to carry out a complex task, that can be divided into several simple subtask, greater efficiency and operational scheme is realized by the team of autonomous vehicles dividing into subgroup fashion.

Recently, a variety of control strategies have been presented to solve cluster synchronization problem. As one of the efficient strategies in control for complex dynamical networks, pinning adaptive control has been widely investigated, in which only a very few fraction of nodes are controlled, and it also provides a systematic approach for automatic on-line tuning of controller parameters. Noted that it is economical and effective in large scale networks. In [12], a cluster synchronization pattern of a general network was realized by pinning control, and adaptive control strategy was also introduced. Then, the cluster synchronization problem for the networks by intermittent pinning control was further explored in [11]. By using a decentralized adaptive pinning control strategy, cluster synchronization of undirected complex dynamical networks was investigated in [14]. In [15], the authors

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studied clustering behavior in connected networks consisting of diffusively coupled agents with different linear self-dynamics, delays and negative couplings, respectively. The cluster synchronization in the complex dynamical networks with non-identical nodes by a local control method and an adaptive strategy for the coupling strengths of the networks was discussed in [14]. Under pinning control, cluster synchronization in directed networks of partial-state coupled linear systems was investigated in [17]. In [18], cluster synchronization problem of complex dynamical networks was discussed by using of pinning control scheme, if the network topology is directed and weakly connected.

It is worth mentioning that in current literatures, most works have considered cluster synchronization of the networks, in which the non-linear dynamics of each node is assumed to satisfy QUAD condition or Lipschitz condition (see [12–14,16–19] and their references for details). In practice, many non-linear systems can be categorized as Lurie-type dynamics, such as Chua's circuits [26], Goodwin models [27] and Swarm models [25]. With some assumptions on the topology of network, the cluster synchronization for delayed Lur'e dynamical networks in both continuous-time and discrete-time dynamics was studied in [20] by using pinning control strategy. In [21], the cluster synchronization problem of Lurie dynamical networks was converted into solving a series of much lower-dimensional LMIs, which depend on the global information of the topology of the network such as all eigenvalues of the adjacency matrix of the network.

Motivated by the above analysis, this paper mainly focuses on the cluster synchronization problem of a kind of complex dynamical networks with Lurie-type non-linear dynamics. Based on the community structure of network, a pinning adaptive control strategy is presented, in which the adaptive parameters are only updated according to the state information of the corresponding neighbors. Some simple criteria for cluster synchronization are presented for the networks with external disturbances and time delays. At first, by using the proposed pinning adaptive control strategy, some conditions guaranteeing the cluster synchronization are obtained for the networks with or without external disturbances. Compared with the existing results in [20,6], the results in this paper do not require the eigenvalues of the adjacency matrix of the networks, and the criteria in terms of simple lower-dimensional linear matrix inequalities, which are easy to verify and may greatly reduce the computation complexity significantly if the number of nodes in the network is large. Moreover, the Lurie networks with time-varying delays are also considered, and a sufficient condition for achieving cluster synchronization is derived. The numerical simulations are exploited to demonstrate the effectiveness of the pinning adaptive control strategies proposed in this paper.

The rest of this paper is organized as follows. Section 2 formulates the fundamental problem, and introduces some necessary preliminaries. In Section 3, the cluster synchronization problem in the Lurie dynamical networks without and with time delays are discussed, respectively. Numerical simulations are given in Section 4 to illustrate and verify the effectiveness of the theoretical results. Finally, Section 5 concludes the paper.

2. Preliminaries

Notations. Let $\mathcal{R}^n$ and $\mathcal{R}^{m\times n}$ be the n dimensional Euclidean space and the set of $m\times n$ real matrices, respectively. $I_n\in\mathcal{R}^{n\times n}$ denotes the identity matrix. 0 stands for zero value or zero matrix with compatible dimensions. $\mathcal{L}_2[0,\infty)$ denotes the space of square integrable vector functions over $[0,\infty)$. For a symmetric matrix P, the notation $P>0(<0)$ means that the matrix P is positive definite (negative definite). The superscript T means transpose for real matrices and $*$ refers as conjugate transpose for complex matrices. $A\otimes B$ denotes the Kronecker product of matrices A and B with compatible sizes.

