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# Pinning sampled-data control for synchronization of complex networks with probabilistic time-varying delays using quadratic convex approach ☆

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ABSTRACT

This paper addresses pinning sampled-data synchronization problem for complex dynamical networks with probabilistic time-varying coupling delays and control packet loss. The sampling period considered here is assumed to be less than a given bound. By introducing a Bernoulli distributed stochastic variable, the information of probabilistic time-varying delay is transformed into the deterministic time-varying delay with stochastic parameters. A new Lyapunov–Krasovskii functional (LKF) is constructed and by using quadratic convex approach, reciprocal convex technique and Jensen's inequality, sufficient conditions for the synchronization of complex networks are derived. Based on the average dwell-time method and delay-probability-distribution-dependent condition, the synchronization criterion is derived in terms of linear matrix inequalities (LMIs). Finally, numerical examples are provided to illustrate the effectiveness of the proposed techniques.



## 1. Introduction

During last decades, complex dynamical networks (CDNs) have received increasing research interest in different fields for its large size and complex topology [1–4]. Complex networks exist extensively in many practical applications such as the Internet, ecosystems, scientific citation web, biological neural networks, and electrical power grids. Synchronization is one of the most significant and interesting collective behavior of CDNs and has received increasing research attention for example, the large scale and complex networks of chaotic oscillators, potential applications in many other areas, including information science, secure communication, and biological systems [5–9]. In [5], local and global synchronization of CDNs have been studied and synchronization criteria ensuring delay-independent and delay-dependent synchronization were derived. Chaos synchronization of general CDNs with coupling delays has been focused in [8] and some delay-independent and delay-dependent criteria for exponential synchronization of CDNs have been derived.

Time-delays are present in many physical processes due to the period of time it takes for the events to occur. It is well known that time-delays may cause uncertain dynamic behavior of networks such as oscillation and instability. Therefore, time-delays should be taken into account in order to simulate more practical networks. Examples of time-delay systems are biological modeling, chemical engineering systems, electrical networks and many others. Synchronization problem for CDNs with time-varying delays has been widely investigated in [10–17]. In [11], synchronization problem for some general CDNs with time-varying delays has been analyzed. Both time-varying delays in the network couplings and time-varying delays in the dynamical nodes were considered. The synchronization problem for continuous/discrete general CDNs with time-varying delays has been investigated in [12] and the delays were assumed to vary in an interval where the lower and upper bounds are known. In [16], authors have analyzed the synchronization problem for CDNs with interval time-varying coupling delays, some delay-dependent synchronization conditions for the controlled complex dynamical networks have been presented. The synchronization problem for CDNs with additive time-varying coupling delays via non-fragile control has been investigated in [17]. Furthermore, in a real system, time-delay often exists in a random form, that is some values of the time-delay are very large but the probability of its occurrence is very small. Additionally, its probabilistic characteristic such as Bernoulli distribution can also be obtained by statistical methods. Therefore, it is necessary to

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investigate systems with probabilistic time-varying delays. The synchronization problem for complex networks with probabilistic time-varying delays has been studied in the literature, see [18–23]. The synchronization criteria for Markovian jumping stochastic complex networks with distributed time-delays and probabilistic interval discrete time-varying delays have been established in [18]. The synchronization stability for discrete-time complex networks with probabilistic interval time-varying delays has been analyzed in [19] by utilizing stochastic analysis and Kronecker product technique. Recently, in [21], global synchronization stability for CDNs with stochastic disturbances and probabilistic interval time-varying delays has been discussed. In [22], authors have studied the non-fragile synchronization control for Markovian jumping CDNs with probabilistic time-varying coupling delays, delay-probability-distribution-dependent conditions were derived in terms of LMIs.

At the same time, complex networks usually have a large number of nodes and it is difficult to add controllers to all nodes. Pinning control method can reduce the number of controlled nodes, which is a natural approach to control a complex network by pinning a part of the nodes in the network. Synchronization criteria for different CDNs using pinning control have been investigated in [24–28]. In [24], synchronization problem for hybrid coupled CDNs with non-delayed and delayed coupling by the pinning control strategy has been studied. In [26], synchronization problem has been analyzed by linearizing complex networks with multiple coupling delays to some time-delayed subsystems and pinning control scheme has been employed. In [27], pinning-controlled synchronization of a general CDNs with hybrid coupling has been discussed. Also, the network can be composed of coupled identical nonlinear oscillators with or without internal time-delay.

