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Periodic synchronization control of discontinuous delayed networks by using extended Filippov-framework[☆]

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ABSTRACT

This paper is concerned with the periodic synchronization problem for a general class of delayed neural networks (DNNs) with discontinuous neuron activation. One of the purposes is to analyze the problem of periodic orbits. To do so, we introduce new tools including inequality techniques and Kakutani's fixed point theorem of set-valued maps to derive the existence of periodic solution. Another purpose is to design a switching state-feedback control for realizing global exponential synchronization of the drive–response network system with periodic coefficients. Unlike the previous works on periodic synchronization of neural network, both the neuron activations and controllers in this paper are allowed to be discontinuous. Moreover, owing to the occurrence of delays in neuron signal, the neural network model is described by the functional differential equation. So we introduce extended Filippov-framework to deal with the basic issues of solutions for discontinuous DNNs. Finally, two examples and simulation experiments are given to illustrate the proposed method and main results which have an important instructional significance in the design of periodic synchronized DNNs circuits involving discontinuous or switching factors.

1. Introduction

In the field of artificial neural networks, discontinuities are typical phenomena due to the control actions of some interesting engineering tasks. There are many different classes of discontinuous neural network systems and implemented integration circuits such as neural network possessing discontinuous neuron activation (Forti, Grazzini, Nistri, & Pancioni, 2006; Forti & Nistri, 2003; Forti, Nistri, & Papini, 2005), neuron system with McCulloch–Pitts nonlinearity (Huang & Wu, 2001), discontinuous Chen system (Chen & Ueta, 1999), memristor-based Chua's circuit (Adhikari, Yang, Kim, & Chua, 2012; Chua, 1971; Sprott, 2000), discontinuous Sprott circuit (Filippov, 1988) and so on. Especially, the neural networks with discontinuous activations have been proved really useful as ideal models to solve linear or nonlinear programming problems, constrained optimization problems, and various control problems. Because the dynamical behaviors of this class of network system is described by ordinary differential equation (ODE) with discontinuous state on the right-hand side, the traditional theoretical framework has been shown to be invalid for dealing with the solutions of ODE with discontinuous right-hand side. In order to overcome this difficulty, Filippov developed a new theoretical framework based on differential inclusions and redefined the concept of solution named Filippov-solution (Papini & Taddei, 2005). Actually, by using the Filippov regularization method (i.e., constructing appropriate Filippov set-valued map), the solution of ODE with discontinuous right-hand side could be transformed into a solution of differential inclusion. By doing so, many useful results concerning the basic properties of solutions in the sense of Filippov and more complex dynamic phenomena to ODE with discontinuous right-hand side can be obtained. In 2003, Forti et al. firstly introduced the Filippov differential inclusion framework to study the dynamical behaviors of neural network models with discontinuous neuron activations (Forti & Nistri, 2003). This motivated the later investigations on neural networks with discontinuous neuron activations (see, for example, Allegretto, Papini, & Forti, 2010, Cai, Huang, Guo, & Chen, 2012, Huang & Guo, 2009, Huang, Cai, Zhang, & Duan, 2013; Huang, Wang, & Zhou, 2009, Liu & Cao, 2009, Liu, Cao, & Yu, 2012; Liu, Chen, Cao, & Lu, 2011, Lu & Chen, 2008, Qin,

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Xue, & Wang, 2013 and Yang & Cao, 2013). However, the theoretical results on periodic synchronization of neural network systems possessing discontinuous neuron activations are few.

Periodic synchronization, which means that the dynamical behaviors of coupled periodic systems achieve the same time-spatial state, can be found in a wide variety of science and engineering fields involving periodic factors, such as secure communication, meteorology, and information processing. Actually, one can control the periodic drive–response system states to converge some periodic orbit by way of periodic synchronization. Therefore, an unknown periodic dynamical system can be understood from the well-known periodic dynamical system by periodic synchronization control. Up to now, much attention has been paid to analyze periodic synchronization problems (Suzuki & Imai, 2004; Zhou, Huang, Qi, Yang, & Xie, 2005; Zou & Zhan, 2008). In particular, in the field of neural networks, some attempts have been made to investigate the periodic synchronization problems for dynamical neuron system possessing periodic coefficients and discontinuous property. In Liu, Cao, and Huang (2010), complete periodic synchronization was considered for the delayed neural networks with discontinuous activation functions by using non-smooth Lyapunov method and linear matrix inequality. Authors of Wu, Li, Ding, Zhang, and Yao (2014); Wu, Li, Zhang, and Yao (in press); Wu, Zhang, Ding, Guo, and Wang (2013) investigated different types of periodic synchronization problems for memristor-based neural networks modeled by state-dependent discontinuous or switching systems. Note that the human brain is often in periodic oscillatory or chaos state. So the periodic synchronization analysis of discontinuous drive–response network system is an important step for understanding the function of human brain and further enables us to simulate the human brain under periodic environment. However, the analysis of periodic synchronization issues of neural networks with discontinuous activation is not a simple task and there still lacks effective analysis methods. Such periodic synchronization analysis is faced with four difficulties as follows:

(1) How to ensure the existence of periodic orbits for discontinuous neural networks?
(2) What kind of controller should be designed such that the periodic synchronization can be realized? If we add a switching term to the classical controller, whether the uncertain differences between the Filippov solutions of the drive and response network systems can be well handled?
(3) If time delays are considered, how to extend the Filippov-framework for dealing with the solutions of discontinuous delayed network systems? What role do inequality techniques play?
(4) How to propose some sufficient conditions which are applicable to general discontinuous delayed network systems and are easy to be verified?

