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Periodic solution for state-dependent impulsive shunting inhibitory CNNs with time-varying delays

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ABSTRACT

In this paper, we consider existence and global exponential stability of periodic solution for state-dependent impulsive shunting inhibitory cellular neural networks with time-varying delays. By means of B-equivalence method, we reduce these state-dependent impulsive neural networks system to an equivalent fix time impulsive neural networks system. Further, by using Mawhin's continuation theorem of coincide degree theory and employing a suitable Lyapunov function some new sufficient conditions for existence and global exponential stability of periodic solution are obtained. Previous results are improved and extended. Finally, we give an illustrative example with numerical simulations to demonstrate the effectiveness of our theoretical results.

1. Introduction

A model of artificial neural networks named as shunting inhibitory cellular neural networks is introduced by Bouzerdoum and Pinter in Bouzerdoum and Pinter (1993) and has many potential applications in the areas such as pattern recognition, signal and image processing, optimization problems, biology, vision, speech and parallel computations. All of these applications tediously depend on the dynamical characteristics such as stability and periodicity of the designed network. In practical applications, because of the finite speed of the switching and transmission of signals, the time delays are indispensable in the networks. For these reasons, stability and periodicity of shunting inhibitory cellular neural networks with constant or time-varying delays were extensively studied and sufficient conditions for global exponential stability are constructed in Cao (1999), Cao, Chen, and Huang (2005), Chen, Cao, and Huang (2004), Hien, Loan, and Tuan (2008), Huang and Cao (2003), Li, Liu, and Zhu (2005); Li, Meng, and Zhou (2008), Liu (2009), Liu and Huang (2007a, 2007b), Meng and Li (2008), Ou (2009), Wang and Lin (2009), Wu and Fu (2009), Zhang, Yang, Long, and He (2010), Zhong and Liu (2007) and references therein. On the other hand, the instantaneous perturbations and abrupt changes in the voltages at certain instant, which are created by circuit elements, are exemplary of impulsive phenomena that can affect the evolutionary process of the neural networks. Therefore, stability and periodicity of impulsive shunting inhibitory cellular neural networks with delay (see, for example Akhmet & Yılmaz, 2014, Gui & Ge, 2006a, 2006b, 2007, Li & Xing, 2007, Lin & Jun, 2009, Sun, Wang, & Gao, 2009, Wang, Li, & Xu, 2010, Xia, Cao, & Huang, 2007, Yang, 2009, Yang & Cao, 2007, Yang, Zhang, Wu, Chen, & Yang, 2010 and Zhang & Gui, 2009) which are neither purely continuous nor discrete have been widely considered.

The main necessity of the present paper is to find sufficient conditions which guarantee the existence and global exponential stability of periodic solution for neural networks with discontinuities. Besides, in the present paper, different from the most existing studies, we introduce a more general class of shunting inhibitory cellular neural networks including time-varying delays and related to the state-dependent impulsive phenomena. The aim of defining this new class is that the moments of impulses are arbitrary in $\mathbb{R}_+$, that is, solutions with different initial data have different impulse time. As it is mentioned in Liu, Li, and Liao (2011), in real world problems, the impulses of many systems do not occur at fixed time, like for example, population control systems, saving rates control systems, some circuit control systems and so on. These types of systems are called state-dependent impulsive differential systems or impulsive systems with variable-time impulses. For more detailed discussion of real world applications of state-dependent impulsive systems please see Refs. Akhmet (2010) and Yang (2001).

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Therefore, considering the system with non-fix moments of impulses is more general than the fixed time impulses. This results in more theoretical and technical challenges, since simple transformations are not allowed. To the best of our knowledge, these types of impulsive neural networks were considered in Liu et al. (2011), Şaylı & Yılmaz (2014) and Yılmaz (2014) and problems related to stability, almost periodicity and robustness of bidirectional associative memory neural networks were analyzed. One should underline that there are no results on existence and global exponential stability of periodic solution of shunting inhibitory cellular neural networks having variable coefficients with state-dependent impulse and time-varying delays in the literature. Therefore, our results are generalization of the studies (Akhmet & Yılmaz, 2014; Gui & Ge, 2006a, 2006b, 2007; Li & Xing, 2007; Lin & Jun, 2009; Sun et al., 2009; Wang et al., 2010; Xia et al., 2007; Yang, 2009; Yang & Cao, 2007; Yang et al., 2010; Zhang & Gui, 2009) with fix-time impulses to the state-dependent impulse time $t = \theta_k + \tau_k(x)$. To solve the problem we used the technique of the reduction of the analyzed system to a system with fixed moments of impulses by using the B-equivalence method, which was studied widely in Akalın and Akhmet (2005), Akhmet (2005, 2010) and Akhmet and Perestyuk (1990) for ordinary differential equations and applied delay differential equation in Liu and Wang (2006). Then, we find some sufficient constraints by using Mawhin's continuation theorem of coincide degree theory and employing an appropriate Lyapunov function to guarantee the existence and global exponential stability of periodic solution of the considered networks.