The network in this paper includes N nodes and is divided into d communities, where $2\le d\le N$. Let $\{\mathcal{C}_1,\mathcal{C}_2,\dots,\mathcal{C}_d\}$ denote the d communities of the network satisfying (i) $\bigcup_{i=1}^{d}\mathcal{C}_i=\{1,2,\dots,N\}$; (ii) $\mathcal{C}_i\cap\mathcal{C}_j=\varnothing$ for $i\neq j$. The function μ_i denotes the index of the community in which node i lies, that is, $\mu_i=j$ if node i belongs to the jth community for $i=1,2,\dots,N$, $j=1,2,\dots,d$. For any pair of indices i and j, node i and node j belong to different communities if and only if $\mu_i\neq\mu_j$.

An undirected graph $\mathcal{G}$ is a pair of $(\mathcal{V},\mathcal{E})$, where the set of nodes $\mathcal{V}=\{1,\dots,N\}$, and set of edges $\mathcal{E}\subseteq\{(i,j):i,j\in\mathcal{V}\}$. The edges (i,j) and (j,i) in the undirect graph $\mathcal{G}$ are considered to be the same. If there is an edge between two nodes, then the two nodes are called adjacent. A path in this network from node i_1 to node i_l is a sequence of ordered edges of the form $(i_k,i_{k+1}),k=1,\dots,l-1$. A graph is connected if there exists a path between every pair of distinct nodes. In this paper, it is assumed that the graph $\mathcal{G}$ is connected. $G=[a_{ij}]\in\mathcal{R}^{N\times N}$ is adjacency matrix describing the coupling configuration of the network. If there is a connection between node i and node j $(i\neq j)$, then $a_{ij}=a_{ji}>0$;otherwise, $a_{ij}=a_{ji}=0$. Assume that $a_{ii}=0$ for all i. The Laplacian matrix $L=[l_{ij}]\in\mathcal{R}^{N\times N}$ is defined as $l_{ij}=-a_{ij},j\neq i$, and $l_{ii}=\sum_{j=1,j\neq i}^{N}a_{ij}$, $i=1,2,\dots,N$.

Lemma 1. *For matrices A, B, C and D with compatible sizes, one has*

(i) $(A\otimes B)^T=A^T\otimes B^T$;
(ii) $(A+B)\otimes C=A\otimes C+B\otimes C$;
(iii) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$.

Lemma 2 (*Chen et al. [2]*). *Suppose that* $M=\mathrm{diag}\{m_1,m_2,\dots,m_N\}>0$ *with at least one* $m_i>0$. *Then all eigenvalues of the matrix* $L+M$ *are positive if* $\mathcal{G}$ *is connected.*

Proof. This result has been proved in [2]. The following process is a repetition of the previous work. Set $\overline{L}=L+M$. Suppose that λ is an eigenvalue of $\widehat{L}$, and $v=[v_1,v_2,\dots,v_N]^T\in\mathcal{R}^N$ is the corresponding eigenvector. It is clear that if v is an eigenvector, then $-v$ is also an eigenvector. Thus, without loss of generality, we can assume that $v_k=\max_{j=1,2,\dots,N}v_j>0$. For $k=i$,

$$\lambda v_k\sum_{j=1}^{N}\widehat{l}_{kj}=\sum_{j=2}^{N}a_{kj}v_k+m_iv_k-\sum_{j=2}^{N}a_{kj}v_j\ge m_iv_k.$$

Thus $\lambda\ge m_i>0$. For $k\neq i$,

$$\lambda v_k=\sum_{j=1}^{N}\widehat{l}_{kj}v_j=\sum_{j=2}^{N}a_{kj}v_k-\sum_{j=1,j\neq k}^{N}a_{kj}v_j,\ge 0$$

which means $\lambda\ge0$. If $\lambda=0$, $(L+M)v=0$. However, this is impossible. Therefore, $\lambda>0$. □

Lemma 3 (*Schur complement Boyd et al.[28]*). *The linear matrix inequality*

$$\begin{bmatrix} S_{11} & S_{12}\\ S_{12}^* & S_{22}\end{bmatrix}<0$$

with $S_{11}=S_{11}^*$ *and* $S_{22}=S_{22}^*$, *is equivalent to any one of the following statements:*

(i) $S_{11}<0$, $S_{22}-S_{12}^*S_{11}^{-1}S_{12}<0$;
(ii) $S_{22}<0$, $S_{11}-S_{12}S_{22}^{-1}S_{12}^*<0$.

3. Problem formulation

In this paper, we consider a complex network composed of N coupled identical Lurie dynamics systems with external