To the best of the authors' knowledge, periodic synchronization of complex periodic networks coupled with nonidentical periodic neuron systems possessing discontinuous activations is still seldom. Motivated by the above discussions, this paper aims to overcome these four difficulties and achieve periodic synchronization control of delayed network system with discontinuous neuron activation.

The rest of this paper is organized as follows. In Section 2, the model description and preliminaries including some useful definitions and lemmas are briefly given. In Section 3, the fixed point theory of set-value map is employed to analyze the existence of periodic orbits for discontinuous delayed network systems. Several sufficient conditions are derived to guarantee the existence of periodic solutions in corollaries. In Section 4, by designing novel switching state-feedback control, global exponential synchronization of the drive–response network system with periodic coefficients is studied. In Section 5, two examples and simulation experiment are presented to illustrate the proposed methods and theoretical results. Finally, main conclusions reached in this paper are drawn in Section 6.

Notations: Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space. The superscript "T" represents the transpose operator. Given the column vectors $x = (x_1, x_2, \ldots, x_n)^{\mathrm{T}} \in \mathbb{R}^n$ and $y = (y_1, y_2, \ldots, y_n)^{\mathrm{T}} \in \mathbb{R}^n$, $\langle x, y\rangle = x^{\mathrm{T}}y = \sum_{i=1}^{n} x_i y_i$ stands for the scalar product of x and y. If $x \in \mathbb{R}^n$, let us define the norm $\|x\|_1 = \sum_{i=1}^{n} |x_i|$, while $\|x\|$ denotes any vector norm of x. Given a set $\mathbb{E} \subset \mathbb{R}^n$, by meas$(\mathbb{E})$ we mean the Lebesgue measure of set $\mathbb{E}$ in $\mathbb{R}^n$ and $\overline{\mathrm{co}}[\mathbb{E}]$ represents the closure of the convex hull of $\mathbb{E}$. If $z \in \mathbb{R}^n$ and $\delta > 0$, $\mathcal{B}(z, \delta) = \{\hat{z} \in \mathbb{R}^n : \|\hat{z} - z\| \le \delta\}$ denotes the ball of δ about z. Given the function $V : \mathbb{R}^n \to \mathbb{R}$, ∂V means Clarke's generalized gradient of V. By $L^1([0, T), \mathbb{R}^n)$, $T \le +\infty$, we denote the Banach space of the Lebesgue integrable functions $g : [0, T) :\to \mathbb{R}^n$ equipped with the norm $\int_0^T \|g(t)\|\mathrm{d}t$ or $\int_0^T \|g(t)\|_1 \mathrm{d}t$. For any continuous ω-periodic function $g(t)$ defined on $\mathbb{R}$, we set

$$\bar{g} = \frac{1}{\omega}\int_0^{\omega} g(t)\mathrm{d}t, \qquad g^M = \sup_{t\in[0,\omega]} |g(t)|, \qquad g^L = \inf_{t\in[0,\omega]} |g(t)|.$$

2. Model description and preliminaries

In this paper, we consider a general class of time-varying delayed neural networks described by the following functional differential equations:

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -d_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)f_j \times (x_j(t - \tau_j(t))) + J_i(t), \quad i \in \mathbb{N}, \tag{1}$$

where $\mathbb{N} = \{1, 2, \ldots, n\}$, n corresponds to the number of units in the network system (1); $x_i(t)$ represents the state variable of the ith unit at time t; $f_j(\cdot)$ denotes the activation function of jth neuron; $d_i(t)$ represents the self-inhibition of the ith neuron unit at time t; $a_{ij}(t)$ and $b_{ij}(t)$ are connection weights of the jth unit on the ith unit at time t and time $t - \tau_j(t)$, respectively; $\tau_j(t)$ corresponds to the transmission delay at time t; $J_i(t)$ is the neuron input on the ith unit at time t.

Throughout this paper, the neuron activations in network system (1) are assumed to possess the following basic properties:

($\mathcal{H}$1) For every $i \in \mathbb{N}$, $f_i : \mathbb{R} \to \mathbb{R}$ is piecewise continuous. That is, f_i is continuous in $\mathbb{R}$ except on a countable set of isolate points $\{\rho_k^i\}$, where there exist finite right and left limits, $f_i^+(\rho_k^i)$ and $f_i^-(\rho_k^i)$, respectively. Moreover, on any compact interval of $\mathbb{R}$, f_i has at most a finite number of discontinuities.

($\mathcal{H}$2) For each $i \in \mathbb{N}$, there exist nonnegative constants α_i and β_i such that

$$\sup_{\gamma_i \in \overline{\mathrm{co}}[f_i(x_i)]} |\gamma_i| \le \alpha_i |x_i| + \beta_i, \quad \forall x_i \in \mathbb{R},$$

where

$$\overline{\mathrm{co}}[f_i(x_i)] = \left[\min\{f_i^-(x_i), f_i^+(x_i)\}, \max\{f_i^-(x_i), f_i^+(x_i)\}\right].$$

For later discussion, we always assume that $d_i(t)$, $a_{ij}(t)$, $b_{ij}(t)$, $J_i(t)$, $\tau_j(t)$ $(i, j \in \mathbb{N})$ are continuously ω-periodic functions in $\mathbb{R}$ and $d_i(t) > 0 (i \in \mathbb{N})$ for $t \in \mathbb{R}$. Moreover, for all $j \in \mathbb{N}$, the time-varying transmission delay $\tau_j(t)$ is continuous function satisfying

$$0 \le \tau_j(t) \le \tau \ (\text{here } \tau = \max_{1\le j\le n}\{\sup_{t\in[0,\omega]} \tau_j(t)\} \text{ denotes a nonnegative constant}).$$