2. Model description and preliminaries

Let $\mathbb{Z}_+$, $\mathbb{R}_+$ and $\mathbb{R}$ be the sets of positive integers, nonnegative real numbers and real numbers, respectively. Consider the following variable-time impulsive neural networks with time-varying delay:

$$\begin{cases} x'_{ij}(t) = -a_{ij}(t)x_{ij}(t) - \sum\limits_{b^{hl}\in N_r(i,j)} b^{hl}_{ij}(t)f_{ij}(x_{hl}(t))x_{ij}(t) \\ \qquad - \sum\limits_{c^{hl}\in N_r(i,j)} c^{hl}_{ij}(t)g_{ij}(x_{hl}(t-\rho_{hl}(t))) \\ \qquad \times x_{ij}(t) + L_{ij}(t), \quad t \geq 0 \\ \Delta x_{ij}\,|_{t=\theta_k+\tau_k(x)} = e_{ijk}x_{ij} + I_{ijk}(x_{ij}) \end{cases} \tag{2.1}$$

where $k \in \mathbb{Z}_+$, $x \in \mathbb{R}^{m\times n}$, $t \in \mathbb{R}_+$ and c_{ij} denote the cell at the (i,j) position of the lattice, the r-neighborhood $N_r(i,j)$ of c_{ij} is

$$N_r(i,j) = \left\{c^{hl}_{ij} : \max(|h-i|, |l-j|) \leq r, 1 \leq h \leq m, 1 \leq l \leq n\right\},$$

$i = 1,2,\dots,m, j = 1,2,\dots,n$. Also, $\{e_{ijk}\}$ is a bounded sequences such that $(1+e_{ijk}) \neq 0$, $i = 1,2,\dots,m$, $j = 1,2,\dots,n$, $k \in \mathbb{Z}_+$, $\tau_k(x)$ are positive real valued continuous functions defined on $\mathbb{R}^{m\times n}$, $k \in \mathbb{Z}_+$. Moreover, the sequence θ_k satisfies the condition $0 = t_0 < \theta_k < \theta_{k+1}$, $\theta_k \to +\infty$ as $k \to \infty$.

In the system (2.1), $x_{ij}(t)$ denotes the membrane potential of the cell c_{ij} at time t; the function $a_{ij}(t) > 0$ denotes the passive decay rate of the membrane potential of the cell c_{ij} at time t; the continuous bounded nonnegative functions $f_{ij}(x_{hl}(t))$ denotes the measures of activation to its incoming potentials of the cell c_{hl} at time t; the continuous function $\rho_{hl}(t)$ corresponds to the transmission delay along the axon of the (h,l)th cell from the (i,j)th cell and satisfies $0 \leq \rho_{hl}(t) \leq \rho_{hl}$ (ρ_{hl} is a constant); $\rho = \max_{\substack{1\leq h\leq m\\1\leq l\leq n}} \{\rho_{hl}\}$; the continuous bounded nonnegative functions $g_{ij}(x_{hl}(t-\rho_{hl}(t)))$ denotes the measures of activation to its incoming potentials of the cell c_{hl} at time $t-\rho_{hl}(t)$; $L_{ij}(t)$ is the bounded external bias on the (i,j)th cell at time t; $b^{hl}_{ij}(t)$ and $c^{hl}_{ij}(t)$ correspond to the positive bounded synaptic connection weight of the cell c_{hl} on the cell c_{ij} at time t. It will be assumed that $a_{ij}, b^{hl}_{ij}, c^{hl}_{ij}, L_{ij}, I_{ijk}(\cdot) : \mathbb{R} \to \mathbb{R}$, $i = 1,2,\dots,m, j = 1,2,\dots,n$, $k \in \mathbb{Z}_+$ are continuous functions.

Now, we will define the following class of maps and norm:

$C[X,Y] = \{\phi : X \to Y | \phi(\cdot)$ is a continuous mapping from the topological space X to the topological space $Y\}$.

Set $\{x_{ij}(t)\} = (x_{11}(t),\dots,x_{1n}(t),\dots,x_{m1}(t),\dots,x_{mn}(t))$, for $\forall x \in \mathbb{R}^{m\times n}$ its norm is given by $\|x\| = \max_{(i,j)}\{|x_{ij}(t)|\}$.

