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Stochastic exponential synchronization control of memristive neural networks with multiple time-varying delays ☆

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ABSTRACT

As an indispensable part of memristive synaptic weights, the switching jumps can induce instability, oscillation or even chaos to the memristive network system. Based on the available information of the switching jumps, this paper is concerned with the stochastic exponential synchronization of a class of memristive neural networks with multiple time-varying delays. By using stochastic differential inclusions and Lyapunov stability theory, discontinuous state feedback controller which depends upon the switching jumps is proposed. Compared with the previous state feedback scheme, more information of memristive synaptic weights is used to design the synchronous controller which ensures the stochastic exponential synchronization of considered networks. When the information of switching jumps is incomplete, discontinuous adaptive controller which is independent of the switching jumps is also designed, thus the applicability of synchronization is broadened. A numerical example is provided to illustrate the effectiveness and potential of the proposed design techniques.

1. Introduction

Memristor was first theoretically predicted by Chua in 1971 [1]. In late 2008, Hewlett-Packard research team unveiled a two-terminal titanium dioxide nanoscale device that exhibited memristive characteristic [2]. According to the amount, direction and duration of charges passing through it, the resistance of memristor would change. In this way, the memristor remembers information. This new device will be helpful for low-power computation and storage to store information and data without the need of power [3,4]. The nonvolatile nature of memristors makes them an attractive candidate for the next-generation memory technology.

Chua also pointed out that the behavior of memristor is somewhat similar to the synapses in the brain [1]. It is the memory that memristors can be used to simulate biological synapses. By replacing the resistors in the primitive recurrent neural networks with memristors, memristive neural networks (MNNs) were developed in [5–9]. In fact, MNNs are a class of switched nonlinear systems, in which the switching signals depend on the neural states. Compared with traditional electronic neural networks, the nonlinear properties of MNNs are more complex. One main reason for the complexity of MNNs may lie in the threshold sensitive memristor with a nonlinear drift effect [7]. Recent research has a number of promising results of MNNs [10–27].

Recently, chaotic synchronization of MNNs has gained much attention due to its strong applications in diverse areas. In particular, the authors in [14,15,19] showed that memristive chaotic system is more safe in secure communications. Unlike the traditional recurrent neural networks [28–31], MNN is a state-dependent switch system. The drive system and response system may switch asynchronously because of their different states. In other words, the synaptic weights of drive system and response system may be nonidentical before achieving synchronization. Thus, the synchronization control of MNNs is much more complicated than that of tradition electronic neural networks. By employing the differential inclusions, some synchronization criteria for a class of MNNs were established in [19–25]. However, almost all the mentioned results neglected the influence of switching jumps on controller. As an indispensable part of memristive synaptic weights, switching jumps can induce instability, oscillation or even chaos to the memristive network system. Obviously, its importance has not aroused the attention of researchers. Taking the evolution of stability analysis of recurrent neural networks for example, it has experienced a process from delay-independent criteria to delay-dependent ones [32,33]. The research into the stability and synchronization of MNNs will inevitably experience a homologous developing process that switching jumps independent criteria will be extended to switching jumps dependent ones. Thus, both the switching jumps dependent controller and the switching jumps

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independent controller are designed in this paper. Each controller has its own advantages.

On the other hand, a real system is usually replaced by nondeterministic one because of the random uncertainties such as stochastic forces on physical systems and noisy measurements caused by environmental uncertainties [34–36]. This should be also the case for MNNs. For instance, a class of MNNs with stochastic perturbation has been formulated in [11]. However, to the best of our knowledge, there is no research on the stochastic synchronization of MNNs in the existing literature. Motivated by the earlier discussions, our objective of this paper is to study the stochastic synchronization control problem of MNNs. The main contributions of this paper can be summarized as follows:

(1) Both multiple time-varying delays and stochastic perturbation are considered in this paper. By utilizing sign function, discontinuous state feedback controller is designed such that the considered networks can realize stochastic exponential synchronization in p-th moment. Moreover, this controller depends upon the switching jumps.