On the other hand, because of the rapid growth of digital hardware technologies, sampled-data control method has attracted more attention than other control approaches. In sampled-data control systems, control signals are kept constant during the sampling period and are allowed to change only at the sampling instants. These discontinuous control signals which have stepwise form cause big trouble to control or analyze the system. In order to effectively deal with sampled-data control, authors in [29,30] have introduced a concept that discontinuous sampled-data control inputs handle time-varying delayed continuous signals, although applied actual control signals are discontinuous. Synchronization analyses for various CDNs using sampled-data control and networked control systems have been considered in [31–43]. The digital controller has been popularly utilized for controlling complex dynamical systems in industry [39] including neural networks [40]. In order to improve the inter sampling performance, hybrid system models with both continuous and discrete-time signals are generally built through a zero-order hold (ZOH) function. In [32], authors have analyzed the synchronization of CDNs with coupling time-varying delays via sampled-data controller. Further, in [33], authors have investigated the exponential synchronization problem for CDNs with time-varying coupling delay via a sampled-data controller with variable sampling. The sampling period considered was assumed to be time-varying and bounded. Recently, in [34], authors have studied the problem of sampled-data exponential synchronization of CDNs with time-varying coupling delay and uncertain sampling. The sampled-data synchronization problem for complex networks with random coupling strengths, probabilistic time-varying coupling delay, and distributed delay has been analyzed in [35] and the sampling period is assumed to be time varying and bounded. The problem of exponential synchronization for CDNs with Markovian jumping parameters using sampled-data and mode-dependent probabilistic time-varying delays has been investigated in [36]. The problem of exponential synchronization of Markovian jumping CDNs with mode-dependent time-varying coupling delay via a stochastic sampled-data controller has been analyzed in [37]. In [38], pinning sampled-data synchronization for CDNs with probabilistic time-varying coupling delay has been established and the sampling period considered there was assumed to be less than a given bound.

Moreover, in practice in sampled-data systems, control packet can be lost because of several factors, for instance actuator failures, actuator suspensions for power saving, communication interference or congestion and so on. When the control packet from the controller to the actuator is lost, the actuator input to the plant may be set to zero. As frequent control packet loss is unavoidable, it may lead to instability and poor performance of systems and therefore, it is necessary and important to consider the effect of control packet loss in sampled-data control systems. In [44], switched system approach for the stabilization of sampled-data control systems with control inputs missing has been proposed and sufficient conditions for the existence of exponential stability of state feedback controllers have been derived. In [45], an improved stabilization method for sampled-data control systems with control packet loss has been established and new criteria have been proved theoretically to be less conservative than existing results. Also, in [50], authors proposed a reciprocally convex approach to study the stability of systems with time-varying delays. The synchronization problem for CDNs with interval time-varying delay via pinning control approach and less conservative criteria has been established in [51] based on reciprocal convex technique. To the best of authors' knowledge, pinning sampled-data control for synchronization of complex networks with control packet loss and probabilistic time-varying delays has not yet been proposed in the literature.

Motivated by the above analysis, in this paper, the problem of pinning sampled-data control for synchronization of complex networks with probabilistic time-varying delays using quadratic convex approach is investigated. The main contributions of the paper are given as follows:

(i) It is the first time to deal with the problem of synchronization of CDNs using pinning-sampled-data controller with control packet loss. Based on the Lyapunov function method, a novel exponential synchronization criterion is obtained in terms of LMIs. A new integral inequality approach which utilizes the quadratic convex approach and reciprocal convex technique is proposed.
(ii) Based on the Lyapunov theory, a delay-dependent pinning sampled-data synchronization criterion is derived in terms of LMIs. It should be noted that in [35,38] only double integral terms have been considered in the LKF. But in our paper, we construct double integral terms as in $V_{13}(t)$ and $V_{23}(t)$ and using the reciprocal convex combination technique without control packet loss and with control packet loss respectively. In addition, we have constructed triple and four integral terms as in $V_{14}(t)$, $V_{15}(t)$ and $V_{24}(t)$, $V_{25}(t)$. Quadratic convex approach has been applied for these terms in the derivation of synchronization criterion. Also, much information about the lower bound of the delay is used in the constructed LKF in Theorems 1 and 2.
(iii) Moreover, it should be noticed that there were no works in the literature that include exponential terms in the four integral LKF. In our present work, exponential terms are included in four integral LKF as in $V_{15}(t)$ and $V_{25}(t)$.

Finally, numerical examples are given to illustrate the effectiveness of our theoretical results.

*Notations*: $\mathbb{R}^n$ denotes the $n$-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ be the set of all $n \times m$ real matrices. For real symmetric