The system (2.1) is supplemented with initial data given by

$$x(s) = \varphi(s) \tag{2.2}$$

where, $s \in [-\rho, 0]$, $\varphi \in C([-\rho,0], \mathbb{R}^{m\times n})$.

Using the constant ρ, $C([-\rho,0], R^{m\times n})$ is a Banach space with the norm $\|\cdot\|$ given by $\|\varphi\| = \sup_{-\rho\leq s\leq 0}\|\varphi(s)\|$. Also, we assume $\|\varphi(s)\| \leq \tilde{h}$, $\tilde{h} \in \mathbb{R}_+$.

In the present study, we do not necessitate smoothness and monotonicity of the activation functions $f_{ij}(.)$ and $g_{ij}(.)$, $i = 1,2,\dots,m, j = 1,2,\dots,n$.

From now on the following assumptions will be needed throughout the paper:

(A1) Each $a_{ij}(\cdot)$ is positive, continuous and bounded, that is, there exist $\underline{a}_{ij}$ and $\bar{a}_{ij}$ such that

$$0 < \underline{a}_{ij} \leq a_{ij}(\cdot) \leq \bar{a}_{ij}$$

additionally, we denote $\underline{a} = \min\{\underline{a}_{ij}\}$, $\bar{a} = \max\{\bar{a}_{ij}\}$ where $i = 1,2,\dots,m, j = 1,2,\dots,n$.

(A2) There exists a Lipschitz constant $\ell > 0$ such that

$$|f_{ij}(x) - f_{ij}(y)| + |g_{ij}(x) - g_{ij}(y)| + |I_{ijk}(x) - I_{ijk}(y)| \\ \leq \ell|x-y|, \quad \forall x,y \in \mathbb{R}$$

and

$$\|\tau_k(x) - \tau_k(y)\| \leq \ell\|x-y\|, \quad \|\tau_k(x)\| \leq \ell, \\ \forall x,y \in \mathbb{R}^{m\times n}, \ k \in \mathbb{Z}_+.$$

(A3) There exists a positive number $\theta \in \mathbb{R}$ such that $\theta_{k+1} - \theta_k \geq \theta$ holds for all $k \in \mathbb{Z}_+$ and the surfaces of discontinuity $\Gamma_k : t = \theta_k + \tau_k(x)$, $k \in \mathbb{Z}_+$ satisfy the following conditions:

$$0 < \theta_k + \tau_k(x) < \theta_{k+1} + \tau_{k+1}(x), \\ |\theta_k| \to +\infty \text{ as } |k| \to \infty, \\ \tau_k((\mathbb{S}+E_k)x + I_k(x)) \leq \tau_k(x), \quad x \in \mathbb{R}^{m\times n},$$

where $\mathbb{S}$ is an $(m\times n)\times(m\times n)$ identity matrix and

$$E_k = \operatorname{diag}(e_{11k},\dots,e_{mnk}) = \begin{pmatrix} e_{11k} & 0 & \cdots & 0 \\ 0 & e_{12k} & \cdots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & e_{mnk} \end{pmatrix}$$

$$\text{and} \quad \mathrm{I_k} = \begin{pmatrix} I_{11k} \\ I_{12k} \\ \cdots \\ I_{mnk} \end{pmatrix};$$

(A4) $\ell(\tilde{r_1}\tilde{h} + \tilde{r_2}) < 1$, where

$$\tilde{r_1} = \max_{\substack{1\leq i\leq m\\1\leq j\leq n}} \sup_{t\in\mathbb{R}_+} \left(\bar{a} + \sum_{b^{hl}\in N_r(i,j)} b^{hl}_{ij}(t)\tilde{p_1} \right. \\ \left. + \sum_{c^{hl}\in N_r(i,j)} c^{hl}_{ij}(t)\tilde{p_2}\right) < +\infty,$$

$$\tilde{r_2} = \max_{\substack{1\leq i\leq m\\1\leq j\leq n}} \sup_{t\in\mathbb{R}_+} \left(|L_{ij}(t)|\right), \ \tilde{p_1} = \max_{\substack{1\leq i\leq m\\1\leq j\leq n}} \sup_{t\in\mathbb{R}_+} \left(f_{ij}(x_{hl}(t))\right),$$

$$\tilde{p_2} = \max_{\substack{1\leq i\leq m\\1\leq j\leq n}} \sup_{t\in\mathbb{R}_+} \left(g_{ij}(x_{hl}(t-\rho_{hl}(t)))\right)$$

with $\|x(t)\| \leq \tilde{h}$.