(2) Discontinuous adaptive controller is also established. This controller is independent of the switching jumps and can be used when the switching jumps are not well known.

(3) The proposed synchronization schemes are still feasible when the derivative of time-varying delay is more than one.

Notation: Throughout this paper, $[\cdot,\cdot]$ represents the interval. $co[a,b]$ represents the closure of the convex hull generated by real numbers a and b. $\mathfrak{C}([-\tau,0],R)$ represents the Banach space of continuous functions. $\max\{\cdot\}$ and $\min\{\cdot\}$ denote the maximum and minimum values, respectively. Let $(\Omega,\mathfrak{F},\mathfrak{P})$ be a complete probability space with natural filtration $\{\mathfrak{F}\}_{t\ge 0}$, $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure $\mathfrak{P}$.

2. Model description and preliminaries

Now,we consider the following MNN model with different multiple time-varying delays:

$$dx_i(t)=\Big[-d_i(x_i(t))x_i(t)+\sum_{j=1}^{n}a_{ij}(x_i(t))f_j(x_j(t))+\sum_{k=1}^{N}\sum_{j=1}^{n}b_{ij}^k(x_i(t-\tau_{ki}(t)))g_j(x_j(t-\tau_{kj}(t)))+I_i(t)\Big]\,dt,\quad i=1,2,\ldots,n, \tag{1}$$

where $x_i(t)$ denotes the state variable of the ith neuron at time t; $I_i(t)$ is the external input to the ith neuron, $\tau_{kj}(t)$ is the time-varying delay with $0\le\tau_{kj}(t)\le\tau$; N denotes the number of delayed connection weights; f_j and g_j are the activation functions; $d_i(x_i(t))$, $a_{ij}(x_i(t))$, $b_{ij}^k(x_i(t-\tau_{ki}(t)))$ represent memristive synaptic weights, respectively. Referring to the works in [11–15,22–25], we let

$$d_i(x_i(t))=\begin{cases}\underline{d}_i, & |x_i(t)|\le T_i,\\ \overline{d}_i, & |x_i(t)|>T_i,\end{cases}\qquad a_{ij}(x_i(t))=\begin{cases}\underline{a}_{ij}, & |x_i(t)|\le T_i,\\ \overline{a}_{ij}, & |x_i(t)|>T_i,\end{cases}$$

$$b_{ij}^k(x_i(t-\tau_{ki}(t)))=\begin{cases}\underline{b}_{ij}^k, & |x_i(t-\tau_{ki}(t))|\le T_i,\\ \overline{b}_{ij}^k, & |x_i(t-\tau_{ki}(t))|>T_i,\end{cases}$$

in which switching jumps $T_i>0$, $\overline{d}_i>0$, $\underline{d}_i>0$, $\overline{a}_{ij}$, $\underline{a}_{ij}$, $\overline{b}_{ij}^k$, $\underline{b}_{ij}^k$ are constants. The initial conditions of system (1) are given by $x_i(t)=\phi_i(t)\in\mathfrak{C}([-\tau,0],R)$, $i=1,2,\ldots,n$.

In this paper, we consider system (1) as the drive system, the response system with stochastic perturbation is designed as

$$dy_i(t)=\Big[-d_i(y_i(t))y_i(t)+\sum_{j=1}^{n}a_{ij}(y_i(t))f_j(y_j(t))+\sum_{k=1}^{N}\sum_{j=1}^{n}b_{ij}^k(y_i(t-\tau_{ki}(t)))g_j(y_j(t-\tau_{kj}(t)))+I_i(t)+u_i(t)\Big]\,dt+\sum_{j=1}^{n}h_{ij}(e_j(t),e_j(t-\tau_{1j}(t)),\ldots,e_j(t-\tau_{Nj}(t)))\,d\omega_j(t), \tag{2}$$

where $y_i(t)$ is the state of the response system with initial conditions $y_i(t)=\varphi_i(t)\in\mathfrak{C}([-\tau,0],R)$, $e_i(t)=y_i(t)-x_i(t)$ is the synchronization error signal; $u_i(t)$ is the controller to be designed; h_{ij} is the noise intensity function; $\omega_{ij}(t)$ is a Brownian motion defined on a complete probability space $(\Omega,\mathfrak{F},\mathfrak{P})$ satisfying $\mathbb{E}\{d\omega_j(t)\}=0$ and $\mathbb{E}\{d\omega_j^2(t)\}=dt$, and

$$d_i(y_i(t))=\begin{cases}\underline{d}_i, & |y_i(t)|\le T_i,\\ \overline{d}_i, & |y_i(t)|>T_i,\end{cases}\qquad a_{ij}(y_i(t))=\begin{cases}\underline{a}_{ij}, & |y_i(t)|\le T_i,\\ \overline{a}_{ij}, & |y_i(t)|>T_i,\end{cases}$$

$$b_{ij}^k(y_i(t-\tau_{ki}(t)))=\begin{cases}\underline{b}_{ij}^k, & |y_i(t-\tau_{ki}(t))|\le T_i,\\ \overline{b}_{ij}^k, & |y_i(t-\tau_{ki}(t))|>T_i.\end{cases}$$

Remark 2.1. In systems (1) and (2), we assume that the memristive synaptic weights $b_{ij}^k(\cdot)$ are closely related to the time-varying delays. This is because that the switching of synaptic weights may also be affected by the time delays in delayed MNN system. For this reason, some homologous MNN models have been constructed in [10,19,21], and some stability and synchronization conditions have been established, respectively. However, to the best our knowledge, there is no research on the stochastic exponential synchronization issue for MNNs with multiple time-varying delays and stochastic perturbation.

Let $d_i^*=\min\{\underline{d}_i,\overline{d}_i\}$, $d_i^{**}=\max\{\underline{d}_i,\overline{d}_i\}$, $a_{ij}^*=\min\{\underline{a}_{ij},\overline{a}_{ij}\}$, $a_{ij}^{**}=\max\{\underline{a}_{ij},\overline{a}_{ij}\}$, $b_{ij}^{k*}=\min\{\underline{b}_{ij}^k,\overline{b}_{ij}^k\}$, $b_{ij}^{k**}=\max\{\underline{b}_{ij}^k,\overline{b}_{ij}^k\}$. By applying the theories of set-valued maps and stochastic differential inclusions [10–13], from (1) and (2), we have

$$dx_i(t)\in\Big\{-co[d_i(x_i(t))]x_i(t)+\sum_{j=1}^{n}co[a_{ij}(x_i(t))]f_j(x_j(t))+\sum_{k=1}^{N}\sum_{j=1}^{n}co[b_{ij}^k(x_i(t-\tau_{ki}(t)))]g_j(x_j(t-\tau_{kj}(t)))+I_i(t)\Big\}\,dt,\quad i=1,2,\ldots,n, \tag{3}$$

and

$$dy_i(t)\in\Big\{-co[d_i(y_i(t))]y_i(t)+\sum_{j=1}^{n}co[a_{ij}(y_i(t))]f_j(y_j(t))+\sum_{k=1}^{N}\sum_{j=1}^{n}co[b_{ij}^k(y_i(t-\tau_{ki}(t)))]g_j(y_j(t-\tau_{kj}(t)))+I_i(t)+u_i(t)\Big\}\,dt+\sum_{j=1}^{n}h_{ij}(e_j(t),e_j(t-\tau_{1j}(t)),\ldots,e_j(t-\tau_{Nj}(t)))\,d\omega_j(t), \tag{4}$$

where

$$co[d_i(x_i(t))]=\begin{cases}\underline{d}_i, & |x_i(t)|<T_i,\\ [d_i^*,d_i^{**}], & |x_i(t)|=T_i,\\ \overline{d}_i, & |x_i(t)|>T_i,\end{cases}$$